MATH-111(en) FALL 2024
Linear Algebra Annina Iseli

Graded Exercise 3

December 3rd, 2024

Duration: 25–30 min

Rules for hand in:

- You can only hand in one piece of paper (i.e. keep your solution short but complete!)
- In the top right corner of your piece of paper, you write the the two digits of the month you were born as well as your initials. (Example: as I am born in April, I would write: 04 AI).
- Handwriting must be legible (else your solution won't be graded).
- **Hand in before class on Tuesday, December 3rd or December 11th.**

Problem 1

Let $v_1, \ldots, v_n \in \mathbb{R}^n$ be linearly independent vectors.
Let A be a diagonalizable matrix in $\mathbb{R}^{n\times n}$ so that the vectors $v_1, \ldots, v_n$ are eigenvectors of A for the eigenvalues $\alpha_1, \ldots, \alpha_n$ respectively.
Let B be a diagonalizable matrix in $\mathbb{R}^{n\times n}$ so that the vectors $v_1, \ldots, v_n$ are eigenvectors of B for the eigenvalues $\beta_1, \ldots, \beta_n$ respectively.

Prove that $A - B$ is diagonalizable and satisfies $\det(A-B) = (\alpha_1 - \beta_1)\cdots(\alpha_n - \beta_n)$.

Problem 2

Let $A \in \mathbb{R}^{n\times n}$ and let 0 be the $n\times n$ zero matrix.

Show that if A is diagonalizable and $A^2 = 0$, then $A = 0$.

BE AWARE : "Graded" means that we provide you with feedback / corrections on your submission. It does not mean that you will receive a grade. Moreover, this exercise does not count towards your final grade of this course and participation is not mandatory.

For better feedback, this time, we will assign letters to each graded problem:
A = *good solution, only minor mistakes or imperfect (but still clear) notation.*
B = *your solution catches relevant aspects but also has considerable flaws or gaps.*
C = *your solution was mostly wrong and/or there were many substantial mistakes.*