In class

Exercise 1 A researcher is interested in the relative merits of diets compared to cholesterol lowering drugs. For each of the 65 subjects who started the study with high cholesterol, she records the total level of blood cholesterol (in mg per deciliter) after 6 months of study participation. Patients are divided into 5 groups : a control group (C) receiving a placebo, a vegetarian diet group (V), a low-fat diet group (PG), a low-dose group of drugs (DF) and a group with a high dose of drugs (DE). Summaries of the data :

Groupe	Mean	SD	n_{groupe}
C	240	1.22	25
V	225	1.18	10
PG	230	1.10	10
DF	215	1.02	10
DE	200	1.11	10

ANOVA table :

Source	df	SS	MS	F	p
Treatments					4.8×10^{-11}
Error	60		80		
Total		11681.4			

(a) What assumptions must be made to obtain a valid p-value using ANOVA ?

(b) Complete the table.

(c) On the basis of these data, is there evidence that at least one of the group averages differs ? Justify your answer by performing an appropriate hypothesis test. State null and alternative hypotheses. What is your conclusion, if $\alpha = 0.01$? Interpret the result.

(d) What percentage of the variability is explained by the difference between treatments ?

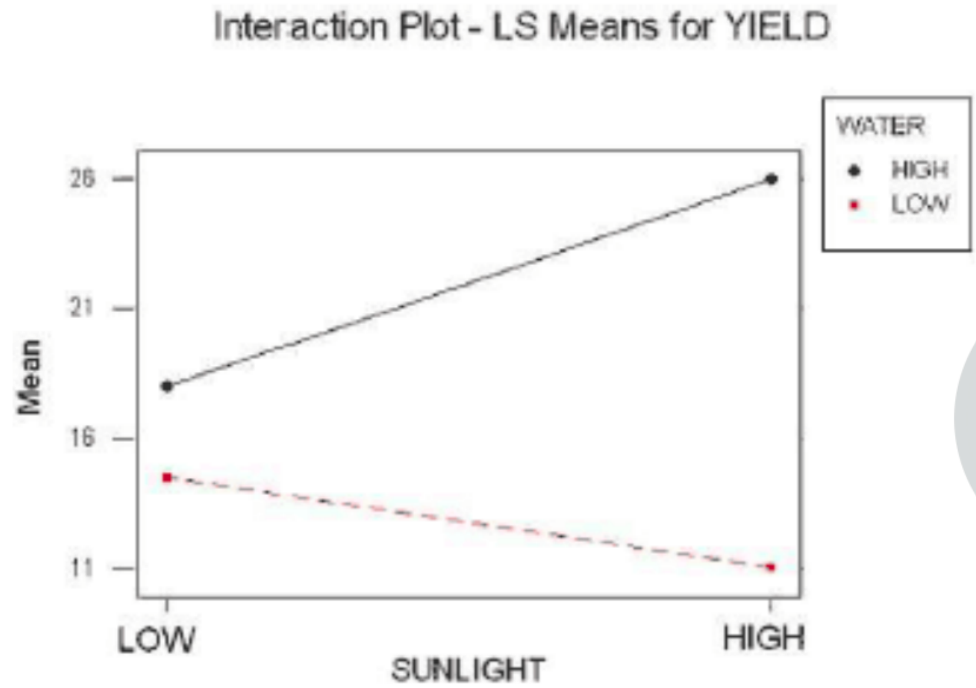
(e) Calculate the test statistics t_{obs} for the 10 comparison tests (each pair).

(f) Which pairs of averages are different at the level (nominal – ie without adjustment) of significance $\alpha = 0.01$?

(g) According to the Bonferroni method, find the threshold (critical value) for each test if you want an overall (global) threshold of $\alpha = 0.01$. Which pairs of averages are different at (global) significance level $\alpha = 0.01$?

Exercise 2 Does the amount of sunlight and watering affect the growth of geraniums ? The plant growth of 16 plants is measured in centimeters. Each combinaton of sunlight and water (high / low) has 4 plants ; e.g., High water with high sunlight has 4 plants ranging in length from 21 to 30cm. The (partially complete) ANOVA table is below, as well as an interaction plot.

Source	df	SS	MS	F	$P(> F)$
Water	(a)	342.2	(g)	(k)	0.000365 ***
Sunlight	(b)	20.2	(h)	(l)	0.256272
(n)	(c)	132.2	(i)	(m)	0.010152 *
Error	(d)	(f)	(j)		
Total	(e)	665.6			



(a)-(n) Complete the table.

(o) Write out the **complete** (theoretical) model, including any model assumptions.

(p) Does the amount (level) of watering affect the growth of potted geraniums? Carry out the relevant hypothesis test, making sure to write out each of the 5 steps, and *interpret your results* (use $\alpha = 0.05$).

(q) Does the amount (level) of sunlight affect the growth of potted geraniums? Carry out the relevant hypothesis test, making sure to write out each of the 5 steps, and *interpret your results* (use $\alpha = 0.05$).

(r) Does the effect of the level of sunlight depend on level of watering? Carry out the relevant hypothesis test, making sure to write out each of the 5 steps, and *interpret your results* (use $\alpha = 0.05$).

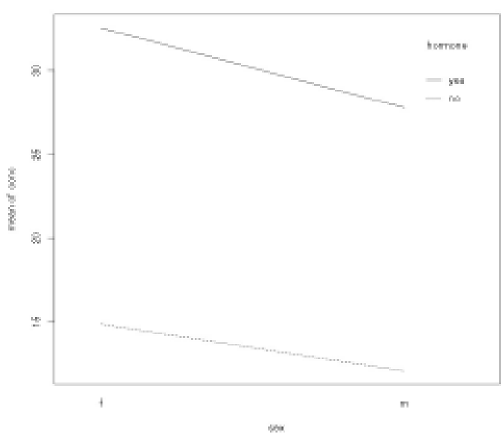
(s) Does it make sense to drop the variable 'sunlight' from the model? **Explain.**

At home

Exercise 1 We want to test if plasma calcium concentration (mg/100 ml) of male and female birds is affected by hormone treatment. The R output from the experiment and interaction plot are given below. You may assume that the design is balanced.

```
interaction.plot(sex,hormone,conc)
```

```
              Df Sum Sq Mean Sq F value   Pr(>F)
sex            1   70.3    70.3   3.733   0.0713 .
hormone        1 1386.1  1386.1  73.585 2.22e-07 ***
sex:hormone    1    4.9     4.9   0.260   0.6170
Residuals     16  301.4    18.8
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```



(a) How many birds are there in the study? How many males? How many females?

(b) How many hormone treatments are there?

(c) How many birds are in each sex $\times$ hormone group?

(d) How many interaction terms can be estimated? How do you know?

(e) Write out the **complete** (theoretical) model, including any model assumptions.

(f) Is there any graphical evidence of an interaction effect? **Explain.**

(g) Write out the 3 possible hypothesis tests that you could do in this experiment. Which NULL hypotheses are REJECTED at the $\alpha = 10\%$ level?

(h) Using the results of part (g), write out the final (theoretical) model.

Exercise 2 This example comes from a statement by Texaco, Inc. to the Air and Water Pollution Subcommittee of the US Senate Public Works Committee on June 26, 1973. Mr. John McKinley, President of Texaco, cited an automobile filter developed by Associated Octel Company as effective in reducing pollution.

However, questions had been raised about the effects of filters on vehicle performance, fuel consumption, exhaust gas back pressure, and silencing. On the last question, he referred to a dataset as evidence that the silencing properties of the Octel filter were at least equal to those of standard silencers.

This is an experiment in which the treatment *filter type* with levels *standard* and *octel* are randomly assigned to the experimental units (cars). Three types of cars are used : small, medium, or large car. The outcome is the quantitative (continuous) variable *noise*.

Six cars are tested for each *size* $\times$ *filter type* combination. A partial ANOVA table is given below :

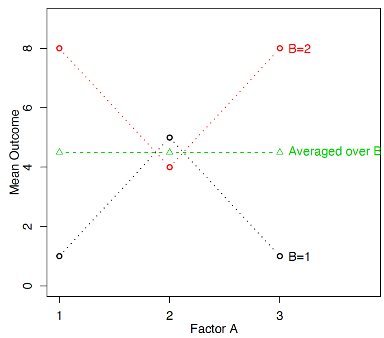
Source	df	SS	MS	F	p
Size			13026		< 2e-16 ***
Filter		1056			0.00373 ***
		804			0.000158 ***
Error			65		
		29874			

(a) Complete the table.

(b) Would an interaction model or an additive model be more appropriate in this situation? Explain.

(c) What percent of total variability is explained by the model you chose in part (b)?

Exercise 3 A typical example of a statistically significant interaction with statistically NON-significant MAIN effects is where, for example, there are three levels of factor A and two levels of factor B, and the pattern of effects of changes in factor A is that the means are in a V shape for one level of B and an inverted V shape for the other level of B. Then the test for the main effect for A is a test of whether the mean outcome at all three levels of A, averaged over both levels of B, are equivalent :



Explain why the non-significance of main effects is misleading here.