EDO

$$-\lambda_{max} \leq \frac{\partial f}{\partial x}(t,x) \leq -\lambda_{min}$$

$$h < \frac{2}{\max_{j=1,\dots,p} |\lambda_j|} = \frac{2}{\rho(A)} \,,$$

$$\forall n = 0,\dots,N_h \quad |u_n - y(t_n)| \leq C(h)$$

$$|y(t_n) - u_n| \leq h t_n \frac{1}{2} \max_{t\in[t_0,t_n]} |y''(t)|$$

$$u_{n+1} - u_n = \frac{h}{2}\left[f(t_n,u_n) + f(t_{n+1},u_{n+1})\right]$$

$$... = \frac{h}{2}\left[f(t_n,u_n) + f(t_{n+1}, u_n + hf(t_n,u_n))\right]$$

Équations non-linéaires

$$\lim_{k\to\infty} \frac{x^{(k+1)} - \alpha}{\left(x^{(k)} - \alpha\right)^d} = \frac{1}{d!}\phi^{(d)}(\alpha)$$

Systèmes linéaires

$$B = P^{-1}(P-A) = I - P^{-1}A \quad , \quad \mathbf{g} = P^{-1}\mathbf{b}.$$

$$P(\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}) = \alpha_k \mathbf{r}^{(k)}$$

$$\alpha_k = \alpha_{opt} = \frac{2}{\lambda_{min}(P^{-1}A) + \lambda_{max}(P^{-1}A)}$$

$$\rho_{opt} = \frac{\lambda_{max}(P^{-1}A) - \lambda_{min}(P^{-1}A)}{\lambda_{min}(P^{-1}A) + \lambda_{max}(P^{-1}A)}$$

$$\begin{aligned} P\mathbf{z}^{(k)} &= \mathbf{r}^{(k)} \\ \alpha_k &= \frac{(\mathbf{z}^{(k)})^T \mathbf{r}^{(k)}}{(\mathbf{z}^{(k)})^T A \mathbf{z}^{(k)}} \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \alpha_k \mathbf{z}^{(k)} \\ \mathbf{r}^{(k+1)} &= \mathbf{r}^{(k)} - \alpha_k A \mathbf{z}^{(k)} \end{aligned}$$

$$\|\mathbf{x}^{(k)} - \mathbf{x}\|_A \leq \left(\frac{\mathrm{Cond}(P^{-1}A) - 1}{\mathrm{Cond}(P^{-1}A) + 1}\right)^k \|\mathbf{x}^{(0)} - \mathbf{x}\|_A$$

$$\begin{aligned} \alpha_k &= \frac{\mathbf{p}^{(k)^T} \mathbf{r}^{(k)}}{\mathbf{p}^{(k)^T} A \mathbf{p}^{(k)}} \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \alpha_k \mathbf{p}^{(k)} \\ \mathbf{r}^{(k+1)} &= \mathbf{r}^{(k)} - \alpha_k A \mathbf{p}^{(k)} \\ P\mathbf{z}^{(k+1)} &= \mathbf{r}^{(k+1)} \\ \beta_k &= \frac{\left(A\mathbf{p}^{(k)}\right)^T \mathbf{z}^{(k+1)}}{\left(A\mathbf{p}^{(k)}\right)^T \mathbf{p}^{(k)}} \\ \mathbf{p}^{(k+1)} &= \mathbf{z}^{(k+1)} - \beta_k \mathbf{p}^{(k)} \end{aligned}$$

$$\|\mathbf{x}^{(k)} - \mathbf{x}\|_A \leq \frac{2c^k}{1+c^{2k}} \|\mathbf{x}^{(0)} - \mathbf{x}\|_A \,, \quad c = \frac{\sqrt{K_2(P^{-1}A)} - 1}{\sqrt{K_2(P^{-1}A)} + 1}$$

$$\frac{\|\mathbf{x}^{(k)} - \mathbf{x}\|}{\|\mathbf{x}\|} \leq \mathrm{Cond}\,(P^{-1}A) \frac{\|P^{-1}\mathbf{r}^{(k)}\|}{\|P^{-1}\mathbf{b}\|}$$

$$\mathrm{Cond}(C) = \sqrt{\frac{\lambda_{\max}(C^TC)}{\lambda_{\min}(C^TC)}} \quad \text{ou} \quad \frac{\lambda_{\max}(C)}{\lambda_{\min}(C)}$$

Interpolation

$$\varphi_k(x) = \prod_{j=0, j\neq k}^{n} \frac{(x - x_j)}{(x_k - x_j)}.$$

$$\max_{x\in I} |E_n f(x)| \leq \frac{1}{4(n+1)} (h)^{n+1} \max_{x\in I} |f^{(n+1)}(x)|,$$

$$\max_{x\in I} \mid E_1^H f(x) \mid \leq \frac{H^2}{8} \max_{x\in I} |f''(x)|.$$

$$\max_{x\in I} \mid E_n^H f(x) \mid \leq \frac{H^{n+1}}{4(n+1)} \max_{x\in I} |f^{(n+1)}(x)| \,.$$

Intégration numérique

$$J^{xx}(f) = (b-a) f\left(\frac{a+b}{2}\right); \quad H \sum_{k=0}^{N-1} f(\overline{x}_k)$$

$$J^{xx}(f) = (b-a)\frac{f(a)+f(b)}{2}; \quad \frac{H}{2}\sum_{k=0}^{N-1}\left[f(x_k) + f(x_{k+1})\right]$$

$$J^{xx}(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right];$$

$$\frac{H}{6}\sum_{k=0}^{N-1}\left[f(x_k) + 4f(\overline{x}_k) + f(x_{k+1})\right]$$

$$|I(f) - I^c_{xx}(f)| \leq \frac{b-a}{24} H^2 \max_{x\in[a,b]} |f''(x)|$$

$$|I(f) - I^c_{xx}(f)| \leq \frac{b-a}{12} H^2 \max_{x\in[a,b]} |f''(x)|$$

$$|I(f) - I^c_{xx}(f)| \leq \frac{b-a}{180\cdot 16} H^4 \max_{x\in[a,b]} |f''''(x)|$$