

## GROUP THEORY 2024 - 25, EXERCISE SHEET 11

Since last week's exercise sheet was more challenging and longer than usual, take the opportunity of this shorter one to catch up on any unfinished sheet, correct them, and review the lecture material!

**Exercise 1.** (hard) *To always do in every course!*

Review the lecture and understand/fill in the gaps in the proofs.

**Exercise 2.** (easy)

Show that nilpotent groups have non-trivial centers.

**Exercise 3.** (easy) Let  $G$  be a nilpotent group. If  $H$  is a minimal normal subgroup of  $G$  then show that  $H \subseteq Z(G)$ .

**Exercise 4.** (medium) For what values of  $n$  are the following groups nilpotent:

(1)  $S_n$

(2)  $D_{2n}$

**Exercise 5.** (medium) Prove that a finite group  $G$  is nilpotent if and only if each maximal subgroup of  $G$  is normal in  $G$ .

**Hint:** Exercise 7 of sheet 9.

**Exercise 6.** (hard) Let  $G$  be a finite group and let  $J$  denote the intersection of all maximal subgroups of  $G$ .

(1) Prove that  $J$  is a normal subgroup of  $G$ .

(2) Show that  $J$  is a nilpotent group.