Exercise Set #1

"Discrete Mathematics" (2025)

Exercise 6 is to be submitted on Moodle before 23:59 on February 24th, 2025

E1. How many symmetric $n \times n$ matrices are there with entries chosen from the numbers $[q]$?

E2. Let $n \in \mathbb{N}$ be even. How many permutations does the set $[n]$ have such that

- **(a)** the sum of the first two elements is odd?
- **(b)** the last two elements sum up to n.

E3. In how many different ways can the letters of the word $\mathsf{MATHEMATICALLY}$ be arranged such that

- **(a)** the word starts always with MA?
- **(b)** the three A's are adjacent?
- **(c)** the word MATH is always included?
- **(d)** the word MAT is included twice?

E4. In how many different ways can we distribute 40 bottles of water to 10 kids and 5 adults, such that each kid gets at least one bottle?

E5. Let $n, r, k \in \mathbb{N}$ and $n \geq r \geq k$. Prove the following two equations:

(a)

$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}.$$

(b)

$$\binom{n}{k} = \sum_{j=k-1}^{n-1} \binom{j}{k-1}.$$

E6. (**Exercise to submit**)
Laura has 11 balls (all different). First, she splits them into two piles; then she picks one of the piles with at least two elements, and splits it into two; she repeats this until each pile has only one element. In how many different ways can she carry out this procedure?
Hint: Imagine the procedure backward.