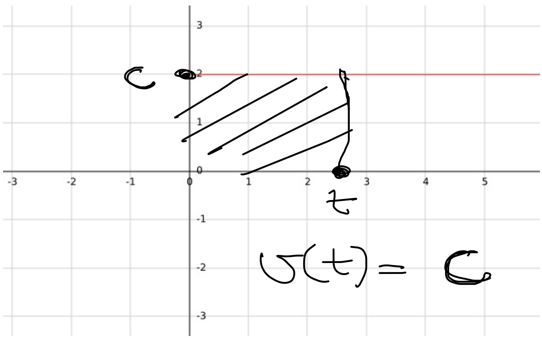
ANALYSIS 1
(ENGLISH)
LECTURE 24

INTEGRATION

$v(t)$ = velocity of a particle at time t

$p(t) = ?$ $p(0) = 0$

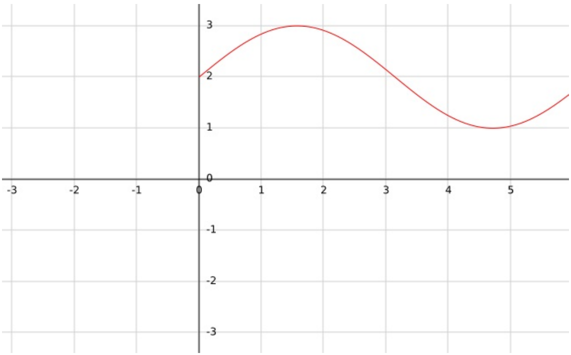


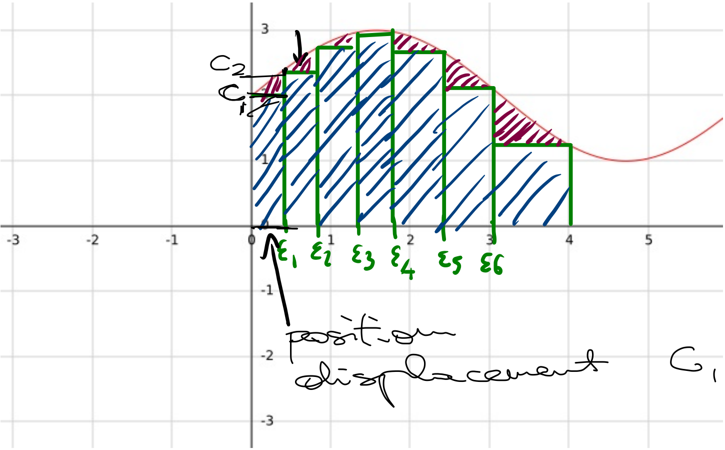
$p(t) = c \cdot t$

INTEGRATION

$v(t) =$ velocity of a particle at time t

$p(t) = ?$ $\qquad p(0) = 0$



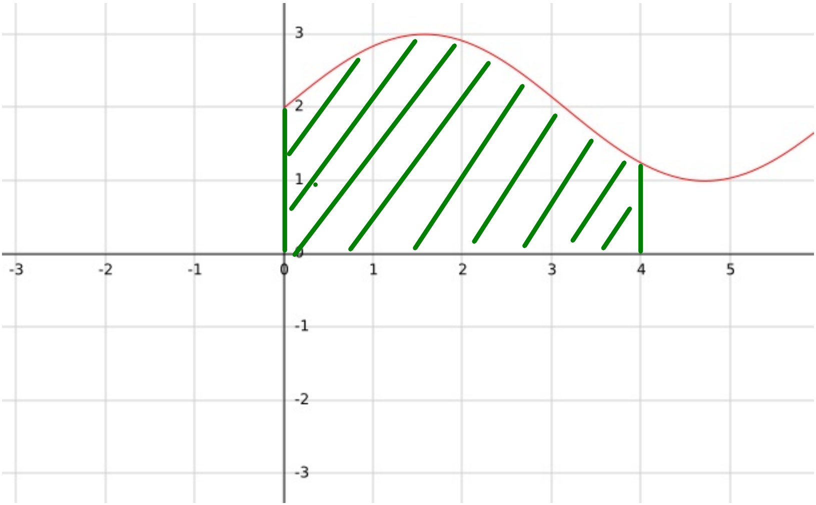


$$C_1 \cdot \varepsilon_1 + C_2(\varepsilon_2 - \varepsilon_1) +$$

$$+ C_{i+1}(\varepsilon_{i+1} - \varepsilon_i)$$

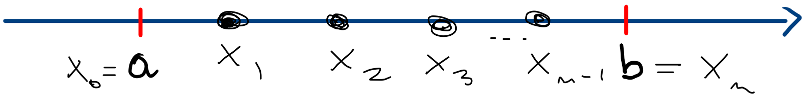
Q: ① HOW TO GENERALIZE THIS

② WHAT ARE WE DOING?



$$[a,b] \subseteq \mathbb{R}$$

σ = PARTITION OF $[a,b]$ =



$$\sigma = \{\underset{a}{\underset{\|}{x_0}} < x_1 < x_2 < \dots < x_{n-1} < x_n = b\}$$

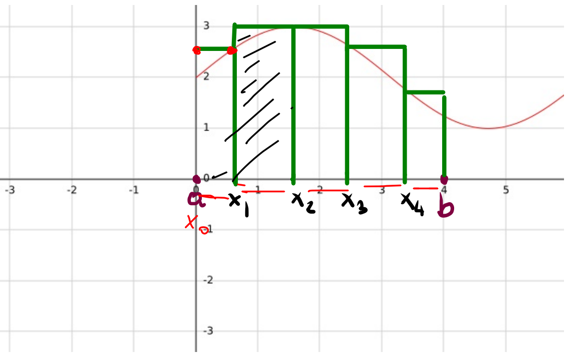
MESH(σ) = $\max |x_{i+1} - x_i|$

$f : [a,b] \longrightarrow \mathbb{R}$,

FIX σ = PARTITION OF $[a,b]$ =

$$\{x_0 = a < x_1 < x_2 < x_3 < \dots < x_n = b\}$$

UPPER DARBOUX SUM of σ:



OVER $[x_i, x_{i+1}]$ WE TAKE THE CONST. APPROX. GIVEN BY

$$\sup_{[x_i, x_{i+1}]} f \geq f(x) \quad \ni \forall x$$

$f : [a,b] \to \mathbb{R}$,

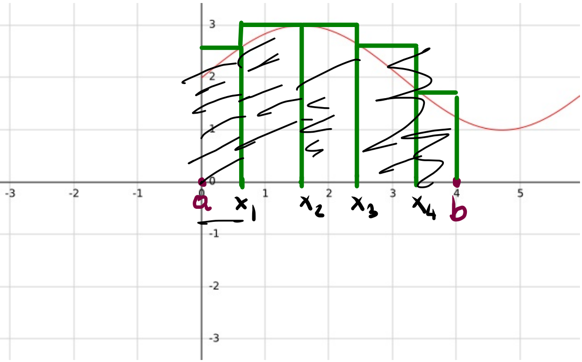
FIX σ = PARTITION OF $[a,b]$ =

$\{x_0 = a < x_1 < x_2 < x_3 < \ldots < x_n = b\}$

UPPER DARBOUX SUM of σ:

$$\overline{S}_\sigma = \sum_{i=0}^{n-1} (x_{i+1} - x_i) \cdot f_i$$

$$f_i = \sup_{[x_i, x_{i+1}]} f$$

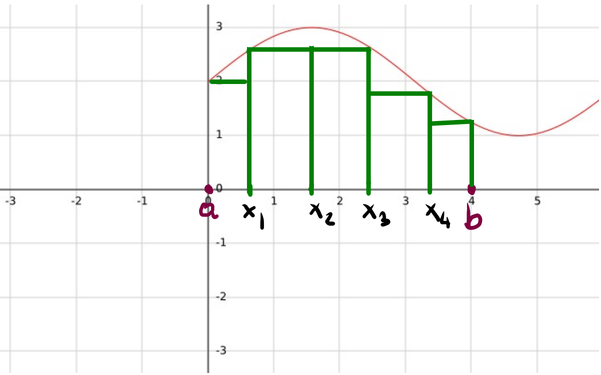


$f : [a,b] \longrightarrow \mathbb{R}$,

FIX σ = PARTITION OF $[a,b]$ =

$\{ x_0 = a < x_1 < x_2 < x_3 < \dots < x_n = b \}$

LOWER DARBOUX SUM of σ:



$\inf_{[x_i, x_{i+1}]} f =: f_i($

$f : [a,b] \longrightarrow \mathbb{R}$,

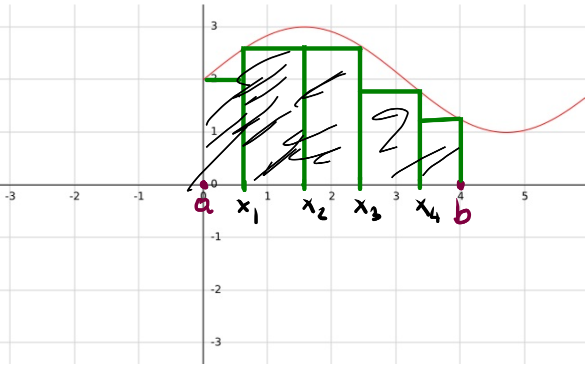
FIX σ = PARTITION OF $[a,b]$ =

$\{x_0 = a < x_1 < x_2 < x_3 < \dots < x_n = b\}$

LOWER DARBOUX SUM of σ :

$$\underline{S}_\sigma = \sum_{i=0}^{n-1} (x_{i+1} - x_i)\, \underline{f}_i$$

$$\underline{f}_i = \inf_{[x_i, x_{i+1}]} f$$



$$f : [a, b] \to \mathbb{R}$$

$$\sigma \to \underline{S}_\sigma \qquad \text{LOWER DARBOUX SUM OF } f \text{ WRT } \sigma$$

$$\sigma \to \overline{S}_\sigma \qquad \text{UPPER DARBOUX SUM.}$$

σ PARTITION of $[a,b]$

$$\underline{S}_\sigma \leq \overline{S}_\sigma$$

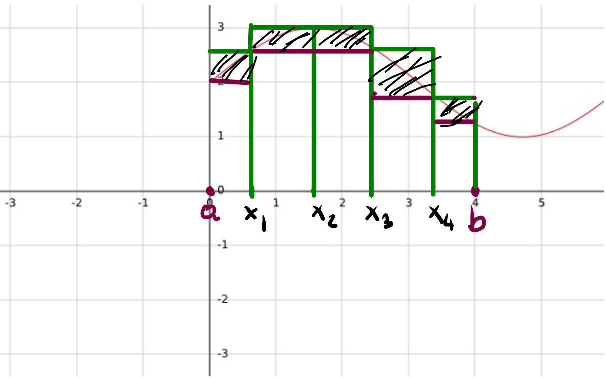
$$\Uparrow$$

$$\sup_{[x_i, x_{i+1}]} f \geq \inf_{[x_i, x_{i+1}]} f$$

$f : [a, b] \to \mathbb{R}$, σ PARTITION OF $[a, b]$

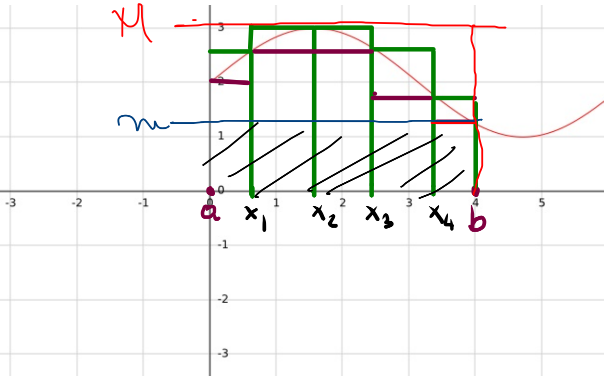
$$\underline{S}_\sigma \leq \overline{S}_\sigma$$

$$\overline{S}_\sigma - \underline{S}_\sigma = ////$$



$f : [a, b] \to \mathbb{R}$, σ PARTITION OF $[a, b]$

$$m \cdot (b-a) \leq \underline{S}_\sigma \leq \overline{S}_\sigma \leq M(b-a)$$



$$M \doteq \sup_{[a,b]} f$$

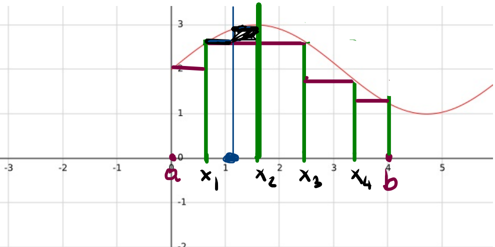
$$m \doteq \inf_{[a,b]} f$$

$$f : [a, b] \to \mathbb{R}$$

$$\begin{tikzcd} & \underline{S}_\sigma \\ \sigma \arrow[ur] \arrow[dr] & \\ & \overline{S}_\sigma \end{tikzcd}$$

$$\underset{\displaystyle\inf_{[a,b]} f}{\underset{\parallel}{m}}\,(b-a) \leq \underline{S}_\sigma \leq \overline{S}_\sigma \leq \underset{\displaystyle\sup_{[a,b]} f}{\underset{\parallel}{M}}\,(b-a)$$

$f : [a, b] \to \mathbb{R}$

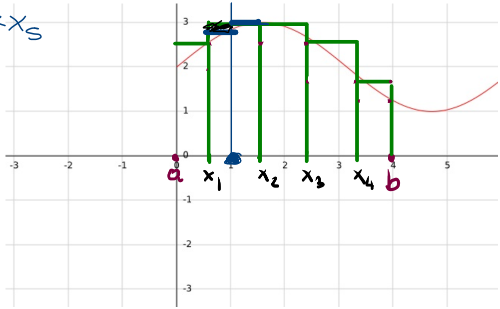


IF WE REFINE THE PARTITION σ W/ A NEW PARTITION σ'

$\implies \overline{S}_{\sigma'} < \overline{S}_{\sigma}$

$x_0 < x_1 < \bullet < x_2 < x_3 < x_4 < x_5$

$\underline{S}_{\sigma'} > \underline{S}_{\sigma}$



DEF $f : [a,b] \to \mathbb{R}$

- LOWER DARBOUX INTEGRAL

$$\underline{S} \doteq \sup \left\{ \underline{S}_\sigma \;\middle|\; \sigma \text{ is A PARTITION of } [a,b] \right\}$$

- UPPER DARBOUX INTEGRAL

$$\overline{S} \doteq \inf \left\{ \overline{S}_\sigma \;\middle|\; \sigma \text{ is A PARTITION of } [a,b] \right\}$$

- f is INTEGRABLE if

$$\overline{S} = \underline{S}$$

<u>DEF</u> $f : [a,b] \to \mathbb{R}$

f IS INTEGRABLE (RIEMANN INTEGR.) IF

$$\overline{S} = \underline{S}$$

DEF $f : [a,b] \to \mathbb{R}$

f IS INTEGRABLE (RIEMANN INTEGR.) IF

$$\overline{S} = \underline{S}$$

RMK $\forall \sigma$ PARTITION OF $[a,b]$

$$\underline{S}_\sigma \le \underline{S} \le \overline{S} \le \overline{S}_\sigma$$

$\underline{S} \doteq \sup\limits_{\sigma \text{ partitions}} \underline{S}_\sigma$ $\qquad \overline{S} \doteq \inf \overline{S}_\sigma$

DEF $f : [a,b] \to \mathbb{R}$

f IS INTEGRABLE IF

(RIEMANN INTEGR.)

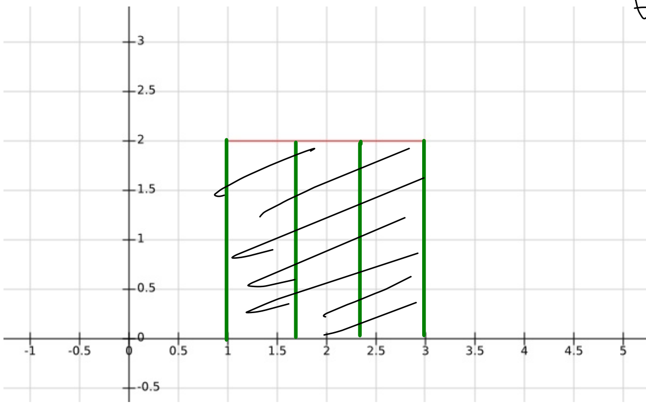
$$\overline{S} = \underline{S}$$

NOTATION: $\displaystyle\int_a^b f(x)\,\underline{\underline{dx}} = \underline{S} = \overline{S}$

$$\int f(x)\,dx$$

EXAMPLE

$f:[1,3]\to\mathbb{R} \quad f(x)=2$



$\forall$ 6 partition of $[a,b]$

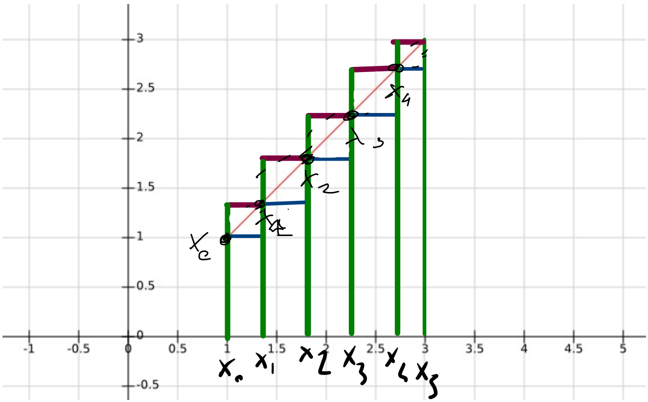
$\overline{S}_6 = \underline{S}_6$
$\quad\quad\quad\;\; \parallel$
$\quad\quad\quad\;\; 4$

$$\int_1^3 f(x)\,dx = 2\cdot(3-1) = 4$$

EXAMPLE

$f(x) = x$, $\quad G = \{1 < x_1 < x_2 < \cdots < x_n = 3\}$

STR. INCR

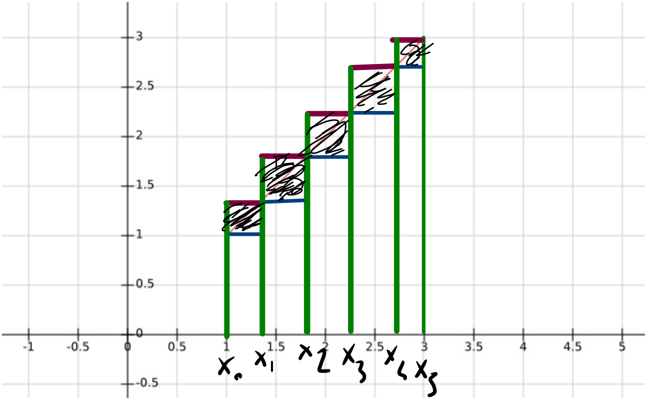


$$\underline{S}_6 = \sum_{i=0}^{n-1} x_i \underline{(x_{i+1} - x_i)}$$

$$\overline{S}_6 = \sum_{i=0}^{n-1} x_{i+1} \underline{(x_{i+1} - x_i)}$$

EXAMPLE

$f(x) = x$, $6 = \{1 < x_1 < x_2 < \cdots < x_n = 3\}$



$\underline{S}_6 =$

$$\sum_{i=0}^{n-1} \overline{x_i}\,(x_{i+1} - x_i)$$

$\overline{S}_6 =$

$$\sum_{i=0}^{n-1} x_{i+1}\,(x_{i+1} - x_i)$$

$$\overline{S}_6 - \underline{S}_6 = //// = \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2$$

TAKE $\underline{\underline{\sigma_n}} = \{x_0 = 1 < x_1 = 1 + \frac{2}{n} <$

$< x_2 = 1 + 2 \cdot \frac{2}{n} <$

$\vdots$

$< x_i = 1 + i \cdot \frac{2}{n} <$

$\vdots$

$< x_n = 1 + n \cdot \frac{2}{n} = 3 \}$

$$\Longrightarrow \underline{S_{\sigma_n}} = \sum_{i=0}^{n-1} \overbrace{\left(1 + \frac{2i}{n}\right)}^{x_i} \overbrace{\left(\frac{2}{n}\right)}^{x_{i+1} - x_i} =$$

$$= \sum_{i=0}^{n-1} \frac{2}{n} + \sum_{i=0}^{n-1} \left(\frac{2}{n}\right)^2 \cdot i =$$

$$= \overset{n}{2} + \underbrace{\left(\frac{2}{n}\right)^{\cancel{2}}}_{2} \left(\frac{\cancel{n}(n-1)}{\cancel{2}}\right) \xrightarrow{n \to \infty} 4$$

TAKE $\sigma_n = \{ x_0 = 1 < x_1 = 1 + \frac{2}{n} <$

$< x_2 = 1 + 2 \cdot \frac{2}{n} <$

$\vdots$

$< x_i = 1 + i \cdot \frac{2}{n} <$

$\vdots$

$< x_n = 1 + n \cdot \frac{2}{n} = 3 \}$

$$\Longrightarrow \overline{S}_{\sigma_n} = \sum_{i=0}^{n-1} \overset{x_{i+1}}{\left(1 + \frac{2(i+1)}{n}\right)} \overset{x_{i+1} - x_i}{\left(\frac{2}{n}\right)} =$$

$$= \sum_{i=0}^{n-1} \frac{2}{n} + \sum_{i=0}^{n-1} \left(\frac{2}{n}\right)^2 \cdot (i+1)$$

$$= 2 + \left(\frac{2}{n}\right)^{\cancel{2}} \left(\frac{\cancel{n}(n+1)}{\cancel{2}} - 1\right) \xrightarrow{n \to \infty} 4$$

PARTIT'S: σ_n , $n \in \mathbb{N}$

$$\underline{S}_{\sigma_n} \leq \underline{S} \leq \overline{S} \leq \overline{S}_{\sigma_n} \quad \forall n$$

$$\underline{S}_{\sigma_n} = 2 + \frac{2}{n}(n-1) \xrightarrow{n \to \infty} 4$$

$$\overline{S}_{\sigma_n} = 2 + \left(\frac{2}{n}\right)^2 \left(\frac{n(n+1)}{2} - 1\right) \xrightarrow{n \to \infty} 4$$

$$\Downarrow$$

$$\underline{S} = \overline{S} = 4$$

$$\underline{S} = \int_1^3 x \, dx = 4$$

Q: WHAT FUNCTIONS ARE INTEGRABLE?

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PROP $f : [a,b] \to \mathbb{R}$ IS C^0

closed bounded interval

$\implies f$ IS INTEGRABLE

$$\lim_{x \to a^+} f(x) = \cancel{f(a)}$$

$$\lim_{x \to b^-} f(x) = f(b)$$

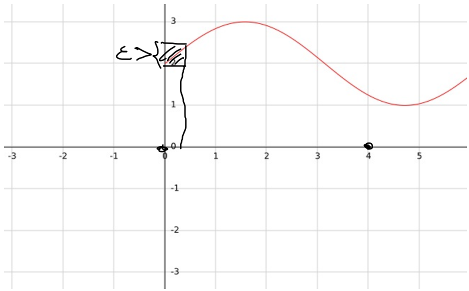
Q: WHAT FUNCTIONS ARE INTEGRABLE?

PROP $f : [a,b] \to \mathbb{R}$ IS C^0

$\implies f$ IS INTEGRABLE

PROOF

$f : [a,b] \to \mathbb{R}$ IS $C^0 \implies f$ UNIF. C^0



$\forall \varepsilon > 0, \exists \delta_\varepsilon > 0 \quad > 0$

S.T.

IF $|x-y| < \delta_\varepsilon$

$\Downarrow$

$|f(x) - f(y)| < \varepsilon \quad (y)| < \varepsilon$

ALGEBRAIC PROPERTIES

f IS INTEGRABLE ON $[a,b]$

& IS INTEGRABLE ON $[b,c]$

$\Downarrow$

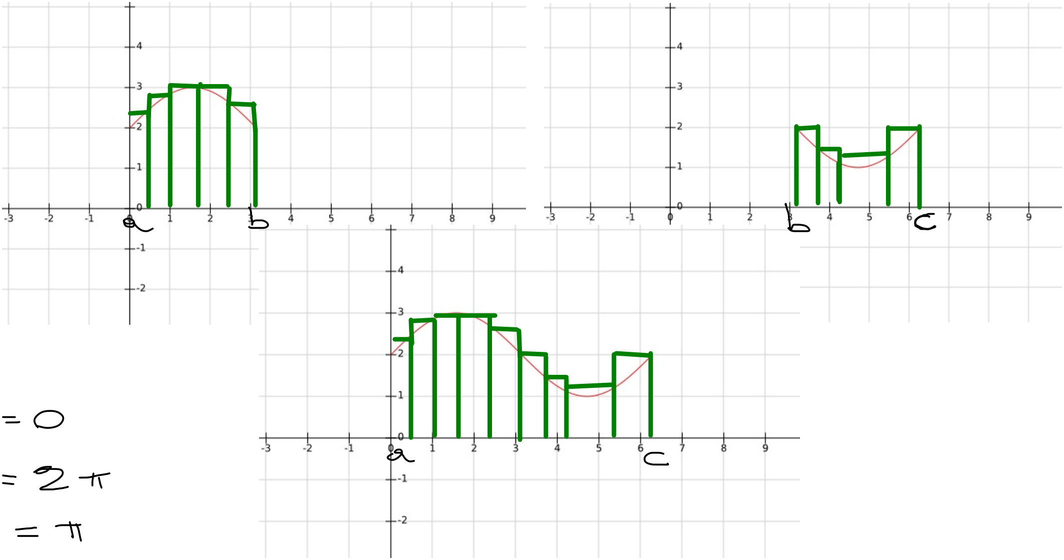
$[a,b] \cup [b,c]$

$\quad\quad //$

f IS INTEGRABLE ON $[a,c]$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



$a = 0$

$c = 2\pi$

$b = \pi$

CONVENTION

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

ALGEBRAIC PROPERTIES

PROP $g, f : [a,b] \to \mathbb{R}$ INTEGRABLE

$\alpha, \beta \in \mathbb{R}$

① $\displaystyle\int_a^b \underbrace{(\alpha f + \beta g)(x)}_{\alpha \cdot f(x) + \beta \cdot g(x)} dx = \alpha \cdot \int_a^b f(x)\,dx + \beta \cdot \int_a^b g(x)\,dx$

② IF $\underline{f \leq g}$ $\Rightarrow \displaystyle\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$

$f(x) \leq g(x) \quad \forall x \in [a,b]$

③ $\displaystyle\int_a^b |f(x)|\,dx \geq \left| \int_a^b f(x)\,dx \right|$

ANTIDERIVATIVES

DEF $f : [a,b] \to \mathbb{R}$, $f \in C^0([a,b])$

$G : [a,b] \to \mathbb{R}$ IS AN ANTI-DERIVATIVE IF OF f

1. G IS C^0 on $[a,b]$
2. G IS DIFF. on $]a,b[$
3. $G'(x) = f(x)$ $\forall x \in]a,b[$

f	e^x	$\cos(x)$	$\sin(x)$	$1/x$	$x^{n \in \mathbb{N}}$
G	e^x	$\sin(x)$	$-\cos(x)$	$\log(\lvert x \rvert)$	$\frac{x^{n+1}}{n+1}$
↑					

ANTIDER. of f

$$G'(x) = f(x)$$

$$\left(\frac{x^{n+1}}{n+1}\right)' = \frac{1}{\cancel{n+1}} \cdot \cancel{(n+1)}\, x^n$$

NOTATION: $\int f(x)\,dx = G$

$f = e^x \qquad G(x) = e^x + C$

IF $f : [a, b] \to \mathbb{R}$ C^0

& G IS AN ANTI-DERIV.

$\Downarrow$

$\tilde{G}(x) \doteq G(x) + C$ IS AN ANTI-DERIV.

1. G C^0 on $[a, b] \Rightarrow \tilde{G}$ IS C^0 on $[a, b]$
2. G DIFF on $]a, b[\Rightarrow \tilde{G}$ IS DIFF on $]a, b[$
3. $\underbrace{\tilde{G}'(x)}_{= G'(x)} = f(x)$ $\forall x \in (a, b)$

IF $f : [a, b] \to \mathbb{R}$ C^0

& $G, \tilde{G}$ ARE ANTIDERIV. FOR f

$$g(x) := G(x) - \tilde{G}(x) = C \leftarrow \text{constant.}$$

$$g'(x) = \underbrace{G'(x)}_{= f(x)} - \underbrace{\tilde{G}'(x)}_{= f(x)} = 0 \quad \forall x \in]a, b[$$

$$g' = 0 \text{ on } (a, b) \overset{?}{\Longrightarrow} g = C$$

THM IF $g : [a,b] \to \mathbb{R}$ is C^0 & $g'(x) = 0$ $\forall x \in (a,b)$
THEN g is CONSTANT.

Pf ASSUME g IS NOT CONST.

THEN $R(g) = [m, M]$. $m < M$

$$\Rightarrow \exists\, c, d \in\,]a,b[\text{ s.t } g(c) \neq g(d)$$

MVT $0 \neq \boxed{\dfrac{g(c) - g(d)}{(c-d)}} \overset{MVT}{=} g'(h)$, $h \in\,]c,d[\,\subseteq\,]a,b[$

$g'(h) = 0$

$\Rightarrow\Leftarrow$

THM [FUNDAMENTAL THM OF CALCULUS]

$f : [a,b] \to \mathbb{R}$. C^0

THEN $G(x) = \int_a^x f(t)\,dt$ IS AN ANTI-DERIVATIVE OF f

$G(a) = \int_a^a f\,dt$
"
0

$G(x)$ = value of the area of the shaded region of f over $[a, x]$

THM [FUNDAMENTAL THM OF CALCULUS, 2nd EDITION]

$f : [a,b] \longrightarrow \mathbb{R}$. C^0

ASSUME $G : [a,b] \longrightarrow \mathbb{R}$ IS AN ANTI-DERIVATIVE OF f

THEN $\int_a^b f(x)\,dx = G(b) - G(a)$

f	e^x	$\cos(x)$	$\sin(x)$	$1/x$	x^n
G	e^x	$\sin(x)$	$-\cos(x)$	$\log(\lvert x\rvert)$	$\frac{x^{n+1}}{n+1}$

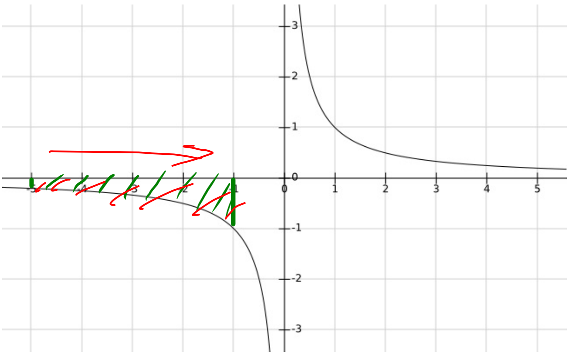
$$\int_a^b \overset{f(x)}{\overset{\shortparallel}{e^x}}\,dx \underset{\substack{\text{FTC}\\ \text{2nd ed.}}}{=} e^b - e^a$$

f	e^x	$\cos(x)$	$\sin(x)$	$1/x$	x^n
G	e^x	$\sin(x)$	$-\cos(x)$	$\log(\lvert x\rvert)$	$\dfrac{x^{n+1}}{n+1}$

$$\int_{-5}^{-1} \frac{1}{t}\,dt = \cancel{\log(\lvert -1\rvert)} - \log(\lvert -5\rvert)$$

$$= -\log(5) < 0$$

$$= \log\left(\frac{1}{5}\right)$$



$$\int_a^b e^{\lambda x}\, dx \underset{\text{FTC 2nd ed}}{=} \frac{1}{\lambda}\left(e^{\lambda \cdot a} - e^{\lambda \cdot b}\right)$$

$G = ?$ for $f(x) = e^{\lambda x}$

$$\left(e^{\lambda x}\right)' = \lambda \cdot e^{\lambda x}$$

$$\left(\frac{e^{\lambda x}}{\lambda}\right)' = \frac{1}{\lambda} \cdot \left(e^{\lambda x}\right)' = e^{\lambda x}$$

ANTID. for $f(x) = e^{\lambda x}$

$$G(x) = \frac{e^{\lambda x}}{\lambda}$$

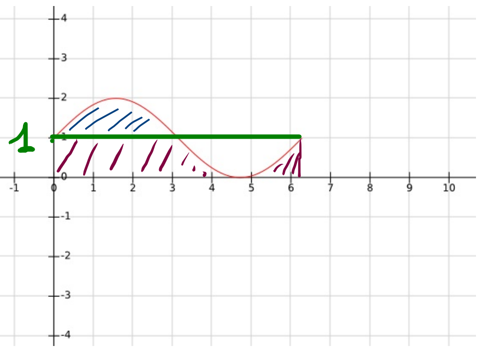
<u>THM</u> [MEAN VALUE THM FOR INTEGRALS]

$$f:[a,b]\to\mathbb{R} \quad C^0 \Rightarrow \exists\, c\in[a,b]$$

$$\text{S.T. } \int_a^b f(x)\,dx = f(c)\cdot(a-b)$$

$f:[0,2\pi]\to\mathbb{R}$

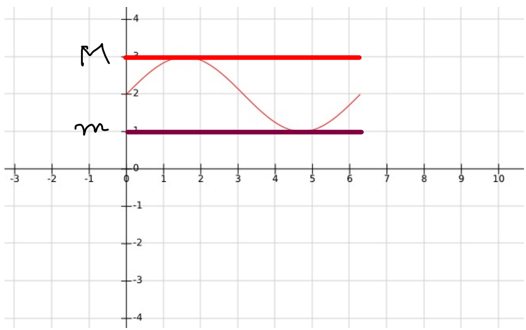
$f(x)=\sin(x)+1$



PROOF $f : [a,b] \to \mathbb{R}$ is C^0

$$\Rightarrow \quad R(f) = [m, M] \quad \Rightarrow$$

$$m \cdot (b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$



THM [FUNDAMENTAL THM OF CALCULUS]

$f : [a,b] \to \mathbb{R}$. C^0

THEN $G(x) = \int_a^x f(x)\,dx$ IS AN ANTI-DERIVATIVE OF f

PROOF $G(x) \doteq \int_a^x f(x)\,dx$

NEED TO SHOW: $G'(x_0) = f(x_0) \quad \forall x_0 \in \,]a,b[$

NEED TO SHOW: $G'(x_0) = f(x_0) \ \forall x_0 \in]a,b[$

$$\lim_{x \to 0} \frac{G(x) - G(x_0)}{x - x_0} = \lim_{x \to 0} \frac{\int_{x_0}^{x} f(x)\, dx}{x - x_0} =$$

THM [FUNDAMENTAL THM OF CALCULUS, 2nd EDITION]

$f : [a,b] \longrightarrow \mathbb{R}$. C^0

ASSUME $G : [a,b] \longrightarrow \mathbb{R}$ IS AN ANTI-DERIVATIVE OF f

THEN $$\int_a^b f(x)\,dx = G(b) - G(a)$$

PROOF

$$G(x) = C + \int_a^x f(t)\,dt$$

$$\Rightarrow \quad G(b) - G(a)$$

$$\|$$

$$C + \int_a^b f(t)\,dt \;-\; \left[C + \int_a^a f(t)\,dt \right]$$

TECHNIQUES OF COMPUTATION:

1. INTEGRATION BY PARTS

LEIBNIZ FORMULA: $(fg)' = f'g + g'f$

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1. INTEGRATION BY PARTS

LEIBNIZ FORMULA: $(fg)' = f'g + g'f$

$$\int_a^b (fg)'(t)\,dt = \int_a^b (f'g)(t)\,dt + \int_a^b (fg')(t)\,dt$$

TECHNIQUES OF COMPUTATION:

1. INTEGRATION BY PARTS

LEIBNIZ FORMULA: $(fg)' = f'g + g'f$

$$\int_a^b (fg)'(t)\,dt = \int_a^b (f'g)(t)\,dt + \int_a^b (fg')(t)\,dt$$

$$\overset{\text{FTC}}{=} (f\cdot g)(b) - (f\cdot g)(a)$$

F(x) IS ANTIDER. FOR F'(x)

$f : \mathbb{R} \to \mathbb{R}$, $f \in \underline{C^1(\mathbb{R})}$

$$\int_a^b \underline{\underline{f'(x)}}\, dx$$

$$\left.\begin{array}{l} f\big|_{[a,b]} : [a,b] \longrightarrow \mathbb{R} \quad C^0 \\ f \text{ is DIFF. on } (a,b) \\ f'(x) \end{array}\right\}$$

$\Downarrow$

f is AN ANTIDER. for f'

INTEGRATION BY PARTS

$g, f : E \to \mathbb{R}$ DIFF.

$[a, b] \subseteq E$

$$\int_a^b f(t)g'(t)\,dt$$

$$\parallel$$

$$f(b)g(b) - f(a)g(a) - \int_a^b f'(t)g(t)\,dt$$

INTEGRATION BY PARTS

$g, f : E \longrightarrow \mathbb{R}$ DIFF.

$[a, b] \subseteq E$

$$\int_a^b f(t)\,g'(t)\,dt$$

$$\parallel$$

NOTATION:

$$f(x)\,g(x)\Big|_{x=a}^{x=b} - \int_a^b f'(t)\,g(t)\,dt$$

$$h(x)\Big|_{x=a}^{x=b} := h(b) - h(a)$$

EXAMPLE

$$\int_a^b x e^x \, dx$$

FORMULA: $\int_a^b f(x) g'(x) \, dx = \boxed{(f \cdot g)(x) \Big|_{x=a}^{x=b}} - \int_a^b f'(x) g(x) \, dx$

$f(x) = x \qquad g'(x) = e^x \qquad \int_a^b 1 \cdot e^x \, dx$

$f'(x) = 1 \qquad g(x) = e^x \qquad$ " $e^b - e^a$

EXAMPLE

$$\int_a^b x e^x \, dx$$

FORMULA : $\displaystyle \int_a^b f(x)\, g'(x)\, dx = (f \cdot g)(x) \Big|_{x=a}^{x=b} - \int_a^b f'(x)\, g(x)\, dx$

$$f = x \qquad f' = 1$$

$$g = e^x \qquad g' = e^x$$

$$\int_a^b x e^x = x e^x \Big|_{x=a}^{x=b} - \int_a^b \underline{\underline{1 \cdot e^x}}\, dx = b e^b - a e^a - \left(e^b - e^a\right)$$

EXAMPLE

$$\int_a^b \sin(x) e^x \, dx =$$

FORMULA: $\int_a^b f(x) g'(x) dx = (f \cdot g)(x) \Big|_{x=a}^{x=b} - \int_a^b f'(x) g(x) dx$

$f(x) = \sin(x)$ $\qquad$ $g'(x) = e^x$

$f'(x) = \cos(x)$ $\qquad$ $g(x) = e^x$

EXAMPLE

$$\int_a^b \underset{g'}{\sin(x)} \; \underset{f}{x} \; dx =$$

FORMULA: $\displaystyle\int_a^b f(x)\,g'(x)\,dx = (f\cdot g)(x)\Big|_{x=a}^{x=b} - \underline{\int_a^b f'(x)\,g(x)\,dx}$

$f' = 1$ $\quad (-\cos(x))$

$$\int_a^b x \sin x = -x\cos(x)\Big|_{x=a}^{x=b} + \int_a^b \cos(x)$$

$$\parallel$$

$$-x\cos(x)\Big|_{x=a}^{x=b} + \sin(b) - \sin(a)$$

HARD PART: HOW DO WE CHOOSE f, g' ?

RECIPE THAT WORKS OFTEN:

- EXPONENTIALS
- TRIGONOMETRIC FCTS
- POLYNOMIALS & RAT'L FCTS
- LOGARITHMS
- INVERSE FCTS OF TRIGONOMETRIC FCTS