Exercice 1.
On a

$$\nabla\left(p+\frac{1}{2}\rho u\cdot u\right)=\left(\partial_{x_1}p+\rho\left(u_1\partial_{x_1}u_1+u_2\partial_{x_1}u_2\right),\partial_{x_2}p+\rho\left(u_1\partial_{x_2}u_1+u_2\partial_{x_2}u_2\right)\right)$$

et donc

$$\begin{aligned}I=\int_a^b&\left(\left(\partial_{x_1}p+\rho\left(u_1\partial_{x_1}u_1+u_2\partial_{x_1}u_2\right)\right)(\gamma_1(t),\gamma_2(t))\right)\gamma_1'(t)\\&+\left(\left(\partial_{x_2}p+\rho\left(u_1\partial_{x_2}u_1+u_2\partial_{x_2}u_2\right)\right)(\gamma_1(t),\gamma_2(t))\right)\gamma_2'(t)dt.\end{aligned}$$

Puisque

$$\gamma_1'(t)=u_1(\gamma_1(t),\gamma_2(t))\qquad\gamma_2'(t)=u_2(\gamma_1(t),\gamma_2(t)),$$

$$\begin{aligned}I&=\int_a^b\left(\left[\partial_{x_1}p+\rho\left(u_1\partial_{x_1}u_1+u_2\partial_{x_1}u_2\right)\right]u_1+\left[\partial_{x_2}p+\rho\left(u_1\partial_{x_2}u_1+u_2\partial_{x_2}u_2\right)\right]u_2\right)(\gamma_1(t),\gamma_2(t))dt\\&=\int_a^b\big(\underbrace{\left[\partial_{x_1}p+\rho\left(u_1\partial_{x_1}u_1+u_2\partial_{x_2}u_1\right)\right]}_{=0\text{ par }(1)}u_1+\underbrace{\left[\partial_{x_2}p+\rho\left(u_1\partial_{x_1}u_2+u_2\partial_{x_2}u_2\right)\right]}_{=0\text{ par }(2)}u_2\big)(\gamma_1(t),\gamma_2(t))dt\\&=0.\end{aligned}$$