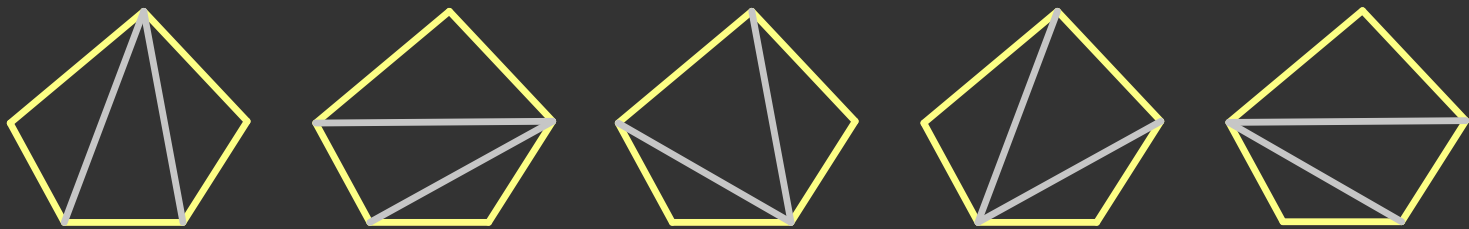


More Catalan numbers

Cutting polygons into triangles

Definition: A diagonal triangulation of a polygon is a partition of the polygon into triangles by non-intersecting diagonals.



Lemma: Let $n \geq 3$. The number of diagonal triangulations a convex n -gon is the Catalan number b_{n-2} .

Parenthesis

$(a(b(c)(d(e)f))$

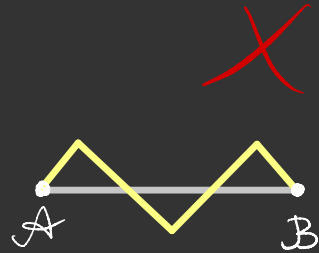
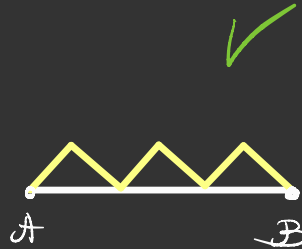
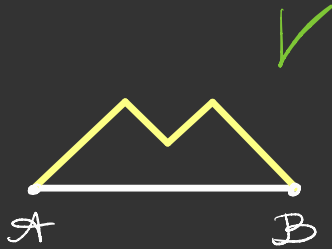
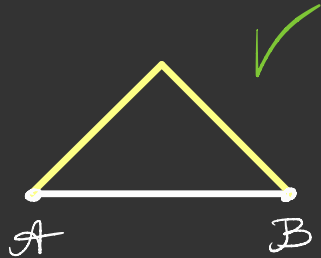
A regular bracket structure satisfies the following two conditions

- the number of left and right brackets is the same
- the number of left brackets in any starting segment is not less than the number of right brackets

Lemma: The number of regular bracket structures with n pairs of brackets is the Catalan number b_n .

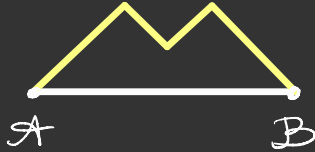
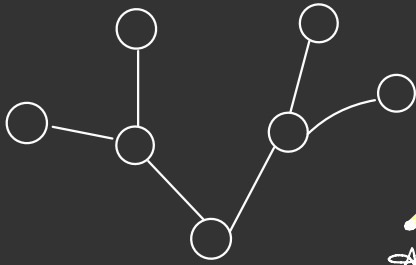
Ups and Downs

Definition: A Dyck path is a continuous broken line consisting of vectors $(1,1)$ and $(1,-1)$ starting at the origin $A=(0,0)$ and ending at the x -axis and always staying in the upper half-plane.



Lemma: The number of Dyck paths consisting of $2n$ steps is the Catalan number b_n

Exercise: Prove the lemmas.



$(a (b c)(d e) f)$

