## Exam training I

**Exercise 1.** Find a holomorphic function $f\colon \mathbb{C} \to \mathbb{C}$ such that its real part is:

$$u(x, y) = x^2 - y^2 + e^{-x}cosy$$

**Exercise 2.** Using the Cauchy Theorem and the Cauchy integral formula compute the following integrals

(1)
$$\int_\Gamma \frac{z^3 + 2z^2 + 2}{z - 2i} dz \qquad \text{where} \qquad \Gamma = \left\{ z \in \mathbb{C} \mid |z - 2i| = \frac{1}{4} \right\}$$

(2)
$$\int_\Gamma \frac{3z^2 + 2z + sin(z + 1)}{(z - 2)^2} dz \qquad \text{where} \qquad \Gamma = \{ z \in \mathbb{C} \mid |z - 2| = 1 \}$$

**Exercise 3.** For the following functions, compute the Laurent expansion around $z_0$ and determine whether $z_0$ is a regular or singular point; in the latter case, say what is the order of pole at $z_0$. Recall that if $f$ is holomorphic at $z_0$ its Laurent expansion is simply the Taylor expansion.

(1) $f(z) = \frac{z}{1+z^2}$ and $z_0 = 1$
(2) $f(z) = \frac{z^2+2z+1}{1+z}$ and $z_0 = -1$
(3) (Bonus) $f(z) = \frac{z^2+z+1}{z^2-1}$ and $z_0 = 1$

**Exercise 4.** Compute the following real integrals:

(1)
$$\int_0^{2\pi} \frac{cos(\theta)sin(2\theta)}{5 + 3cos(2\theta)} d\theta$$

(2)
$$\int_{-\infty}^{+\infty} \frac{x^2}{1 + x^6} dx$$

**Exercise 5.** With the help of the Table of Laplace transforms, find the Laplace transform of

$$f(t) = e^{-2t}(3cos(6t) - 5sin(6t))$$

**Exercise 6.** For $\lambda \in \mathbb{R}$ and $y_0, y_1$ arbitrary values, find the solutions to to

$$\begin{cases} y"(t) + \lambda y(t) = 0, t > 0 \\ y(0) = y_0 \;\; y'(0) = y_1 \end{cases}$$