0) Let $(X_t)_{t\geq 0}$ be a rate 2 Poisson process with $X_0 = 0$ with jump times $\{J_i\}_{i\geq i}$. Let $\{H_i\}_{i\geq i}$ be i.i.d. random variables (independent also of the Poisson process X) obtained by rolling a fair die (i.e. uniformly distributed on the first six positive integers). Let for $t \geq 0$

$$Y_t = \sum_{i:J_i\leq t} I_{H_i is\ even}$$

and

$$Z_t = \sum_{i:J_i\leq t} I_{H_i=6}, \quad W_t = \sum_{i:J_i\leq t} I_{H_i=5},$$

a)What is $P(X_1 = 1|X_2 = 3)$? What is $P(Y_1 = 1|Y_2 = 3)$?

b) Give an expression (as a sum) of $P(X_t = 2Y_t)$. Deduce $P(X_t = 2Y_t|X_t \text{ is even })$, $P(X_t > 2Y_t|X_t \text{ is even })$.

c) What is $P(Z_1 = 1|Y_2 = 5)$, $P(W_1 = 1|Y_2 = 5)$?

1) For $P(t)$ the semigroup generated by

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix}$$

give two differential equations for $P_{12}(t)$ $(I = \{0, 1, 2\})$. Give an expression for $P(t)$

2) Consider a Q matrix for $I = \{0, 1, 2\}$

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Calculate $\mathbb{P}_{01}(t)$. Deduce $P(t)$.

3) For a Markov chain, show that for each $I \in I, \mathcal{P}^i(X \text{ makes at least 2 jumps in } [0, t])$ is $o(t)$.

4) For Markov chains on $I = \mathcal{N}$ which of the following can have explosions?

(i) $\forall\ i\ q_{ii+1} = i + 1$and for $i > 0,\ q_{i0} = 1\ q_{ij} \leq 0$ forall other i, j.

(ii) $\forall\ i\ q_{ii+1} = (i + 1)^3, q_{ii} = -(i + 1)^3$

(iii) $\forall\ i\ q_{ii+1} = (i + 1)^3$ and $i > 0,\ q_{i0} = (i + 1)^2\ q_{ij} \leq 0$ forall other i, j.

5) Consider the continuous time Markov chain on $\mathbb{Z}$ with jump chain $P_{ii+1} = 1 - P_{ii-1} =$
$2/3$ if $i > 0$
$1/3$ if $i > 0$
0 if $i = 0$
Suppose that $q_i = e^i$. Whar is the chance that there is an explosion if $X_0 = 1$?