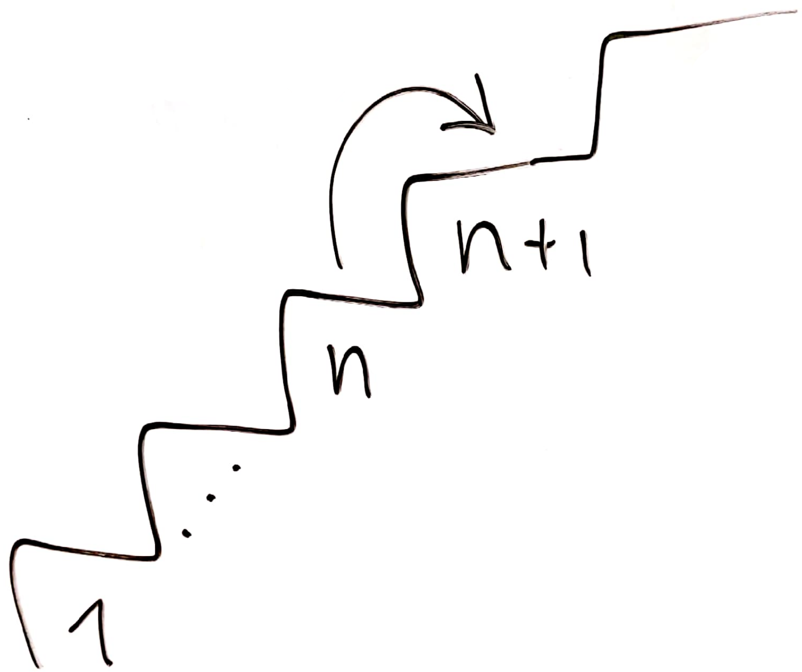
Proofs by induction



Let P be some property of natural numbers.

(a) 1 has property $\mathcal{P}$

(b) if $n \in \mathbb{N}$ has property $\mathcal{P}$ then $n+1$ also has property $\mathcal{P}$.

Principle of induction:

If both (a) and (b) are true, then all natural numbers have property $\mathcal{P}$.

Proofs by induction

Let S be a statement about natural numbers.

Proof of S by induction consists of 2 parts:

1. Basis of induction:

   Prove that S holds for number 1

2. The step of induction.

   induction hypothesis

   Assume that S holds for some $n \in \mathbb{N}$

   and deduce that S holds for $n+1$.

Example

$$1+3+5+\ldots+(2n-1)=?$$

$n=1 \quad 1 = 1 = 1^2$

$n=2 \quad 1+3 = 4 = 2^2$

$n=3 \quad 1+3+5 = 9 = 3^2$

$$\boxed{\underline{\text{Proposition}}: \quad 1+3+\ldots+(2n-1)=n^2}$$

<u>Proof:</u> By induction on n

① <u>Basis of induction</u>. $n=1 \quad 1=1^2 \Rightarrow$ OK.

② Assumption: $1+3+\ldots+(2n-1)=n^2$

$$1+3+\ldots+(2(n+1)-1)=\underbrace{(1+3+\ldots+(2n-1))}_{=n^2 \text{ By induction hypothesis}}+(2n+1)=n^2+2n+1=(n+1)^2$$

Non-examples:

"Theorem" For all $n \in \mathbb{N}$ the number $n \cdot (n+1)$ is odd.

"Proof": By induction

Assume that $n(n+1)$ is odd

$(n+1)(n+2) = n(n+1) + 2(n+1)$ is also odd

$1 \cdot 2 = 2$ is even ⚠

$\blacksquare$

Non- examples

"Theorem" All odd integer numbers bigger then 1 are prime.

"Proof"

3 is prime

5 is prime

7 is prime

We continue like this by induction $\Rightarrow$ ∎

⚠ 9 is composite. Where is a mistake?

I wish you only correct proofs by induction.