Analyse II

MT, MX

Série 8

Réponses

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1. **a)** $\dfrac{\partial f}{\partial x}(x,y) = 3x^2y + 6x\,,\qquad \dfrac{\partial f}{\partial y}(x,y) = x^3 + 3y^2\,,$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 6xy + 6\,,\qquad \frac{\partial^2 f}{\partial y \partial x}(x,y) = 3x^2 = \frac{\partial^2 f}{\partial x \partial y}(x,y)\,,\qquad \frac{\partial^2 f}{\partial y^2}(x,y) = 6y\,.$$

b) $\dfrac{\partial g}{\partial x}(x,y) = 2\arcsin(y)\,,\qquad \dfrac{\partial g}{\partial y}(x,y) = \dfrac{2x}{\sqrt{1-y^2}}\,,$

$$\frac{\partial^2 g}{\partial x^2}(x,y) = 0\,,\qquad \frac{\partial^2 g}{\partial y \partial x}(x,y) = \frac{2}{\sqrt{1-y^2}} = \frac{\partial^2 g}{\partial x \partial y}(x,y)\,,\qquad \frac{\partial^2 g}{\partial y^2}(x,y) = \frac{2xy}{(1-y^2)^{3/2}}\,.$$

c) $\dfrac{\partial h}{\partial x}(x,y) = \dfrac{1}{y} - \dfrac{y}{x^2}\,,\qquad \dfrac{\partial h}{\partial y}(x,y) = \dfrac{1}{x} - \dfrac{x}{y^2}\,,$

$$\frac{\partial^2 h}{\partial x^2}(x,y) = \frac{2y}{x^3}\,,\qquad \frac{\partial^2 h}{\partial y \partial x}(x,y) = -\frac{1}{x^2} - \frac{1}{y^2} = \frac{\partial^2 h}{\partial x \partial y}(x,y)\,,\qquad \frac{\partial^2 h}{\partial y^2}(x,y) = \frac{2x}{y^3}\,.$$

2. **a)** $\dfrac{\partial f}{\partial x}(0,0) = 0$

$\dfrac{\partial f}{\partial y}(0,0) = 0$

b) $\dfrac{\partial^2 f}{\partial x \partial y}(0,0) = 1$

$\dfrac{\partial^2 f}{\partial y \partial x}(0,0) = -1$

3. $2x - 2y + z = 0$

4. **a)** $3x + y - 10z + 10 = 0$

b) $L(x,y) = 2 + \frac{3}{10}(x-2) + \frac{1}{10}(y-4)$

c) $L(1.9, 4.2) = \frac{199}{100} = 1.99$

5. $x - 2y + z - 1 = 0$

6. f est différentiable en $(0,0)$

7. **a)** V **b)** F **c)** F **d)** V **d)** F

8. **a)**

$$\frac{\partial f}{\partial x}(x,y) = 8x\sin(x+y) + 4x^2\cos(x+y)$$

$$\frac{\partial f}{\partial y}(x,y) = 4x^2\cos(x+y)$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = (8-4x^2)\sin(x+y) + 16x\cos(x+y)$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = 8x\cos(x+y) - 4x^2\sin(x+y) = \frac{\partial^2 f}{\partial y \partial x}(x,y)$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = -4x^2\sin(x+y)$$

b) $\dfrac{\partial g}{\partial x}(x,y) = \dfrac{y}{1+x^2y^2}$

$\dfrac{\partial g}{\partial y}(x,y) = \dfrac{x}{1+x^2y^2}$

$\dfrac{\partial^2 g}{\partial x^2}(x,y) = -\dfrac{2xy^3}{(1+x^2y^2)^2}$

$\dfrac{\partial^2 g}{\partial x\partial y}(x,y) = \dfrac{1-x^2y^2}{(1+x^2y^2)^2} = \dfrac{\partial^2 g}{\partial y\partial x}(x,y)$

$\dfrac{\partial^2 g}{\partial y^2}(x,y) = -\dfrac{2x^3y}{(1+x^2y^2)^2}$

c) $\dfrac{\partial h}{\partial x}(x,y) = 8(xy+y^2)y = 8(xy^2+y^3)$

$\dfrac{\partial h}{\partial y}(x,y) = 8(xy+y^2)(x+2y) = 8(x^2y+3xy^2+2y^3)$

$\dfrac{\partial^2 h}{\partial x^2}(x,y) = 8y^2$

$\dfrac{\partial^2 h}{\partial x\partial y}(x,y) = 8(2xy+3y^2) = \dfrac{\partial^2 h}{\partial y\partial x}(x,y)$

$\dfrac{\partial^2 h}{\partial y^2}(x,y) = 8(x^2+6xy+6y^2)$

d) $\dfrac{\partial f}{\partial x}(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$, $\quad \dfrac{\partial f}{\partial y}(x,y) = \dfrac{y}{\sqrt{x^2+y^2}}$

$\dfrac{\partial^2 f}{\partial x^2}(x,y) = \dfrac{y^2}{(x^2+y^2)^{3/2}}$, $\quad \dfrac{\partial^2 f}{\partial y^2}(x,y) = \dfrac{x^2}{(x^2+y^2)^{3/2}}$

$\dfrac{\partial^2 f}{\partial x\partial y}(x,y) = -\dfrac{xy}{(x^2+y^2)^{3/2}} = \dfrac{\partial^2 f}{\partial y\partial x}(x,y)$

e) $\dfrac{\partial g}{\partial x}(x,y) = -\dfrac{x}{(x^2+y^2)^{3/2}}$, $\quad \dfrac{\partial g}{\partial y}(x,y) = -\dfrac{y}{(x^2+y^2)^{3/2}}$

$\dfrac{\partial^2 g}{\partial x^2}(x,y) = \dfrac{2x^2-y^2}{(x^2+y^2)^{5/2}}$, $\quad \dfrac{\partial^2 g}{\partial y^2}(x,y) = \dfrac{2y^2-x^2}{(x^2+y^2)^{5/2}}$

$\dfrac{\partial^2 g}{\partial x\partial y}(x,y) = \dfrac{3xy}{(x^2+y^2)^{5/2}} = \dfrac{\partial^2 g}{\partial y\partial x}(x,y)$

f) $\dfrac{\partial h}{\partial x}(x,y) = \tan(xy) + (x+y)y\big[1+\tan^2(xy)\big]$, $\quad \dfrac{\partial h}{\partial y}(x,y) = \tan(xy) + (x+y)x\big[1+\tan^2(xy)\big]$

$\dfrac{\partial^2 h}{\partial x^2}(x,y) = 2y\big[1+\tan^2(xy)\big]\big[1+(x+y)y\tan(xy)\big]$

$\dfrac{\partial^2 h}{\partial y^2}(x,y) = 2x\big[1+\tan^2(xy)\big]\big[1+(x+y)x\tan(xy)\big]$

$\dfrac{\partial^2 h}{\partial x\partial y}(x,y) = 2(x+y)\big[1+\tan^2(xy)\big]\big[1+xy\tan(xy)\big] = \dfrac{\partial^2 h}{\partial y\partial x}(x,y)$

9.
$$\begin{cases} \dfrac{\partial p}{\partial V}(V,T) = -\dfrac{RT}{(V-b)^2} + \dfrac{2a}{V^3} \\ \dfrac{\partial p}{\partial T}(V,T) = \dfrac{R}{V-b} \end{cases}$$

$$\begin{cases} \dfrac{\partial^2 p}{\partial V^2}(V,T) = \dfrac{2RT}{(V-b)^3} - \dfrac{6a}{V^4} \\ \dfrac{\partial^2 p}{\partial V \partial T}(V,T) = -\dfrac{R}{(V-b)^2} = \dfrac{\partial^2 p}{\partial T \partial V}(V,T) \\ \dfrac{\partial^2 p}{\partial T^2}(V,T) = 0 \end{cases}$$

10. **a)**
$$\frac{\partial f}{\partial x}(x,y,z) = 3x^2y^2z$$
$$\frac{\partial f}{\partial y}(x,y,z) = 2x^3yz - 2yz^3$$
$$\frac{\partial f}{\partial z}(x,y,z) = x^3y^2 - 3y^2z^2$$

b)
$$\frac{\partial g}{\partial x}(x,y,z) = y\cos(xy)$$
$$\frac{\partial g}{\partial y}(x,y,z) = x\cos(xy)$$
$$\frac{\partial g}{\partial z}(x,y,z) = -2z$$

c)
$$\frac{\partial h}{\partial x}(x,y,z) = -\frac{x}{(x^2+y^2+z^2)^{3/2}}$$
$$\frac{\partial h}{\partial y}(x,y,z) = -\frac{y}{(x^2+y^2+z^2)^{3/2}}$$
$$\frac{\partial h}{\partial z}(x,y,z) = -\frac{z}{(x^2+y^2+z^2)^{3/2}}$$

11. $4x + 4y + 9z = 25$

12. L'équation du plan tangent est $z = \dfrac{2}{x_0 y_0}\left(3 - \dfrac{x}{x_0} - \dfrac{y}{y_0}\right)$.

13. 3.06