MATH-207(d) Licht – Analysis IV
Additional exercises for exam

Exercice 1. Which of the following functions are complex differentiable over a subset of $\mathbb{C}$? If complex differentiable, compute their complex derivative

$$\begin{aligned} f(x+iy) &= x + x^3 - 3xy^2 + i(3x^2y - y^3), \\ g(x+iy) &= x^2 + 2xyi - y^2, \\ h(x+iy) &= \frac{e^{ix-y} - e^{-ix+y}}{e^{ix-y} + e^{-ix+y}}, \\ y(x+iy) &= x^2 - y^2. \end{aligned}$$

Exercice 2. Consider the curve $\gamma : [0, pi] \to \mathbb{C}$ with $\gamma(t) = 2e^{it}$. Compute the integral

$$\int_\gamma z^3 \, \mathrm{d}z, \tag{1}$$

Exercice 3. Suppose $y : [0, \infty) \to \mathbb{R}$ satisfies

$$y'''(t) + y(t) = e^{-t}, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1$$

Find the Laplace transform $\mathcal{L}(y)$.

Exercice 4. Given the curve

$$\gamma : [0, 2\pi] \to \mathbb{C}, \quad \theta \mapsto 2e^{\theta \mathrm{i}}$$

compute the curve integrals

$$\int_\gamma \frac{(z+1)^3}{z} \, \mathrm{d}z, \quad \int_\gamma \frac{\sin(z)}{z} \, \mathrm{d}z, \quad \int_\gamma \frac{\cosh(z)}{z} \, \mathrm{d}z.$$

Exercice 5. Find the preimage under the Laplace transform of the function

$$F(z) = \frac{3}{(z-1)^2} + \frac{5}{(z-3)^2 - 1}.$$

Exercice 6. Compute the Laplace transform of the following functions:

$$\begin{aligned} f(t) &= t^3 + \sin(10t) + e^{6t}\cos(5t) + \int_0^t e^{-t}\cosh(t)\mathrm{d}t \\ g(t) &= \int_0^t \int_0^u e^{-2v}\sinh(v)\mathrm{d}v \, \mathrm{d}u \\ h(t) &= \int_0^t \sin(s)\cos(t-s)\mathrm{d}s, \\ j(t) &= e^{-2t} - 2te^{-2t} \end{aligned}$$

The functions are understood to be zero for non-positive $t < 0$.

Exercice 7. Compute the Laurent series of the following function f at $z_0 = 0$.

$$f(z) = \frac{\sin(z)}{e^z}.$$