École Polytechnique Fédérale de Lausanne
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Discrete Optimization (Spring 2025)

Assignment 5

1) Suppose you are given an oracle algorithm, which for a given polyhedron

$$P = \{\bar{x} \in \mathbb{R}^n : \bar{A}\bar{x} \leqslant \bar{b}\}$$

gives you a feasible solution or asserts that there is none. Show that using a single call of this oracle one can obtain an optimum solution for the LP

$$\max\{c^T x : x \in \mathbb{R}^n; Ax \leqslant b\}$$

assuming that the LP is feasible and bounded.

2) Determine the dual program for the following linear program:

$$\begin{aligned} \min \quad & 3x_1 + 2x_2 - 3x_3 + 4x_4 \\ & 2x_1 - 2x_2 + 3x_3 + 4x_4 \leqslant 3 \\ & x2 + 3x3 + 4x4 \geqslant -5 \\ & 2x_1 - 3x_2 - 7x_3 - 4x_4 = 2 \\ & x_1 \geqslant 0 \\ & x_4 \leqslant 0 \end{aligned}$$

3) Let $P = \{x \in \mathbb{R}^n : Ax \leqslant b\}$ be a bounded, non-empty polyhedron. Formulate a linear program that computes the largest ball inside P.

4) Consider the following linear program:

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{subject to } & 2x_1 + x_2 \leqslant 6 \\ & x_1 + 2x_2 \leqslant 8 \\ & 3x_1 + 4x_2 \leqslant 22 \\ & x_1 + 5x_2 \leqslant 23 \end{aligned}$$

Show that $(4/3, 10/3)$ is an optimal solution by using weak duality.