Exam topics “Numerical Analysis”, Spring 2025

1. Representation of numbers

    - Model of floating point numbers: Definition 1.6, Lemma 1.8 (smallest/largest number), Lemma 1.9, unit roundoff.
    - Theorem 1.11 (rounding a number)
    - Definition 1.14 (model of elementary operations)
    - Perform round-off error analysis and interpret result for simple expressions composed of elementary operations.
    - Cancellation: Understand and avoid phenomenom

2. Numerical Integration

    - Theorems 2.2 and 2.3 (statement *without* proofs)
    - Derivation of Newton-Cotes formula (2.11) from polynomial interpolation; midpoint / trapezoidal / Simpson rule.
    - Definition 2.4 (order)
    - Error of midpoint / trapezoidal / Simpson rule: It is important to remember the order (e.g., $(b-a)^3$ or $(b-a)^4$); no need to remember constant. Also, you need to understand the idea of the proof for the trapezoidal rule (proof of Theorem 2.5 ii).
    - Composite rules from Newton-Cotes formula, Theorem 2.6 for comp. trapezoidal rule (statement *and* understanding of proof technique)
    - Definition 2.10 (Legendre polynomials), Theorem 2.14 (statement and general proof idea)

3. Polynomial Interpolation

    - Lagrange representation (3.1) and its consequences (existence/uniqueness)
    - Definition 3.1 (Chebyshev polynomial), Theorem 3.2 (recursion: statement and proof), Lemma 3.3 (statement and proof), Lemma 3.5, Theorem 3.6
    - Definition 3.7 and its consequence for sensitivity of polynomial interpolation

4. Linear systems – small matrices
    - Algorithms 4.3 and 4.4 (subsitution)
    - Computation of LU factorization (without and with pivoting)
    - Theorem 4.8 (existence of LU)
    - Definition 4.21 (condition of a function)
    - Proposition 4.22 (including basic proof idea), Theorem 4.23 (sensitivity of linear systems)
5. Linear systems – large matrices
    - Idea of splitting methods, convergence of general splitting method (relation to spectral radius of B)
    - Theorem 5.3 (statement and proof)
    - Richardson method, Theorem 5.4 (statement and proof) and Remark 5.5
    - gradient method, Theorem 5.6 (statement and proof), Derivation of α_k
    - CG: Knowledge of basic properties (meaning of $\mathbf{r}^{(k)}$, $\mathbf{x}^{(k)}$), Theorem 5.10 (convergence)
6. Regression and Least Squares
    - Formulation of linear least-squares problem (6.9), Theorem 6.3 (statement and proof)
    - Theorem 6.7 (statement and proof)
7. Fourier Transform
    - DFT: Definition (7.9), Lemma 7.5 (statement and proof), Reformulation as matrix-vector product (7.11)
    - Theorem 7.7 and derivation of Algorithm 7.8 (FFT) from Theorem 7.7
8. Unless stated otherwise, you should be familiar with the proof techniques for the lemmas and theorems listed above. You will be asked to prove small results using these techniques (on the level of the exercises and the 2023 exam). However, you will not be asked to reproduce a proof from the lectures.

9. Python:
    - There will be short Python snippets, which perform a numerical analysis algorithm and which you have to understand / complete / correct.