MATH-111(en) FALL 2024
Linear Algebra Annina Iseli

Graded Exercise 2

November 12th, 2024

Duration: approximately 30 min

Rules for hand in:

- You can only hand in one piece of paper (i.e. keep your solution short but complete!)
- In the top right corner of your piece of paper, you write the the two digits of the month you were born as well as your initials. (Example: as I am born in April, I would write: 04 AI).
- Handwriting must be legible (else your solution won't be graded).
- Hand in: Tuesday, Nov 12th, before class starts.

Problem 1

Let $A \in \mathbb{R}^{m\times n}$ be a matrix such that its reduced echelon form has exactly k zero rows. Determine the dimension of $\mathrm{Ker}(A)$ and of $\mathrm{Col}(A)$ in terms of m, n and k.

Problem 2

Let $T : \mathbb{P}_3 \to \mathbb{R}^4$ be a linear transformation defined by

$$T(1) = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \quad T(x) = \begin{pmatrix} -1 \\ 2 \\ 2 \\ 2 \end{pmatrix}, \quad T(x^2) = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 3 \end{pmatrix}, \quad T(x^3) = \begin{pmatrix} -3 \\ 5 \\ 3 \\ 3 \end{pmatrix}$$

(a) Find a basis for $\mathrm{Ker}(T)$.

(b) Determine whether $\begin{pmatrix} 0 \\ 0 \\ 2 \\ -1 \end{pmatrix}$ is in $\mathrm{Ran}(T)$.

BE AWARE : "Graded" means that we provide you with feedback / corrections on your submission. It does not mean that you will receive a grade. Moreover, this exercise does not count towards your final grade of this course and participation is not mandatory.

For better feedback, we will assign letters to each graded problem:
A = *good solution, only minor mistakes or imperfect (but still clear) notation.*
B = *your solution catches relevant aspects but also has considerable flaws or gaps.*
C = *your solution was mostly wrong and/or there were many substantial mistakes or gaps.*