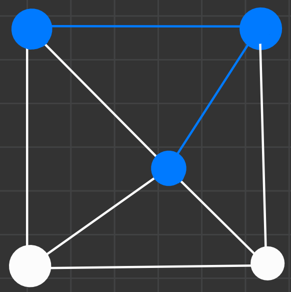
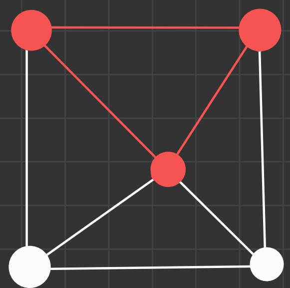


Subgraphs

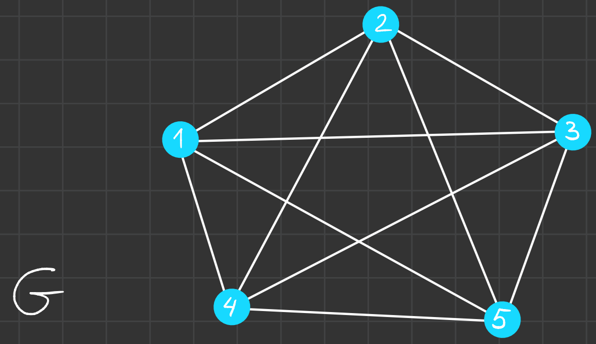
vs

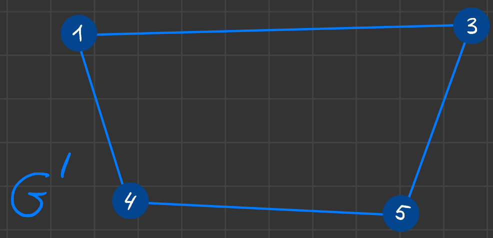
Induced subgraphs



Definition: A **subgraph** of a graph $G = (V, E)$ is a graph $G' = (V', E')$ such that $V' \subseteq V$ and $E' \subseteq E$

Example:

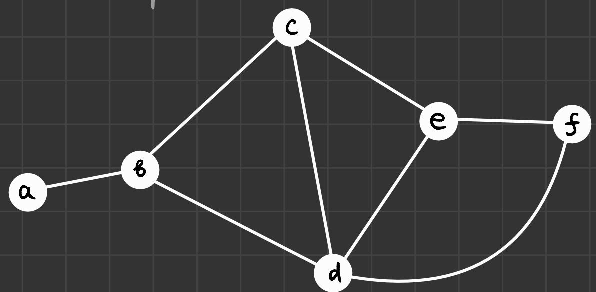


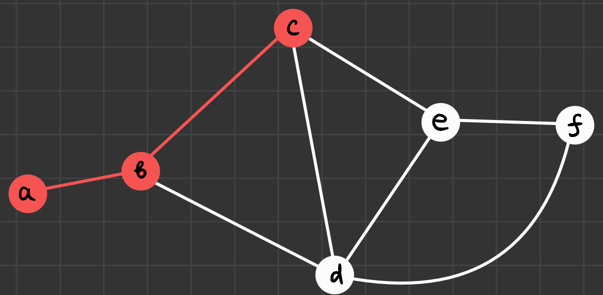


Definition: Graph $G' = (V', E')$ is an **induced subgraph** of $G = (V, E)$ if

$$V' \subseteq V \quad \text{and} \quad E' = E \cap \binom{V'}{2}$$

Example:

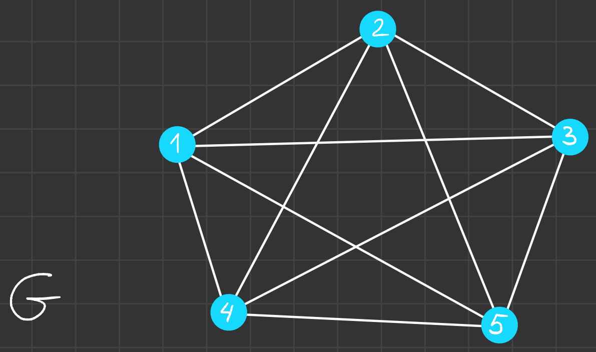


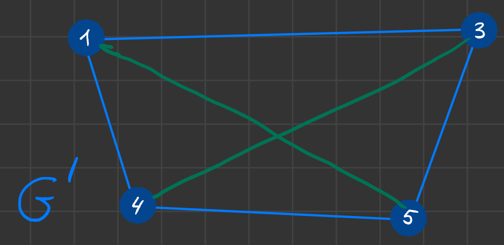


$V = \{a, b, c, d, e, f\}$

$V' = \{a, b, c\} \Rightarrow E' = \{\{a,b\},\{b,c\}\}$

Example:



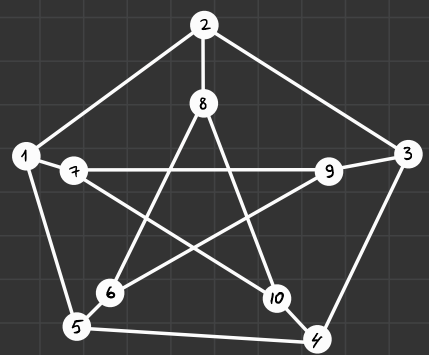


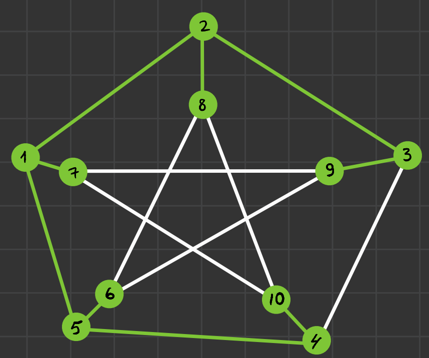
G'' is an induced subgraph of G

Definition: Let $G = (V, E)$ be a graph. An arbitrary tree of the form (V, E'), where $E' \subseteq E$, is called a **spanning tree** of the graph G

So a spanning tree is a subgraph of G that is a tree and contains all vertices of G

Example:





Lemma: Every connected graph contains a spanning tree.

Proof: Let $G = (V, E)$ be a connected graph. We construct a spannig tree of G by the following algorithm:

1. Start with an empty subgraph $T = \emptyset$
2. Pick an edge $e \in E$ such that $e \notin E(T)$
3. Consider the new subgraph T' obtainded from T by adding e

i.e. $V(T') = V(T) \cup \underbrace{\{v_1, v_2\}}_{=e}$ $\quad E(T') = E(T) \cup \{e\}$

4. If T' does not contain cycles set $T := T'$
5. If there is no edge $e \in E$ s.t 2 and 3 hold then stop.

Now we check that the output of this algorithm is a spanning tree of G.

- T contains all vertices of G

  otherwise we can add one more edge
- T is connected

  otherwise we can add one more edge
- T contains no cycles

  by Step 4

$\Rightarrow$ T is a spanning tree $\blacksquare$