GM – Probabilités et Statistique – Exercise Set 7

In class

Exercise 1 Let $Y_1, \ldots, Y_n$ iid $N(\mu, \sigma^2)$.

(a) Find the bias of $\hat{\mu} = \overline{x}$

(b) Find the bias of $\hat{\sigma}^2 = \dfrac{\sum(x_i - \overline{x})^2}{n}$

(use the fact that the bias of $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2 = 0$, c.-à-d. $E(S^2) = \sigma^2$).

(c) Find the asymptotic bias of $\hat{\sigma}^2$ from part (b)).

Exercise 2 Let $X_1, \ldots, X_n$ iid $Pois(\lambda)$ random variables, and $Y_1, \ldots, Y_m$ iid $Pois(2\lambda)$ random variables, indépendants de $X_1, \ldots, X_n$. Find

(a) $L(\lambda)$

(b) $\ell(\lambda)$

(c) $\hat{\lambda}_{MLE}$

(d) $J(\lambda)$

(e) $I(\lambda)$

(f) Is $\hat{\lambda}_{MLE}$ biased ?

(g) Give an approximate 95% confidence interval for λ.

At home

Exercise 1 Let $X_1, \ldots, X_n$ iid $Pareto(a, b)$; that is, $f(x) = \dfrac{a\, b^a}{x^{a+1}},\ a > 0,\ b > 0,$ $b \leq x < \infty$. Suppose that the value of b is known.

Calculate $L(a)$, $\log L(a)$, $\hat{a}_{MLE}$. Verify that you have found a maximum.

Exercise 2 Soient $Y_1, \ldots, Y_n$ *iid* $Exp(\lambda)$; c.-à-d., $f(y) = \lambda\, e^{-\lambda y},\ \lambda > 0$.

(a) Calculer $L(\lambda)$, $\log L(\lambda)$, $\hat{\lambda}_{MLE}$. Verify that you have found a maximum.

(b) Calculate $J(\lambda)$ and $I(\lambda)$, and give the form of an approximate 95% CI for λ.

Exercise 3 Let $Y_1, \ldots, Y_n$ iid $Bernoulli(p)$. Find the MLE of $p(1-p)$.