

EXERCISE SHEET 13

Analysis II-MATH-106 (en) EPFL

Spring Semester 2024-2025

May 19, 2025

Exercise 1. Solve the following differential equations:

- i) $xy' + y = \sqrt{x}.$
- ii) $(\sin x)y' + (\cos x)y = \sin(x^2).$
- iii) $xy' - 4y = x^5 e^x.$
- iv) $x(\ln x)y' + y = \frac{1}{\ln^2 x}.$

Exercise 2. Solve the following initial value problems:

- i) $x^3y' + 3x^2y = \cos x, \quad y(\pi) = 0.$
- ii) $2xy' + y = 6x, \quad x > 0, \quad y(4) = 20.$
- iii) $(x^2 + 1)y' + 3x(y - 1) = 0, \quad y(0) = 2.$

Exercise 3. Solve the following differential equations using the method for Bernoulli differential equations:

- i) $xyy' - y^2 + x^2 = 0, \quad x, y > 0.$
- ii) $xy' + y = -xy^2.$
- iii) $y' + \frac{2y}{x} = \frac{y^3}{x}.$

Exercise 4. Define the differential equation $y' = 5\sqrt[5]{y^4}.$

- a) Verify that $y : \mathbb{R} \rightarrow \mathbb{R}$ defined by $y(x) = 0$ for $x \leq 0$ and $y(x) = x^5$ for $x > 0$ is a solution.
- b) Determine all the solutions to this equation.
- c) Give a solution such that both $y(-3) = -1$ and $y(2) = 1$.

Exercise 5. Give the general solutions to the following differential equations.

- i) $y'' = 0$
- ii) $y'' = 1$