

PRACTICE EXAM

Exercise 0.1.

Revisit the following exercises:

- 1.3
- 2.1, 2.2
- 3.1, 3.2
- 4.1
- 5.1a-c
- 6.1 (easy cases)
- 7.3
- 8.1

Exercise 0.2.

Suppose $y : [0, \infty) \rightarrow \mathbb{R}$ satisfies

$$y'''(t) + y(t) = e^{-t}, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1.$$

Find the Laplace transform $\mathcal{L}(y)$.

Exercise 0.3.

Given the curve

$$\gamma : [0, 2\pi] \rightarrow \mathbb{C}, \quad \theta \mapsto 2e^{\theta i},$$

compute the curve integrals

$$\int_{\gamma} \frac{(z+1)^3}{z} dz, \quad \int_{\gamma} \frac{\sin(z)}{z} dz, \quad \int_{\gamma} \frac{\cosh(z)}{z} dz, .$$

Exercise 0.4.

Find the preimage under the Laplace transform of the function

$$F(z) = \frac{3}{(z-1)^2} + \frac{5}{(z-3)^2 - 1}.$$

Exercise 0.5.

Compute the Laplace transform of the following functions:

$$f(t) = t^3 + \sin(10t) + e^{6t} \cos(5t) + \int_0^t \exp^{-t} \cosh(t) dt,$$

$$g(t) = \int_0^t \int_0^u \exp^{-2v} \sinh(v) dv du,$$

$$h(t) = \int_0^t \sin(s) \cos(t-s) ds, ,$$

$$j(t) = e^{-2t} - 2te^{-2t}.$$

The functions are understood to be zero for non-positive $t < 0$.