GM – Probabilités et Statistique – Exercise Set 5

In class

Exercise 1 A service station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a RV with probability density function

$$f(x) = \begin{cases} 5(1-x)^4 & 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

what must the capacity of the tank be so that the probability of the supply's being exhausted in a given week is 0.01 ?

Exercise 2 A randomly chosen IQ test taker obtains a score that is approximately a normal RV with mean 100 and SD 15. What is the probability that the score of such a person is

(a) above 125 ?

(b) between 90 and 110 ?

Exercise 3 Let X be a normal RV with mean 12 and variance 4. Find the value of c such that $P(X > c) = 0.10$.

At home

Exercise 4 For some constant c, the random variable X has the probability density function

$$f(x) = \begin{cases} cx^n & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find (a) c and (b) $P(X > x)$, $0 < x < 1$.

Exercise 5 Let $X_1, \ldots, X_n$ be independent random variables with distribution $N(\mu, \sigma^2)$. Calculate the expected value and variance of $\overline{X} = \dfrac{X_1 + \ldots + X_n}{n}$.

Exercise 6 Let a, b be constants ; the random variable X has density

$$f(x) = \begin{cases} ax + bx^2 & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

If $E[X] = 0{,}6$, find

(a) a, b

(b) $P(X < \frac{1}{2})$

(c) $Var(X)$

Exercise 7 Bowling scores of Gilles follow an approximately Normal distribution with mean 170 and standard deviation 20, while those of Jacques are approximately Normal with mean 160 and standard deviation 15. If Jacques and Gilles each play one game and, supposing that their scores are independent, approximate the probability that

(a) Jacques beats Gilles.

(b) the sum of their scores exceeds 350.