

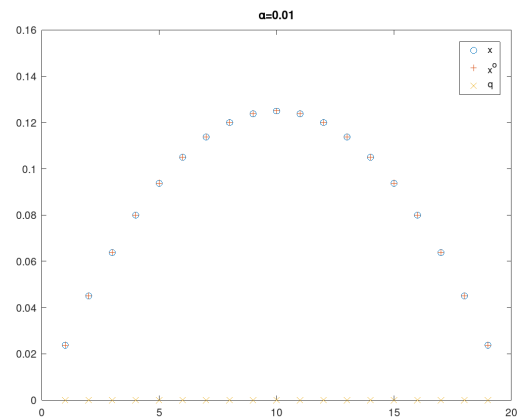
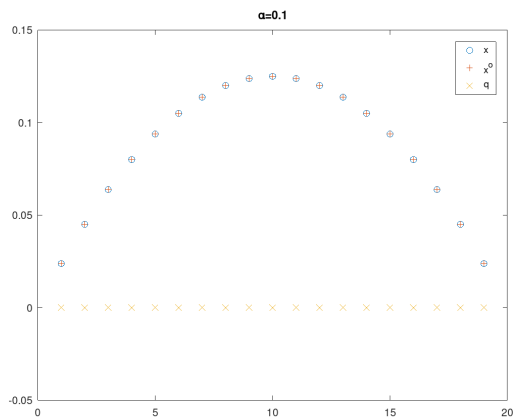
Corrigé 15

Exercice 1

1.a) Le fichier est complété de la manière suivante :

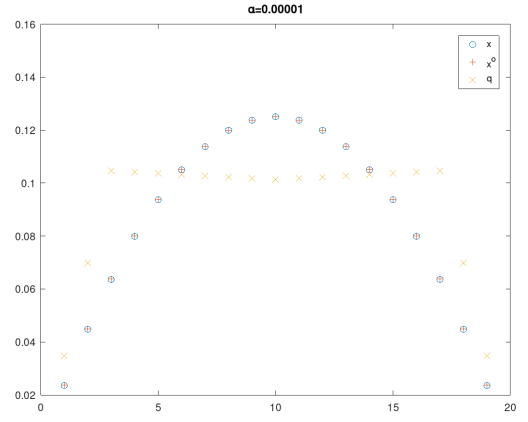
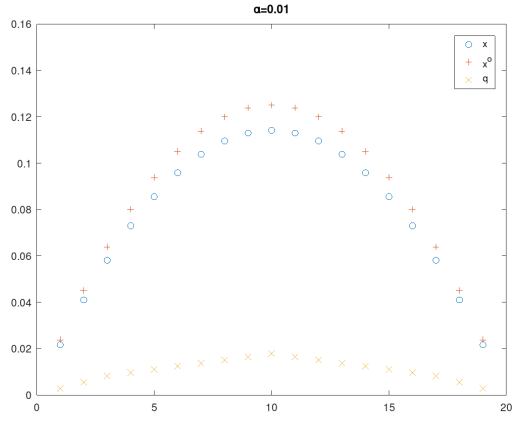
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function err=optcontr(n,m,alpha)
A = (n+1)*(n+1)*(sparse(2:n,1:n-1,-1,n,n)
    + sparse(1:n,1:n,2,n,n) + sparse(1:n-1,2:n,-1,n,n));
matobs = zeros(n,n);
for i=1:m
    index=int8((i-0.5)/m*(n+1));
    matobs(index,index)=1.;
end
mat1 = horzcat(A,-speye(n));
mat2 = horzcat(matobs,alpha*A');
mat = vertcat(mat1,mat2);
xobs=A\ones(n,1);
b = ones(n,1);
rhs = vertcat(b,matobs*xobs);
sol=mat\rhs;
x=sol(1:end/2);
q=sol(end/2+1:end);
plot(x,'o');
hold on;
plot(xobs,'+');
plot(q,'x');
err=max(abs(x-xobs));
end
```

1.b) On vérifie que si on choisit $n = 19$, $m = 3$ et $\vec{b} = \vec{1}$ on a $\vec{x}^* = \vec{x}^o$ et $\vec{q}^* = \vec{0} \forall \alpha > 0$ (par exemple pour $\alpha = 1$ on a $\text{err} = 6.94 \cdot 10^{-17}$).



Si on choisit $\vec{b} = 0.9 \cdot \vec{1}$ on obtient les résultats suivants :

α	err
10^{-1}	0.012314
10^{-2}	0.010851
10^{-3}	0.0048267
10^{-4}	0.00077194
10^{-5}	0.00019896
10^{-6}	0.00012855
10^{-7}	0.00012232
10^{-8}	0.00012168



Exercice 2

2.a) On définit $f : \mathbb{R}^n \rightarrow \mathbb{R}$ par $f(\vec{q}) = \frac{1}{2} \|\vec{q}\|^2$ et $\Omega = \{\vec{q} \in \mathbb{R}^n : B\vec{q} - \vec{c} = \vec{0}\}$.

2.b) Le Lagrangien est défini comme $\mathcal{L}(\vec{q}, \vec{\mu}) = \frac{1}{2} \|\vec{q}\|^2 - \vec{\mu}^T (B\vec{q} - \vec{c})$.

2.c) On obtient $\partial_{\vec{q}} \mathcal{L}(\vec{q}, \vec{\mu}) = \vec{q} - B^T \vec{\mu}$.

2.d) Soit $\vec{q}^* \in \mathbb{R}^n$ et $\vec{\mu}^* \in \mathbb{R}^m$, les conditions KKT sont :

$$\begin{cases} \vec{q}^* - B^T \vec{\mu}^* &= 0, \\ B\vec{q}^* - \vec{c} &= 0. \end{cases}$$