GM – Probabilités et Statistique – Exercise Set 8

In class

Exercise 1 The mean lifetime of a sample of 31 fluorescent lightbulbs fabricated by a factory is estimated at 1570 hours with a standard deviation of $s = 85$ hours.

Test if the mean lifetime for the lightbulbs produced by the factory μ is smaller than 1600, choosing a significance level of 0.05 and also 0.01.

Exercise 2 Let $Y_1, \ldots, Y_n$ be independent and identically distributed (iid) variables according to the Normal distribution with parameters (μ, σ^2) where the expected value μ and the standard deviation σ are *unknown*.

(a) Write out the formula of a Student's confidence interval with confidence level 90% for the parameter μ, as a function of the sample size n, the estimator s of σ, of the random variable $\overline{Y}$ and of the 95%-quantile of the Student distribution with $n-1$ degrees of freedom, noted $t_{n-1,0.95}$.

(b) What is the length of the confidence interval obtained in (a) ?

(c) We have ten measures of the time T_A of flow of an hourglass :

30 29 32 31 29 29 28 31 33 28

Determine the corresponding 90% confidence interval.

Exercise 3 Find the regression equation and RMS error for predicting final score from midterm score, based on the following information (you can assume that the scatter plot is oval-shaped) :

average midterm score = 70 ; SD = 10

average final score = 55 ; SD = 20 ; $r = 0.60$.

Exercise 4 In a large observational study, the regression equation for predicting height from number of years education for 25-34 year old men is given by

$$\text{predicted height } = 0.25 \text{ inches per year } \times \text{ education } + 66.75 \text{ inches}$$

Predict the height of a man with 12 years of education (*i.e.* without university), and with 16 years of education (*i.e.* with university). Does going to university increase a man's height ? Explain.

At home

Exercise 1 The following measurements represent the thickness (in microns) of epitaxial at the center of silicon chips.

1.72	1.67	1.54	1.46	1.62	1.49	1.51	1.46	1.52
1.69	1.54	1.69	1.61	1.39	1.64	1.59	1.36	1.59
1.53	1.63	1.40	1.53	1.76	1.69	1.69		

Calculate a 95% confidence interval for the mean μ.

Exercise 2 We would like to compare two concepts of energy economy in home construction.

Concept 1 : solar energy

Concept 2 : house sheltered in the ground

Let X (respectively Y) represent the annual cost of energy consumption per home using concept 1 (concept 2, respectively).

(a) Use the test of Student (t-test) to test ($\alpha = 0.05$)
$H : \mu_x = \mu_y$
$A : \mu_x \neq \mu_y$
for the following data :

$$n = 12 \quad \bar{x} = \text{Fr. } 400.00 \quad s_x = \text{Fr. } 37.00$$
$$m = 6 \quad \bar{y} = \text{Fr. } 327.00 \quad s_y = \text{Fr. } 36.40$$

(b) What assumptions on the random variables X and Y must we make under (a) ?

Exercise 3 For a certain experiment, the regression equation for predicting the length of a spring with a weight attached is given by

$$\text{predicted length } = 0.05 \text{ cm per kg} \times \text{ weight } + 439.01 \text{ cm}$$

Predict the length of the spring when the weight is 3 kg, and also when it is 5 kg. Does putting more weight on the spring make it longer ? Explain.