EXERCISE SHEET 12

Analysis II-MATH-106 (en) EPFL
Spring Semester 2024-2025
May 12, 2025

Exercise 1. Solve the following differential equations:

i) $y' = -xy$.
ii) $y' + \dfrac{3}{x}y = 0$.
iii) $x(3x+4) - 6(y-1)^2 y' = 0,\ y(0) = 0$.
iv) $yy' - e^{y^2-4x} = 0,\ y(0) = \sqrt{\ln(2)}$.

Exercise 2. Using a separation of variables, solve the differential equation of the Verhulst model for the growth of a population, that is,

$$y'(t) = y(t)(a - by(t)), \qquad a, b > 0\,,$$

for the initial condition $y(0) = y_0$.

Exercise 3. Solve

$$(t^2 + u(t)^2)u'(t) - tu(t) = 0,\ t > 0,\ u(t) > 0,$$

and give the general solution in implicit form.

Exercise 4. Determine the general solution of the differential equation

$$x(x-1)y' - y(y-1) = 0$$

and draw the graph of the maximal solutions for the following initial conditions:

x_0	-1	-1	2	2
y_0	-1	1	4	-4

Exercise 5. Solve the following differential equations:

i) $y'(xy + x^2) + y^2 = 0$.
ii) $y' = \frac{x-y}{x+y}$.
iii) $y' = \frac{x^2+3y^2}{2xy}$.
iv) $xy' - y = \sqrt{x^2+y^2}$.

Exercise 6. Solve the following initial value problems:

i) $y'x^2 + y(x+y) = 0, \quad y(1) = 1$.
ii) $y'(x^2+xy) + 3xy + y^2 = 0, \quad y(2) = 2$.
iii) $(x^2 - 2xy)y' = y^2 - 2xy, \quad y(1) = 2$.