Random Matrix Theory (RMT) Homework 2

March 22, 2025

1. Prove

   Lemma 1. If A is symmetric positive semi-definite, then

   $$\mathrm{tr}(AB) \ \le \ \mathrm{tr}(A)\|B\|$$

   and if B is also positive definite, then

   $$\mathrm{tr}(A^{1/2}BA^{1/2}) \ \le \ \mathrm{tr}(B)\,\|A\|.$$

   Furthermore,

   $$x'ABAx \ \le \ \|A^{1/2}BA^{1/2}\|\,x'Ax \tag{1}$$

   for any positive definite A.

2. Given a Martingale $q_t = E_t[q_T]$ and it martingale differences $q_t - q_{t-1}$, prove that

   $$E[(q_T - E[q_T])^2] \ = \ E[\sum_t (q_t - q_{t-1})^2]$$

   Use this to prove the law of large numbers for Martingales:

   Theorem 1. If $E[(q_t - q_{t-1})^2] < K$ for some K, then $T^{-1}(q_T - E[q_T]) \to 0$ in probability.

3. Pick a rolling window and compute the Stieltjes transform for the US stock market (get the data you can get). Plot its dynamics over time for various ridge penalties.

4. Then, plot the same for the correlation matrix of returns. Discuss the results.