Please note : the **reasoning/justifications** for the steps in your solution are also important (not only the final result).

In class

Exercise 1 $\int_x^1 5(1-y)^4\,dy = 0,01 \implies -(1-y)^5\Big|_x^1 = (1-x)^5 = 0,01 \Rightarrow \boxed{x = 1-(0,01)^{1/5}}$

Exercise 2

1. Let **X** be the person's score (number of points)

2. $X \sim N(\mu = 100,\ \sigma^2 = 15^2)$ [according to problem info]

(a) 3. Probability that the score is above 125 : $P(X > 125)$

4. $P(X > 125) = P\left(\boxed{\frac{X-100}{15}} > \frac{125-100}{15}\right)$ [standardize both sides]

$= P(Z > 1.67) = 1-\underbrace{P(Z \le 1.67)}_{\Phi(1.67)} = 1-0.9525 = \boxed{0.0475}$ [$Z \sim N(0,\ 1)$; table, simp.]

(b) 3. Probability that the score is between 90 and 110 : $P(90 < X < 110)$
(or : $P(90 \le X \le 110)$)

4. $P(90 < X < 110) = P\left(\frac{90-100}{15} < \boxed{\frac{X-100}{15}} < \frac{110-100}{15}\right)$ [standardize]

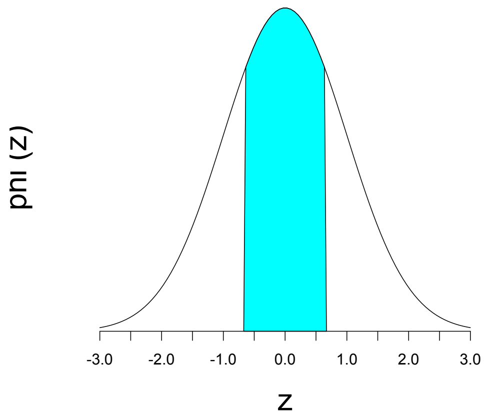
$= P(-0.67 < Z < 0.67) = \underbrace{P(Z < 0.67)}_{\Phi(0.67)} - \underbrace{P(Z < -0.67)}_{\Phi(-0.67)}$ [calcs, $Z \sim N(0,1)$]

$= \underbrace{P(Z < 0.67)}_{\Phi(0.67)} - \underbrace{(1-P(Z < 0.67))}_{\Phi(-0.67)} = 0.7486-(1-0.7486) = \boxed{0.4972 \approx 0.5}$ [table, simp]

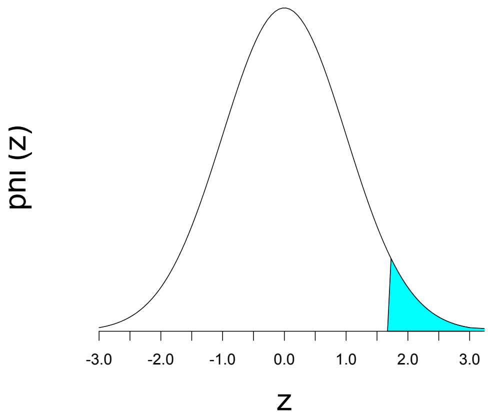
Exercise 3 Find z such that $P(Z > z) = 0.10$:

$\implies P(Z \le z) = 0.90 \implies z = 1.28$ [Normal distribution table]

Then $z = 1.28 = (c-12)/\sqrt{4} \implies c = 12 + \sqrt{4} \times 1.28 \implies \boxed{c = 14.56}$



(a) ex. 2a



(b) ex. 2b

At home

Exercise 1 (a) Using the fact that the integral of the density function must equal 1, we have :

$$1 = \int_0^1 cx^n\,dx = \left.\frac{c\,x^{n+1}}{n+1}\right|_{x=0}^1 = c\left[\frac{1}{n+1} - 0\right] = \frac{c}{n+1} \quad \Rightarrow \quad \boxed{c = n+1}$$

$$\text{(b) } P(X > x) = \int_x^1 f(u)\,du = (n+1)\int_x^1 u^n\,du = \left. u^{n+1}\right|_{u=x}^1 = \boxed{1 - x^{n+1}}$$

Exercise 2

$$E[\overline{X}] = E\left[\frac{X_1 + \ldots + X_n}{n}\right] = \frac{1}{n}\left(E[X_1 + \cdots + X_n]\right) \qquad \text{[subst., factor const. times exp.]}$$

$$= \frac{1}{n}\left(E[X_1] + \cdots + E[X_n]\right) = \frac{n\mu}{n} = \underline{\mu} \qquad \text{[exp. sum indep. RVs]}$$

Exercise 3 (a) (1) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^1 (ax + bx^2)\,dx = \frac{a}{2} + \frac{b}{3}$ [int. dens. = 1, eval. int.]

(2) $\displaystyle E[X] = 0.6 = \int_0^1 x(ax + bx^2)\,dx = \frac{a}{3} + \frac{b}{4}$ [problem info., defn. exp., eval. int.]

Then (1) $\Rightarrow a + 2b/3 = 2 \Rightarrow a = 2 - 2b/3$;

substitution in (2) : $(2 - 2b/3)/3 + b/4 = 0.6 \Rightarrow 1/15 = -b/36 \Rightarrow b = -12/5 = -2.4$;

$\Rightarrow a = 2 - 2(-2.4)/3 \Rightarrow a = 18/5 = 3.6$; thus $\boxed{a = 3.6,\ b = -2.4}$

(b) $\displaystyle P\left(X < \frac{1}{2}\right) = \int_0^{1/2} (3.6x - 2.4x^2)\,dx = \left.(1.8x^2 - 0.8x^3)\right|_{x=0}^{1/2} = \boxed{0.35}$ [prob. cont. RV]

(c) $\displaystyle E[X^2] = \int_0^1 (3.6x^3 - 2.4x^4)\,dx = \left.0.9x^4 - 0.48x^5\right|_{x=0}^1 = 0.9 - 0.48 = 0.42$ [E(g(X)), eval. int.]

$\Rightarrow Var(X) = E[X^2] - (E[X])^2 = 0.42 - 0.36 = \boxed{0.06}$ [alternative formula Var(X)]

Exercise 4 1. Let $\underline{\mathbf{X}}$ = Gilles' score, $\underline{\mathbf{Y}}$ = Jacques' score.

2. $X \sim N(170,\, 20^2)$, $Y \sim N(160,\, 15^2)$ [according to problem info]

(a) 1. Let $\mathbf{Y}-\mathbf{X}$ difference in scores J−G

2. $Y - X \sim N(\underline{\mu} = 160 - 170 = \underline{-10},\ \underline{\sigma^2} = (20)^2 + (15)^2 = \underline{625})$ [lin. comb. normal RVs]

3, 4. $P(Y > X) = P(Y - X > 0)$

$$= P\left(\boxed{\frac{(Y-X)-(-10)}{\sqrt{625}}} > \frac{0-(-10)}{\sqrt{625}}\right) \qquad \text{[standardize both sides]}$$

$$= P(Z > 0.4) = 1 - \underbrace{P(Z \leq 0.4)}_{\Phi(0.4)} \qquad \text{[simp., symmetry]}$$

$$= 1 - 0.6554 \Rightarrow \boxed{P(Y > X) \approx 0.34} \qquad \text{[normal table]}$$

(b) 1. Let **X** + **Y** sum of the scores

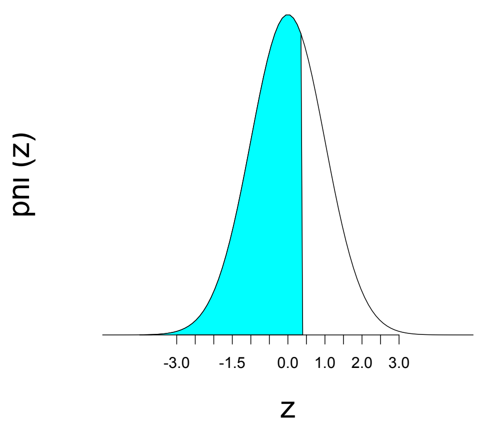
2. $X + Y \sim N(\underline{\mu} = 170 + 160 = \underline{330}, \underline{\sigma^2} = (20)^2 + (15)^2 = \underline{625})$ [lin. comb. normal RVs]

3, 4. $P(X + Y > 350)$

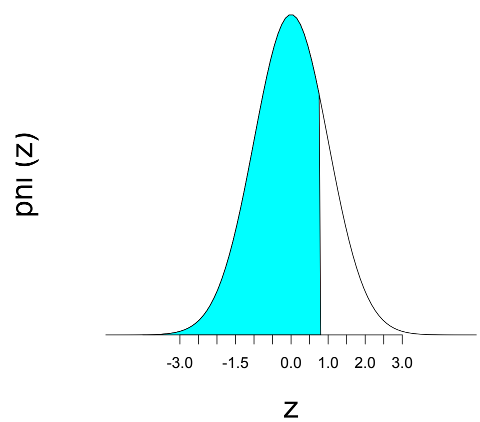
$$= P\left(\boxed{\frac{(X+Y)-330}{\sqrt{625}}} > \frac{350-330}{\sqrt{625}}\right) \qquad \text{[standardize both sides]}$$

$$= P(Z > 0.8) = 1 - \underbrace{P(Z \leq 0.8)}_{\Phi(0.8)} \qquad \text{[simp.]}$$

$$= 1 - 0.7881 \Rightarrow \boxed{P(X + Y > 350) \approx 0.21} \qquad \text{[normal table]}$$



(a) ex. 4a



(b) ex. 4b