Week #12

Algebra V - Galois theory

Dec 6, 2024

In this worksheet, if K is a field and $f \in K[x]$ is an irreducible and separable polynomial (over K), we will call the group $Gal(SF_K(f)/K)$ the Galois group of the polynomial f.

Problem 1. *For each of the following polynomials, compute its Galois group.*

(a) $f(x) = x^3 + (2t + 3)x - 1 \in \mathbb{Q}(t)[x]$.

(b) $f(x) = x^4 - 7x^2 + 1 \in \mathbb{Q}[x]$.

(c) $f(x) = x^4 - 5x + 2 \in \mathbb{Q}[x]$.

(d) $f(x) = x^5 - x - 1 \in \mathbb{Q}[x]$.

(e) $f(x) = x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 \in \mathbb{Q}[x]$.

Problem 2. *In each of the following cases, explain why G is the Galois group of the given polynomial $f(x) = x^4 + ax + b \in \mathbb{Q}[x]$.*

(a) $G = S_4$ and $(a, b) = (1, 1)$.

(b) $G = A_4$ and $(a, b) = (8, 12)$.

(c) $G = D_4$ and $(a, b) = (3, 3)$.

(d) $G = C_4$ and $(a, b) = (5, 5)$.

(e) $G = V$ and $(a, b) = (0, 1)$.