Question 28: *16 points*

Consider the following partial differential equation over the interval $[0, 1]$:

$$\frac{d^2}{d^2t}u(x,t) - 2\frac{d}{dt}u(x,t) = \frac{d^2}{d^2x}u(x,t), \quad 0 < x < 1, \quad t > 0.$$

Suppose we have Dirichlet boundary conditions

$$u(0,t) = u(1,t) = 0, \quad t > 0,$$

and initial data

$$u(x,0) = 0, \quad 0 < x < 1,$$

$$\frac{d}{dt}u(x,0) = 1, \quad 0 < x < 1.$$

Express the solution $u(x,t)$ in terms of a Fourier sine series

$$u(x,t) = \sum_{n=1}^{\infty} b_n(t) \sin(\pi n x).$$