

## EXERCISE SHEET 13

Analysis II-MATH-106 (en) EPFL

Spring Semester 2024-2025

May 19, 2025

**Exercise 1.** Solve the following differential equations:

- i)  $xy' + y = \sqrt{x}$ .
- ii)  $(\sin x)y' + (\cos x)y = \sin(x^2)$ .
- iii)  $xy' - 4y = x^5e^x$ .
- iv)  $x(\ln x)y' + y = \frac{1}{\ln^2 x}$ .

**Exercise 2.** Solve the following initial value problems:

- i)  $x^3y' + 3x^2y = \cos x$ ,  $y(\pi) = 0$ .
- ii)  $2xy' + y = 6x$ ,  $x > 0$ ,  $y(4) = 20$ .
- iii)  $(x^2 + 1)y' + 3x(y - 1) = 0$ ,  $y(0) = 2$ .

**Exercise 3.** Solve the following differential equations using the method for Bernoulli differential equations:

- i)  $xyy' - y^2 + x^2 = 0$ ,  $x, y > 0$ .
- ii)  $xy' + y = -xy^2$ .
- iii)  $y' + \frac{2y}{x} = \frac{y^3}{x}$ .

**Exercise 4.** Define the differential equation  $y' = 5\sqrt[5]{y^4}$ .

- a) Verify that  $y : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $y(x) = 0$  for  $x \leq 0$  and  $y(x) = x^5$  for  $x > 0$  is a solution.
- b) Determine all the solutions to this equation.
- c) Give a solution such that both  $y(-3) = -1$  and  $y(2) = 1$ .

**Exercise 5.** Give the general solutions to the following differential equations.

- i)  $y'' = 0$
- ii)  $y'' = 1$