École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

March 11, 2025

Discrete Optimization (Spring 2025)

Assignment 4

1) Let $P = \{x : Ax \leqslant b\} \subseteq \mathbb{R}^n$ be a polyhedron. Show that x^* is an extreme point $\iff \forall x_1 \neq x_2 \in P, x^* \neq \frac{1}{2}x_1 + \frac{1}{2}x_2$.

2) Suppose that the linear program $\max\{c^T x \colon x \in \mathbb{R}^n,\, Ax \leqslant b\}$ is non-degenerate and B is an optimal basis. Show that the linear program has a unique optimal solution if and only if $\lambda_B > 0$.

3) For each of the following assertion, provide a proof or a counterexample.

   i) An index that has just left the basis B in the simplex algorithm cannot enter in the very next iteration.

   ii) An index that has just entered the basis B in the simplex algorithm cannot leave again in the very next iteration.

4) Consider the following linear program:

$$\begin{aligned} \max \quad & 6a + 9b + 2c \\ \text{subject to } & a + 3b + c \leqslant -4 \\ & b + c \leqslant -1 \\ & 3a + 3b - c \leqslant 1 \\ & a \leqslant 0 \\ & b \leqslant 0 \\ & c \leqslant 0. \end{aligned}$$

   Solve the linear program with the Simplex method and initial vertex $(-1, -1, 0)^T$. For each iteration indicate all the parameters including the optimal value and the proof of optimality.

5) Fill in the blanks to complete the code for the Simplex.py file which runs a simplex algorithm.