

EXERCISE SHEET 3

Analysis II-MATH-106 (en) EPFL

Spring Semester 2024-2025

March 3, 2025

Exercise 1. Find the equation of the level curve of the function $f(x, y)$ that passes through the given point $p = (p_1, p_2)$.

(a) $f(x, y) = 16 - x^2 - y^2$, $p = (2\sqrt{2}, \sqrt{2})$

(b) $f(x, y) = \sqrt{x^2 - 1}$, $p = (1, 0)$

(c) $f(x, y) = \int_x^y \frac{d\theta}{\sqrt{1-\theta^2}}$, $p = (0, 1)$

(Hint: the set $\{(x, y) \in \mathbb{R}^2 : f(x, y) = f(p_1, p_2)\}$)

Exercise 2. Calculate the following limits if they exist:

(i) $\lim_{(x,y) \rightarrow (2,1)} \frac{x^2 - 3y}{x + 2y^2}$

(ii) $\lim_{(x,y) \rightarrow (0,0)} xy \frac{x^2 - y^2}{x^2 + y^2}$

(iii) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^4}$

Exercise 3. Prove that the following limits don't exist:

(i) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$

(ii) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + (2x - y)^2}$

(iii) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

(iv) $\lim_{(x,y) \rightarrow (0,0)} xy \frac{x^2 - y^2}{(x^2 + y^2)^2}$

Exercise 4. Calculate the following limits if they exist:

(i) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$

(ii) $\lim_{(x,y) \rightarrow (0,0)} \frac{1 - \cos(\sqrt{x^2 + y^2})}{x^2 + y^2}$

(iii) $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^3 - 2y^3}{x^2 + y^2}$

(iv) $\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-\frac{1}{\sqrt{x^2 + y^2}}}}{x^2 + y^2}$

(Hint: use polar coordinates)