# GM – Probabilités et Statistique – Exercise Set 6

## In class

**Exercice 1** The joint density for two random variables $X$ and $Y$ is given by

$$f(x,y) = \begin{cases} x+y & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Verify that $f(x,y)$ is a density function.
(b) Determine the marginal densities $f_X(x)$ and $f_Y(y)$.
(c) Are $X$ and $Y$ independent ?
(d) Calculate $E(X)$, $E(Y)$, $Var(X)$, $Var(Y)$.

**Exercice 2** The joint density of $X$ and $Y$ is $f(x,y) = c(x^2 - y^2)e^{-x}, \quad 0 \le x < \infty$, $-x \le y \le x$. Find the conditional cumulative distribution function $F_{Y|X}(y \mid x)$ of $Y$, given that $X = x$.

**Exercice 3** A study has shown that each restaurant client in the canton spends on average 12 francs for a meal, with a standard deviation of 4 francs. A randomly chosen restaurant has taken a random sample of the bills of 49 clients.

(a) Use the central limit theorem (CLT) to calculate the probability that the average value of 49 bills additions is at least 14 francs.
(b) One hundred (independent) restaurants have carried out the same study. That is, each restaurant had to choose (at random) the bills of 49 of its clients and find the average amount. In principle, how many restaurants do you expect to have an average amount of 14 francs or more ?

---

## At home

**Exercice 1** The joint density of the continuous RVs $X$ and $Y$ is

$$f(x,y) = \frac{x}{5} + cy,\ 0 < x < 1,\ 1 < y < 5;\ \ 0 \text{ otherwise.}$$

(a) Find $c$.
(b) Are $X$ and $Y$ independent ?
(c) Find $P(X+Y > 3)$.

**Exercice 2** The joint density of $X$ and $Y$ is

$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \quad 0 < x < 1,\ 0 < y < 2 \ ; \quad = 0 \text{ otherwise.}$$

(a) Verify that this is a joint density.
(b) Determine the marginal density of $X$.
Find : (c) $P(X > Y)$ (d) $E[X]$ (e) $E[Y]$

**Exercice 3** Suppose that a random sample of 64 bonbons is selected. The average weight of these bonbons ($\overline{x}$) is 90 grams, and the standard deviation $s$ is 10 grams.

(a) Give an (approximate) 95% confidence interval for the average weight $\mu$ of the population of bonbons.
(b) State your suppositions.
(c) What sample size is necessary so that the length of the 95% CI for $\mu$ is smaller than 1 ?
(d) Recalculate the interval for a higher confidence level, 99%.
(e) Verify that the interval from (d) is longer than that from (a) ; explain this fact.