Exercise Set 8

Saliba, May 1, 2019

Exercise 1. A *simple birth process* $(X(t))_{t\geq 0}$ on $\{0, 1, 2, \dots\}$ is a generalisation of a Poisson process by introducing a correlation between the parameter λ and the actual state of the process. More precisely, if the process is in state i, it will go to state $i+1$ after an exponential random time of parameter $0 \leq \delta_i < \infty$. This process is also a Markov process (using similar arguments as those used to prove that a Poisson process is Markovian.)

(i). Find the Q-matrix corresponding to the simple birth process.

(ii). Let $X(0) = 0$ and T_i be the time when the ith jump occurs. Find an example of a simple birth process such that $\lim_{i\to\infty} T_i < \infty$ a.s. This phenomena is called "the explosion". Find a general condition on the δ_i's so that the process explodes almost surely in a finite amount of time.

(iii). Do we have an explosion in the Poisson process case?

(iv). More generally, use the strong law of large numbers to show that if $\sup_{i\in\mathbb{N}} \delta_i < \infty$, then $\lim_{i\to\infty} T_i = \infty$ almost surely.

Exercise 2. A radioactive source emits particles according to a Poisson process of rate λ. The particles are spread in random directions independently from each other. A Geiger counter placed next to the source measures a fraction p of the emitted particles. What is the distribution of the number of particles detected by time t?

Exercise 3. The arrival times of the bus number 1 are modeled by a Poisson process with an average frequency of one bus per hour, whereas the arrival times of the bus number 7 are modeled by a Poisson process independent of the first one with frequency of 7 buses per hour.

(1) What is the probability that we see exactly 3 buses (no matter which buses) in one hour?

(2) What is the probability that we see exactly 3 buses 7 while we are waiting for the bus 1?

Exercise 4. Hockey teams 1 and 2 score goals at times of Poisson processes with rates 1 and 2. Suppose that $N_1(0) = 3$ and $N_2(0) = 1$.

a) What is the probability that $N_1(t)$ will reach 5 before $N_2(t)$ does?

b) Answer part a) for Poisson processes with rates λ_1 and λ_2.