

Question 28: *16 points*

A 2x16 grid of squares representing a 1D lattice. The top row contains 16 white squares labeled 0 through 8. The bottom row contains 16 squares labeled 9 through 16. Squares 9 through 15 are white, and square 16 is black.

Do not write here.

Consider the following partial differential equation over the interval $[0, 1]$:

$$\frac{d^2}{dt^2}u(x,t) - 2\frac{d}{dt}u(x,t) = \frac{d^2}{dx^2}u(x,t), \quad 0 < x < 1, \quad t > 0.$$

Suppose we have Dirichlet boundary conditions

$$u(0, t) = u(1, t) = 0, \quad t > 0,$$

and initial data

$$u(x, 0) = 0, \quad 0 < x < 1,$$

$$\frac{d}{dt}u(x,0) = 1, \quad 0 < x < 1.$$

Express the solution $u(x, t)$ in terms of a Fourier sine series

$$u(x, t) = \sum_{n=1}^{\infty} b_n(t) \sin (\pi n x) .$$