EXERCISE SHEET 10

Analysis II-MATH-106 (en) EPFL
Spring Semester 2024-2025
April 28, 2025

Exercise 1. Calculate the following double integrals:

i) $\iint_D (6x-3y)\dot{x}\dot{y}$, where D is the parallelogram formed by $(2,0),(5,3),(6,7),(3,4)$, by applying the transformation $x=\frac{1}{3}(v-u),\ y=\frac{1}{3}(4v-u)$.

ii) $\iint_D xy^3\dot{x}\dot{y}$, where D is the region bounded by $xy=1,\ xy=3,\ y=2,\ y=6$, by applying the transformation $x=\frac{v}{6u},\ y=2u$.

Exercise 2. i) Evaluate the double integral

$$\iint_D x^3y^3\dot{x}\dot{y},$$

where D is the domain in the first quadrant bounded by the curves $x^2+y^2=5\,,\ x^2+y^2=9\,,$ $x^2-y^2=1$ and $x^2-y^2=4\,.$ Sketch the four curves and the domain of integration D.

ii) Sketch the domain $D=\{(x,y):0\le x\le y\le 4x,\ 1\le xy\le 2\}$ and calculate the double integral

$$\iint_D x^2\,y^2\dot{x}\dot{y}.$$

iii) Evaluate the double integral

$$\iint_D xy^3\dot{x}\dot{y},$$

where

$$D=\left\{(x,y)\in\mathbb{R}^2:\ 1\le x^2+y^2\le 4,\ y\le 0,\ x\le 0\right\}.$$

Exercise 3. Calculate the following double integrals:

i) $\int_0^3\int_{-\sqrt{9-x^2}}^0 e^{x^2+y^2}\dot{y}\dot{x}$.

ii) $\iint_D (4xy-7)\dot{x}\dot{y}$, where $D=\{(x,y)\in\mathbb{R}^2\colon x,y\ge 0,\ x^2+y^2\le 2\}$.

Exercise 4. Calculate the following triple integrals:

i) $\int_0^1\int_0^{z^2}\int_0^{\sqrt{\frac{\pi}{2}}} xy\cos(x^2)\dot{x}\dot{y}\dot{z}$

ii) $\iiint_E 6z^2 \dot{x}\dot{y}\dot{z}$, where E is the region below $4x + y + 2z = 10$ in the first octant.

iii) $\iiint_E yz\dot{x}\dot{y}\dot{z}$, where E is the region bounded by $x = 2y^2 + 2z^2 - 5$ and $x = 1$.

Exercise 5. Calculate the following triple integrals:

i) $\int_0^3 \int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(x + \frac{z}{3}\right) \dot{x}\dot{y}\dot{z}$, using the transformation $u = \frac{2x-y}{2},\ v = \frac{y}{2},\ w = \frac{z}{3}$.

ii) $\iiint_E (x^2y + 3xyz)\dot{x}\dot{y}\dot{z}$, by transforming the region

$$E = \{(x, y, z) \in \mathbb{R}^3 \colon 1 \leq x \leq 2,\ 0 \leq xy \leq 2,\ 0 \leq z \leq 1\}$$

into a rectangular parallelepiped with volume 6.