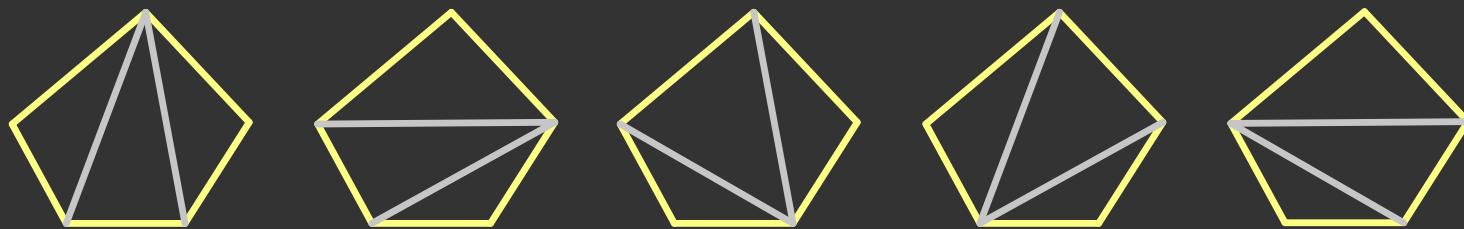


More Catalan numbers

Cutting polygons into triangles

Definition: A **diagonal triangulation** of a polygon is a partition of the polygon into triangles by non-intersecting diagonals.



Lemma: Let $n \geq 3$. The number of diagonal triangulations of a convex n -gon is the Catalan number b_{n-2} .

Parenthesis

(a (b c)(d e) f)

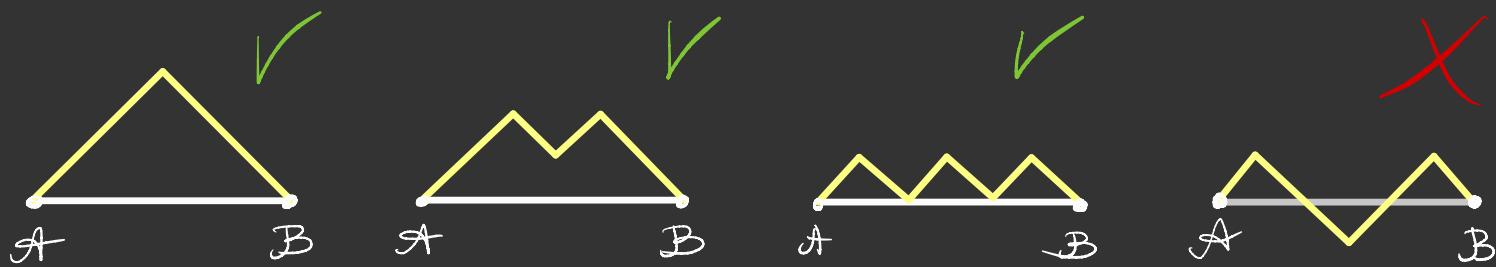
A regular bracket structure satisfies the following two conditions

- the number of left and right brackets is the same
- the number of left brackets in any starting segment is not less than the number of right brackets

Lemma: The number of regular bracket structures with n pairs of brackets is the Catalan number b_n .

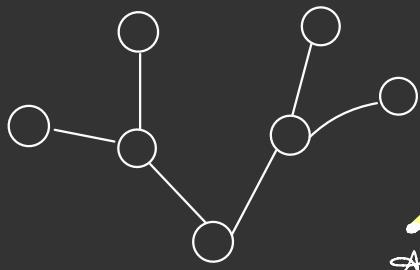
Ups and Downs

Definition: A **Dyck path** is a continuous broken line consisting of vectors $(1, 1)$ and $(1, -1)$ starting at the origin $A = (0, 0)$ and ending at the x -axis and always staying in the upper half-plane.



Lemma: The number of Dyck pathes consisting of $2n$ steps is the Catalan number b_n

Exercise: Prove the lemmas.



(a (b c)(d e) f)

