# GM – Probabilités et Statistique – Corrections 4

**Please note** : the **reasoning/justifications** for the steps in your solution are also important (not only the final result).

## In class

**Exercise 1** (a) $\frac{X-b}{a-b} = 1$ with probability $p$ or 0 with probability $1-p$, thus $X$ is a Bernoulli($p$) random variable.

(b) The variance of a Bernoulli($p$) RV $= p(1-p) = Var\left(\frac{X-b}{a-b}\right)$ [substitution]

$$= \frac{1}{(a-b)^2} Var(X-b) \quad [Var(cY) = c^2 Var(Y)]$$

$$= \frac{1}{(a-b)^2} Var(X) \quad [Var(Y+c) = Var(Y)]$$

$\implies$ thus $Var(X) = (a-b)^2 p(1-p)$.

**Exercise 2** 1. Let **X** the number of typographical errors on the page

2. $\mathbf{X} \sim Pois(\lambda = 0.2)$ [expected value of number of errors = 0.2 according to the problem]

3, 4. $P(\mathbf{X}=0) = \boxed{e^{-0.2}}$ $(\approx 0.82)$ [Poisson pmf, substitution]

**Exercise 3** Let $E$ the even that the remedy has an effect ; $P(E) = 0.75$, $P(E^c) = 0.25$.

1. Let **X** the number of colds

2. $X \sim Pois(\lambda)$, where $\lambda$ depends on the efficacity of the remedy : if it is effective, $\lambda = 3$ ; otherwise, $\lambda = 5$

3. Probability that the remedy has an effect : $P(E \mid X = 2)$

4. It follows from Bayes' rule that $P(E \mid X=2) = \frac{P(X=2 \mid E)P(E)}{P(X=2 \mid E)P(E) + P(X=2 \mid E^c)P(E^c)}$

$$P(X=2 \mid E) = P(X=2 \mid \lambda=3) = e^{-3}\, 3^2/2! \approx 0.2240 \quad \text{[Poisson pmf]}$$

$$P(X=2 \mid E^c) = P(X=2 \mid \lambda=5) = e^{-5}\, 5^2/2! \approx 0.0842 \quad \text{[Poisson pmf]}$$

$$\Rightarrow P(E \mid X=2) = \frac{0.2240 \times 0.75}{0.2240 \times 0.75 + 0.0842 \times 0.25} \approx \underline{\underline{0.89}} \quad \text{[Bayes' rule, subst.]}$$

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## At home

**Exercise 1** $E[X] = np = 6$ ; $Var(X) = np(1-p) = 2.4$.

Then $6(1-p) = 2.4 \implies (1-p) = 0.4$, thus $p = 0.6$ ; $np = 6 \implies 0.6n = 6 \implies n = 10$.

Then, $\boxed{P(X=5) = \binom{10}{5}(0.6)^5(0.4)^5}$.

**Exercise 2** Steps 1 and 2 are the same as in Exercise 2 above ;

3, 4. $P(\mathbf{X} \geq 2) = 1 - P(X < 2) = 1 - P(X=0) - P(X=1) = 1 - e^{-0.2} - 0.2e^{-0.2}$

$= \boxed{1 - 1.2e^{-0.2}}$ $(\approx 0.018)$ [Poisson pmf, substitution]