GM – Probabilités et Statistique – Corrections 3

Please note : the **reasoning/justifications** for the steps in your solution are also important (not only the final result).

In class

Exercise 1 Let **S** the event that the plant survives (that is, it does not die, thus the *complement* of the event that the plant dies, and **A** the event that the neighbor waters it.

According to the problem information :

$$\left.\begin{array}{l} P(A) = 0.90 \\ P(S^c \mid A) = 0.15 \\ P(S^c \mid A^c) = 0.8 \end{array}\right\} \Rightarrow \begin{array}{l} P(A^c) = 1 - 0.90 = 0.10 \\ P(S \mid A) = 1 - P(S^c \mid A) = 1 - 0.15 = 0.85 \\ P(S \mid A^c) = 1 - P(S^c \mid A^c) = 1 - 0.8 = 0.2 \end{array}$$

(a) According to the law of total probability,

$$P(S) = P(S \mid A)P(A) + P(S \mid A^c)P(A^c) = (0.85)(0.9) + (0.2)(0.1) = \boxed{0.785}$$

(b) Using Bayes' rule and the fact that $P(S^c) = 1 - P(S)$ $(= 0.215)$, we have

$$P(A^c \mid S^c) = \frac{P(S^c \mid A^c)P(A^c)}{P(S^c)} = \frac{(0.8)(0.1)}{0.215} = \boxed{\frac{16}{43}} \quad (\approx 0.37)$$

For the remaining exercises, follow **the 4 steps for solving RV problems**, that is :

1. *Identify* la VA
2. Determine the *distribution* (loi) of the RV
3. *Translate* the question
4. *Respond* to the question

Exercise 2

1. Let **Y** = the number of families with 6 children of which 4 or more are girls

2. $Y \sim Bin\left(n = 5, \; p = \frac{11}{32}\right)$

Verification of the 4 conditions :

(i) fixed number : $n = 5$
(ii) Bernoulli trials : 2 possibilities (at least 4 girls/not)
(iii) independent trials : yes, if we suppose that the 5 families are independent
[*] (iv) same probability of 'success' for each trial : $p = \frac{11}{32}$

Supposition : we can model the number of girls in a family with 6 children by the number of 'Heads' obtained when we toss a fair coin 6 times

[*] Calculation of p (according to example 3.6) :

$1'$. Let **X** = number of girls

$2'$. $X \sim Bin(n = 6, \; p_X = 1/2)$

Verification of the 4 conditions : (i) fixed number : $n = 6$

(ii) Bernoulli trials : 2 possibilités (fille/garçon)
(iii) independent trials : yes, if we suppose that the children are independent
(iv) same probability for each child to be a girl : $p_X = 1/2$
[supposition]

3′. Probability that the number of girls **X** is at least 4 : $P(X \geq 4)$

4′. $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

[prob. union mut. excl. events]

$$= \binom{6}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + \binom{6}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + \binom{6}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$$

[binomial dist.]

$$= \left(\frac{1}{2}\right)^6 [15 + 6 + 1] = \frac{22}{64} = \frac{11}{32}$$ [simplification]

3. Probability that the number of families with at least 4 girls **Y** is at least 3 : $P(Y \geq 3)$

4. $P(Y \geq 3) = P(Y = 3) + P(Y = 4) + P(Y = 5)$ [prob. union mut. excl. events]

$$= \binom{5}{3}\left(\frac{11}{32}\right)^3\left(\frac{21}{32}\right)^2 + \binom{5}{4}\left(\frac{11}{32}\right)^4\left(\frac{21}{32}\right)^1 + \binom{5}{5}\left(\frac{11}{32}\right)^5\left(\frac{21}{32}\right)^0$$ [binomial dist.]

$$\approx \boxed{0.226}$$

Exercise 3

1. Let **X** = number of examiners who pass the student, and **F** be the event that the student 'on' (in good form)

2. When he is 'on' : $X \mid F \sim Bin(n, p = 0.8)$; otherwise, we have $X \mid F^c \sim Bin(n, p = 0.4)$

Verification of the 4 conditions :

(i) fixed number : $n = 3$ or 5 (according to the number of judges)
(ii) Bernoulli trials : 2 possibilities (pass/fail)
(iii) independent trials : yes, if we assume that the judges are independent
(iv) same probability of 'success' for each trial : $p = 0.8$ ou 0.4,
depending on whether he is 'on' or pas

According to the information in the problem, $P(\text{en forme}) = 1/3$, and thus $P(\text{'off'}) = 2/3$.

3. The interesting probabilities for the student are :

$P(\text{pass} \mid \text{number of judges} = 3)$ et $P(\text{pass} \mid \text{number of judges} = 5)$.

[*] We note that $P(\text{'on'}) = P(\text{'on'} \mid \text{number of judges} = n)$, $n = 3, 5$ (and thus $P(\text{'off'}) = P(\text{'off'} \mid \text{number of judges} = n)$) because the event 'on' (and 'off') can be assumed to be independent of the number of judges.

4. Using the law of total probability, we have

$$\begin{aligned}
\underline{\underline{P(\text{pass} \mid 3)}} &= P(\text{‘on’} \mid 3)\, P(\text{pass} \mid \text{‘on’}, 3) + P(\text{‘off’} \mid 3)\, P(\text{pass} \mid \text{‘off’}, 3) \\
&= P(\text{‘on’})\, P(\text{pass} \mid \text{‘on’}, 3) + P(\text{‘off’})\, P(\text{pass} \mid \text{‘off’}, 3) && [*] \\
&= P(\text{‘on’})\, P(X \geq 2 \mid F, 3) + P(\text{‘off’})\, P(X \geq 2 \mid F^c, 3) \\
&= P(\text{‘on’})\, [P(X = 2 \mid F, 3) + P(X = 3 \mid F, 3)] && \text{[prob. union mut. excl. events]} \\
&\quad + P(\text{‘off’})\, [P(X = 2 \mid F^c, 3) + P(X = 3 \mid F^c, 3)] \\
&= \frac{1}{3} \sum_{i=2}^{3} \binom{3}{i} (0.8)^i (0.2)^{3-i} + \frac{2}{3} \sum_{i=2}^{3} \binom{3}{i} (0.4)^i (0.6)^{3-i} && \text{[binomial dists.]} \\
&\approx \frac{1}{3} (0.896) + \frac{2}{3} (0.352) \approx \underline{\underline{0.533}} && \text{[calcs., simplification]}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\underline{\underline{P(\text{promu} \mid 5)}} &= \frac{1}{3} \sum_{i=3}^{5} \binom{5}{i} (0.8)^i (0.2)^{5-i} + \frac{2}{3} \sum_{i=3}^{5} \binom{5}{i} (0.4)^i (0.6)^{5-i} && \text{[binomial dists.]} \\
&\approx \frac{1}{3} (0.942) + \frac{2}{3} (0.317) \approx \underline{\underline{0.526}}
\end{aligned}$$

Finally, we note that the probability of passing is bigger with **3 examiners** .

At home

Exercise 1

1. Let $\mathbf{X}$ = the number of correct responses for the 10 tosses
2. $X \sim Bin(n = 10,\ p = 1/2)$

   Verification of the 4 conditions :

   (i) fixed number : $n = 10$
   (ii) Bernoulli trials : 2 possibilities (Head/Tail)
   (iii) independent trials : yes, if we assume that the tosses are independent
   (iv) same probability of ‘success’ for each trial : $p = 1/2$

3. Probability that the number of correct responses $\mathbf{X}$ is at least 7 : $P(X \geq 7)$
4. $P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$ [prob. union events mutually exclusive]

$$\begin{aligned}
&= \sum_{i=7}^{10} \binom{10}{i} \left(\frac{1}{2}\right)^i \left(\frac{1}{2}\right)^{10-i} \\
&= \binom{10}{7} \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + \binom{10}{8} \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + \binom{10}{9} \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + \binom{10}{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 \\
&= \left(\frac{1}{2}\right)^{10} [120 + 45 + 10 + 1] = \frac{176}{2^{10}} = \frac{16 \times 11}{16 \times 64} = \boxed{\frac{11}{64}} \quad (\approx 0.17) \quad \text{[simplification]}
\end{aligned}$$

Exercise 2 [The solution is similar to that of Exercice 2 above.]

1. Let **Y** = number of returned packages

2. $Y \sim Bin\left(n = 3,\ p = 0.0043\right)$

   Verification of the 4 conditions :

   (i) fixed number : $n = 3$
   (ii) Bernoulli trials : 2 possibilities (at most 1 defetive disquette/not)
   (iii) independent trials : yes, if we suppose that the packages are independent
   [*] (iv) same probability of 'success' for each trial : $p = 0.0043$

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[*] Calculation of p :

1′. Let **X** = number of defective disquettes

2′. $X \sim Bin(n = 10,\ p_X = 0.01)$

   Verification of the 4 conditions :

   (i) fixed number : $n = 10$
   (ii) Bernoulli trials : 2 possibilities (defective/not)
   (iii) independent trials : yes, if we suppose that the disquettes are independent
   (iv) same probability of being 'defective' for each disquette : $p_X = 0.01$

3′. Probability that the number of defective disquettes is at most 1 :
   X is at most 1 : $P(X \leq 1)$

4′. $P(X \leq 1) = P(X = 0) + P(X = 1)$ [prob. union mut. excl. events]

$$= \binom{10}{0}(0.99)^{10}(0.01)^0 + \binom{10}{1}(0.99)^9(0.01)^1 \quad \text{[binomial dist.]}$$

$$\approx \underline{0.9957} \quad \text{[simplification]}$$

Therefore, the probability that a package contains at most 1 defective disquette = 0.9957 and thus the probability that a package must be returned = the probability that the package contains more than 1 defective disquette = $1 - 0.9957 = 0.0043$.

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3. Probability that the number of packages with at most 1 defective disquette **Y** equals 1 : $P(Y = 1)$

4. $P(Y = 1) = \binom{3}{1}(0.0043)(0.9957)^2 \approx \boxed{0.013}$ [binomial dist., simplification]