Exercice 1.
c.f. livre.

Exercice 2.
c.f. livre.

Exercice 3.
c.f. livre.

Exercice 4.
c.f. livre.

Exercice 5.

- Puisque F est holomorphe, on a, pour $z = x + iy$

$$F'(z) = \frac{\partial u}{\partial x}(x, y) + i\frac{\partial v}{\partial x}(x, y) = \frac{\partial v}{\partial y}(x, y) - i\frac{\partial u}{\partial y}(x, y).$$

Ainsi,

$$\begin{aligned}
\int_\gamma F'(z)\ dz &= \int_a^b F'(\gamma(t)) \cdot \gamma'(t)\ dt \\
&= \int_a^b \left(\frac{\partial u}{\partial x}(\alpha(t), \beta(t)) + i\frac{\partial v}{\partial x}(\alpha(t), \beta(t))\right) \cdot (\alpha'(t) + i\beta'(t))\ dt \\
&= \int_a^b \left(\frac{\partial u}{\partial x}(\alpha(t), \beta(t))\alpha'(t) + \frac{\partial u}{\partial y}(\alpha(t), \beta(t))\beta'(t)\right)\ dt \\
&+ i\int_a^b \left(\frac{\partial v}{\partial y}(\alpha(t), \beta(t))\beta'(t) + \frac{\partial v}{\partial x}(\alpha(t), \beta(t))\alpha'(t)\ \right) dt \\
&= \int_a^b \left(\frac{d}{dt}\left(u(\alpha(t), \beta(t))\right)\right)\ dt + i\int_a^b \left(\frac{d}{dt}\left(v(\alpha(t), \beta(t))\right)\right)\ dt \\
&= u(\alpha(b), \beta(b)) + iv(\alpha(b), \beta(b)) - u(\alpha(a), \beta(a)) - iv(\alpha(a), \beta(a)) \\
&= F(\gamma(b)) - F(\gamma(a)).
\end{aligned}$$

- On pose $D = \mathbb{C}$ et $F(z) = \dfrac{z^3}{3}$. Puisque γ est fermée, on a $\gamma(a) = \gamma(b)$ et donc, par le point précédent,

$$\int_\gamma z^2 = F(\gamma(b)) - F(\gamma(a)) = 0.$$

Noter qu'on aurait également pu appliquer le théorème de Cauchy et aboutir à la même conclusion.