Stochastic processes Examen BLANC

Spring Semester 2019

Duration : 1h45 (from 13h15 to15h00)

Veuillez svp. commencer chaque exercice sur une nouvelle feuille.

Attempt all the questions

Pleas write first your name and Section :

Exercice	**Points**
1	
2	
3	
4	
Total :	

1) Consider the following Markov chain on $S = \{0, 1, 2, \ldots, 6\}$

$$P = \begin{matrix} 1/2 & 1/4 & 1/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1/3 & 0 & 1/6 & 0 & 1/6 & 0 & 1/6 \\ 1/3 & 1/3 & 0 & 1/6 & 0 & 0 & 1/6 \end{matrix}$$

a) What are the transient states of the chain, what are the communication classes ?

b) If $X_0 = 6$, what is the probability that the chain will visit state 6 exactly 3 times ?

c) Approximately what is P^{400}_{62} ? What approximately is P^{400}_{63} ?

2) For the Markov chain on $\mathbb{Z}_+$ with

$$\begin{aligned} P_{n0} &= 1 - \tfrac{n^\alpha}{(n+1)^\alpha},\ n \geq 1,\ P_{01} = 1 \\ P_{n\ n+1} &= \tfrac{n^\alpha}{(n+1)^\alpha}, \end{aligned}$$

is the chain irreducible, ?, aperiodic ? . Show this.

For which values of α is it recurrent ? positive recurrent ? In the case of positive recurrence Give an expression for the equilibrium distribution ?

3) For the transition matrix on $\{0,1,2\}$ $P = \begin{matrix} 0 & 1/4 & 3/4 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$, is the corresponding chain irreducible, aperiodic ? Justify your answers.

For a Markov chain $(X_n)_{n\geq 0}$ $P = \begin{pmatrix} 0 & 1/3 & 2/3 \\ 1/2 & 1/2 & 0 \\ 0 & 2/3 & 1/3 \end{pmatrix}$, calculate $P(X_3 = 2|X_0 = 0)$.

For a Markov chain $(X_n)_{n\geq 0}$ on $\{0,1,2,3\}$

$P = \begin{pmatrix} 0 & 1/4 & 1/2 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/2 & 0 & 1/4 \end{pmatrix}$, calculate the probability that the chain hits 2 before 3 if $X_0 = 0$. What is the expected number of times 2 is hit before the chain returns to 0 if $X_0 = 0$.

4) A bag contains initially 10 red balls and 8 blue balls. Then two at random are taken out and random variable X_0 is the number of blue balls removed. In succession (and at random) one of the two removed balls is returned to the bag and replaced by one of the 16 balls in the bag. X_n is the number of blue balls after n such operations. Approximately what is $\frac{1}{81}\sum_{i=20}^{100} X_i$.

Is it true that a Markov chain with a unique stationary distribution is irreducible ?