GM – Probabilités et Statistique – Corrections 2

Please note : the **reasoning/justifications** for the steps in your solution are also important (not only the final result).

In class

Exercise 1 (a) By the Generalized basic counting principle, there are 3 (bases) $\cdot$ 2 (cheeses) $\cdot$ 5 (toppings) = 30 possible pizzas.

(b) There are $\binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 10$ combinations of 2 toppings for the pizza for each combination of base and cheese, thus by the Generalized basic counting principle, there are $3 \cdot 2 \cdot \binom{5}{2} = 3 \cdot 2 \cdot 10 = 60$ possible pizzas.

Exercise 2 (a) Without restriction, there are 10 possible choices for president. For each of these, there are 9 possible choices for treasurer, and for each combintion there are 8 possible choices for secretary. By the Generalized basic counting principle, there are thus $10 \cdot 9 \cdot 8 = 720$ possible choices.

(b) By the Generalized basic counting principle, there are $8 \cdot 7 \cdot 6 = 336$ choices with neither A nor B, and $3 \cdot 8 \cdot 7 = 168$ choices where A (but not B) has an office (since there are 3 possibilities for A's office for each combination (of which there are $8 \cdot 7$) of two other people for the other offices). Similarly, there are $3 \cdot 8 \cdot 7 = 168$ choices where B (but not A) has an office. Thus, there are $336 + 2 \cdot 168 = 672$ possible choices.

[You could just as well count the number of committees with both A and B and subtract from 720, the total number of possible committees. There are 3 choices for A, then 2 choices for B, and finally 8 choices for the remaining officer. By the Generalized basic counting principle, there are thus $3 \cdot 2 \cdot 8 = 48$ committees with both of them, thus 720 - 48 = 672 possible choices.]

(c) In the same fashion as for (b), by the Generalized basic counting principle, the number of choices with C and D is $3 \cdot 2 \cdot 8 = 48$: 3 offices possible for C, and for each choice of office for C there are 2 choices of office for D ; there are 8 remaining people possible to take the last office. The number with neither C nor D is $8 \cdot 7 \cdot 6 = 336$ (By the Generalized basic counting principle). Thus the total is $48 + 336 = 384$ possible choices.

(d) There are 3 choices for E, and for each of these there are $9 \cdot 8$ choices for the 2 other officers. By the Generalized basic counting principle, there are $3 \cdot 9 \cdot 8 = 216$ possible choices.

(e) This is the total of the number of committees without F and the number of committees for which F is president. By the Generalized basic counting principle, the number of committees without F is $9 \cdot 8 \cdot 7 = 504$. The number for which F is president is $1 \cdot 9 \cdot 8 = 72$, thus $504 + 72 = 576$ possible choices.

Exercise 3 (a) All $\binom{10}{5}$ exams are equally likely, of which $\binom{7}{5}$ the student is able to solve ; thus, the probability is $\binom{7}{5} / \binom{10}{5} = \frac{1}{12}$.

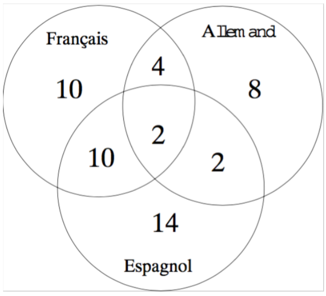
(b) Applying the Generalized basic counting principle, the number of exams for which the students can solve 4 out of 5 questions (and doesn't know how to solve 1) is $\binom{7}{4}\binom{3}{1}$, thus the probability that the student knows how to solve 4 questions is $\binom{7}{4}\binom{3}{1}/\binom{10}{5} = \frac{5}{12}$.

The probability that he knows how to solve at least 4 is the sumof the probabilities of knowing how to solve 4 and of knowing how to solve 5 (these two events are disjoints/ mutually exclusive), thus $\frac{1}{12} + \frac{5}{12} = \underline{\frac{1}{2}}$.

At home

Exercise 1 By the Generalized basic counting principle, there are $5 \cdot 2 \cdot 4 = \underline{40}$ replicates for the experiment.

Exercise 2



Event A : "is taking German (allemand)".
Event F : "is taking French (français)".
Event E : "is taking Spanish (espagnol)".

$$P(E) = \frac{28}{100} \qquad P(A) = \frac{16}{100} \qquad P(F) = \frac{26}{100}$$

$$P(E \cap F) = \frac{12}{100} \qquad P(E \cap A) = \frac{4}{100} \qquad P(F \cap A) = \frac{6}{100}$$

$$P(E \cap A \cap F) = \frac{2}{100}$$

(a) On a
$$\begin{aligned} P(\bar{E} \cap \bar{A} \cap \bar{F}) &= 1 - P(E \cup A \cup F) && \text{[deMorgan, complement]} \\ &= 1 - [P(E) + P(A) + P(F) && \text{[inclusion-exclusion]} \\ &\quad - P(E \cap A) - P(E \cap F) - P(F \cap A) + P(E \cap A \cap F)] \\ &= 1 - \left[\frac{28}{100} + \frac{16}{100} + \frac{26}{100} - \frac{4}{100} - \frac{12}{100} - \frac{6}{100} + \frac{2}{100}\right] = \frac{50}{100} = \underline{\frac{1}{2}} \end{aligned}$$

(b) First, denote for events G and H, the event $G \cap \bar{H} = G \backslash (G \cap H)$ $(*)$ (that is, the part of G not contained in $G \cap H$). You can make a Venn diagram to visualize this fact. Then, in the same fashion as for part (a), we have :

$$\begin{aligned} &P(E \cap \bar{A} \cap \bar{F}) + P(\bar{E} \cap A \cap \bar{F}) + P(\bar{E} \cap \bar{A} \cap F) \\ &= P(E \cap (\overline{A \cup F})) + P(A \cap (\overline{E \cup F})) + P(F \cap (\overline{A \cup E})) && \text{[associativity, deMorgan]} \\ &= [P(E) - \underline{P(E \cap (A \cup F))}] + [P(A) - \underline{P(A \cap (E \cup F))}] + [P(F) - \underline{P(F \cap (A \cup E))}]). && [*] \end{aligned}$$

Now, let's calculate (for example) $\underline{P(E \cap (A \cup F))}$:

$$\underline{P(E \cap (A \cup F))} = P(E) + \boxed{P(A \cup F)} - \underline{\underline{P(E \cup A \cup F)}} \qquad \text{[inclusion-exclusion]}$$

$$\boxed{P(A \cup F)} = P(A) + P(F) - P(A \cap F) \qquad \text{[inclusion-exclusion]}$$

$$\underline{P(E \cup A \cup F)} = P(E) + P(A) + P(F) - P(E \cap A) - P(E \cap F) - P(A \cap F) + P(E \cap F \cap A) \qquad \text{[inclusion-exclusion]}$$

$$\Longrightarrow \underline{P(E \cap (A \cup F))} = P(E) + P(A) + P(F) - P(A \cap F) - [P(E) + P(A) + P(F) - P(E \cap A) - P(E \cap F) - P(A \cap F) + P(E \cap F \cap A)] \qquad \text{[substitution]}$$

$$= P(E \cap A) + P(E \cap F) - P(E \cap F \cap A) \quad \text{[and similarly for the other terms]}$$

Et donc [*] $= [P(E) - P(E \cap A) - \boxed{P(E \cap F)} + \underline{\underline{P(E \cap A \cap F)}}]$

$[P(A) - P(A \cap E) - P(A \cap F) + \underline{\underline{P(E \cap A \cap F)}}]$

$[P(F) - P(F \cap A) - \boxed{P(F \cap E)} + \underline{\underline{P(E \cap A \cap F)}}]$

$$= P(E) + P(A) + P(F) - 2\, P(A \cap F) - 2\, P(A \cap E) - 2\, \boxed{P(F \cap E)} + 3\, \underline{\underline{P(E \cap A \cap F)}}$$

$$= \frac{28}{100} + \frac{16}{100} + \frac{26}{100} - 2 \times \frac{6}{100} - 2 \times \frac{4}{100} - 2 \times \frac{12}{100} + 3 \times \frac{2}{100} = \underline{\frac{32}{100}}$$

(c) Let X_i denote the event that "student i is taking at least one course" $(i = 1, 2)$; then,

$$P(X_1 \cup X_2) = P(X_1) + P(X_2) - P(X_1 \cap X_2) = \frac{1}{2} + \frac{1}{2} - \frac{50}{100} \times \frac{49}{99} = 1 - \frac{49}{198} = \underline{\frac{149}{198}}$$