Introduction

These lecture notes are very much inspired from Kunen's *Set Theory an Introduction to Independence Proofs* [23].

The basic requirements for this course are contained in the "Mathematical Logic" course. Among other things, you should have a clear understanding of each of the following: first order language, signature, terms, formulas, theory, proof theory, models, completeness theorem, compactness theorem, Löwenheim-Skolem theorem.
It **makes no sense** to take this course **without a solid background on first order logic** — see [2, 3, 4, 5, 6, 33].
We use the following notations

Notation 1 (Formulas)**.** *Given $\mathcal{L}$ any first order language, the* set of all *$\mathcal{L}$-formulas is the $\subseteq$-least $X \subseteq {}^{<}\mathcal{L}^{\omega}$ that satisfies:*

- *all atomic formulas belong to X.*
- *If φ and ψ belong to X, then*

$$\neg\varphi,\ (\varphi \wedge \psi),\ (\varphi \vee \psi),\ (\varphi \to \psi) \text{ and } (\varphi \leftrightarrow \psi)$$

  also belong to X.
- *If x is any variable and φ belongs to X, then $\forall x\ \varphi$ and $\exists x\, \varphi$ also belong to X.*

For better readability, we usually omit the outermost parentheses. For instance we write

$$\exists x\, x \in y \longrightarrow x \neq y$$

instead of

$$(\exists x\, x \in y \longrightarrow x \neq y).$$

Notation 2. *Given any first order language $\mathcal{L}$, and any set of $\mathcal{L}$-formulas Γ, and any $\mathcal{L}$-formula φ, we write*

- $\Gamma \models \varphi$ *for "φ holds in all $\mathcal{L}$-structures that satisfy Γ.*
- $\Gamma \vdash \varphi$ *for "Γ proves φ (in classical logic)".*