

# Liquidity, Volume, and Volatility

Vincent Bogousslavsky<sup>1</sup>   Pierre Collin-Dufresne<sup>2</sup>

<sup>1</sup>Boston College

<sup>2</sup>Swiss Finance Institute at EPFL

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# Motivation

We investigate the relation between the **liquidity, volume, and volatility** of individual U.S. stocks since 2002 (post-decimalization)

- ▶ What drives stock market liquidity?
  - ▶ Adverse selection
  - ▶ Inventory risk
- ▶ Dynamics of liquidity, volume, and volatility important for:
  - ▶ Dynamic portfolio allocation  
(Collin-Dufresne, Daniel, and Sağlam (2018))
  - ▶ Costs associated with exiting a position

# Liquidity and Trading Volume: Theory

Theoretically, high trading volume is generally associated with high liquidity ( $\sim$  low spreads)

- ▶ Adverse selection and market breakdown
  - ▶ More uninformed trading alleviates the adverse selection problem (Kyle (1985))
- ▶ Higher volume implies less risk for market makers who can more easily find off-setting trades (Demsetz (1968))
  - ▶ Lower cost of trading leads to more trading
- ▶ Invariance of Transaction Costs Hypothesis (Kyle and Obizhaeva (2016))
  - ▶  $\%spread_{i,t} \propto \left[ \frac{\sigma_{i,t}^2}{P_{i,t} V_{i,t}} \right]^{\frac{1}{3}}$

# Liquidity and Trading Volume: Empirical Evidence

- ▶ Positive volume-liquidity relation supported mostly by **cross-sectional** evidence (Stoll (2000))
- ▶ Only limited (and contradicting) evidence about the **time-series** relation
  - ▶ Spreads widen in response to higher volume (Lee, Mucklow, and Ready (1993))
  - ▶ Positive correlation between changes in spread and volume at the market level (Chordia et al. (2001))
  - ▶ No relation at market level (Johnson (2008))
  - ▶ Few studies control for volatility

# Key Findings

1. Positive association between volume and spread for large stocks, mostly driven by the common component of volume
2. Volatility of high-frequency order imbalances explains (1) and is an important variable for the dynamics of liquidity
3. Volatility of high-frequency order imbalances seems to reflect inventory risk and is priced in the cross-section of weekly returns

# Related Literature

- ▶ Volume and volatility (Clark (1973); Epps and Epps (1976); Tauchen and Pitts (1983); Gallant, Rossi, and Tauchen (1992); Andersen (1996))
- ▶ Spreads (Glosten and Harris (1988); Hasbrouck (1991); Foster and Viswanathan (1993); Bollen, Smith, and Whaley (2004))
- ▶ Liquidity and volume (Lee, Mucklow, and Ready (1993); Chordia, Roll, and Subrahmaniam (2000); Johnson (2008); Barinov (2010))
- ▶ Order imbalance (Chordia, Roll, and Subrahmanyam (2002); Chordia, Hu, Subrahmanyam, and Tong (2018))

# Data and Variables

## *Sample:*

- ▶ U.S. common stocks; 2002-2017
  - ▶ Price > \$5 and market capitalization > \$100 million

## *Main variables:*

- ▶ **Effective spread**:  $2|\ln P_{i,t} - \ln M_{i,t}|$  dollar/share-weighted over the trading day (Holden and Jacobsen (2014))
  - ▶ Similar results with dollar effective spread
- ▶ **Volume**: share turnover (during trading hours)
  - ▶ Similar results with CRSP turnover
- ▶ **Volatility**: average absolute return over the past five trading days or realized volatility
  - ▶ Similar results with  $|r_t|$ ,  $|r_{t,\text{intraday}}|$

# Methodology

Volume and volatility elasticities of spread:

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \epsilon_{i,t}$$

$$\log s_{i,t} = \alpha_i + \beta_\sigma \log \sigma_{i,t} + \epsilon_{i,t}$$

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$

► Levels, changes, and vector autoregressions

► Invariance (Kyle and Obizhaeva (2016)):  $s_{i,t} \propto \left[ \frac{\sigma_{i,t}^2}{P_{i,t} V_{i,t}} \right]^{\frac{1}{3}}$ ,  
where  $V$  is the share volume and  $P$  is the share price

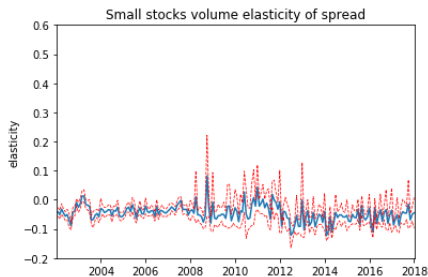
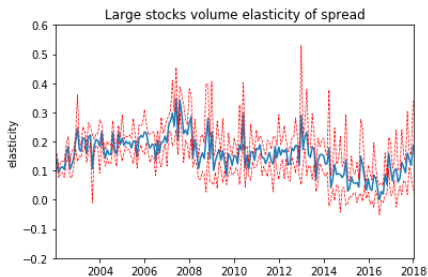
► *Controls*: daily price and market capitalization;  
day-of-the-week and month-of-the-year indicators

► Estimated each month/year on stocks sorted into market capitalization quintiles



# Results for Large vs. Small Stocks Volume

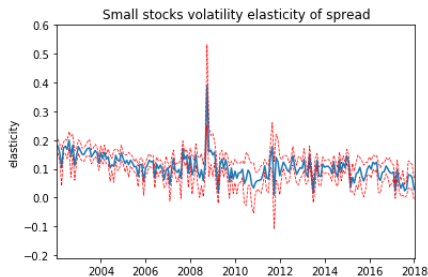
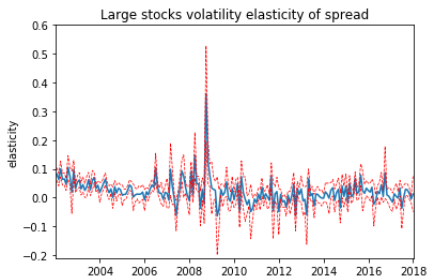
$$\log s_{i,t} = \alpha_i + \beta_{\tau} \log \tau_{i,t} + \beta_{\sigma} \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$



Volume elasticity of spread

# Results for Large vs. Small Stocks Volatility

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$



Volatility elasticity of spread

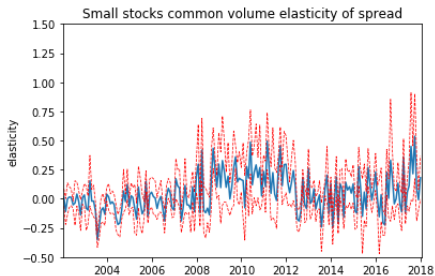
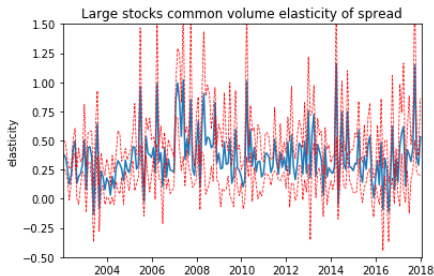
# Decomposing Volume and Volatility

## Systematic vs. idiosyncratic volume and volatility

- ▶ *Adverse selection channel:*
  - ▶ Idiosyncratic volatility is naturally linked to ‘insider information’ and adverse selection
  - ▶ Idiosyncratic volume is more linked to ‘information events’ that trigger more informed trading
  - ▶ Systematic component can be relevant if adverse-selection due to differential interpretation of public news
- ▶ *Inventory risk channel:*
  - ▶ Systematic volume shock consumes liquidity everywhere

# Large vs Small Stocks: **Common Volume** Component

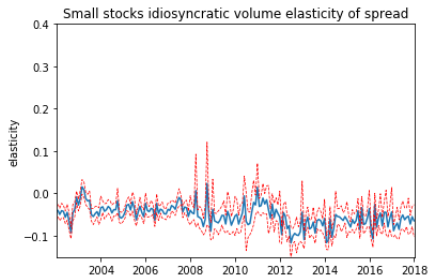
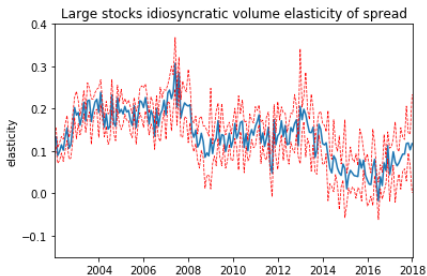
$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau_{i,t}^C + \beta_{\tau,I}\tau_{i,t}^I + \beta_{\sigma,C}\sigma_{i,t}^C + \beta_{\sigma,I}\sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



Common volume elasticity of spread

# Large vs Small Stocks: **Idiosyncratic Volume Comp.**

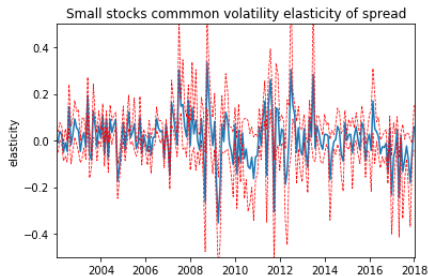
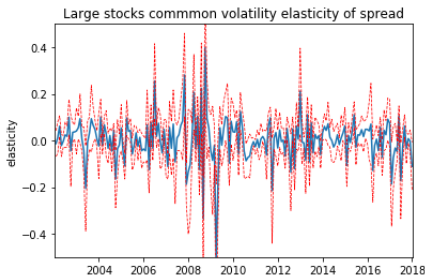
$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau_{i,t}^C + \beta_{\tau,I}\tau_{i,t}^I + \beta_{\sigma,C}\sigma_{i,t}^C + \beta_{\sigma,I}\sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



Idiosyncratic volume elasticity of spread

# Large vs Small Stocks **Common Volatility** Comp.

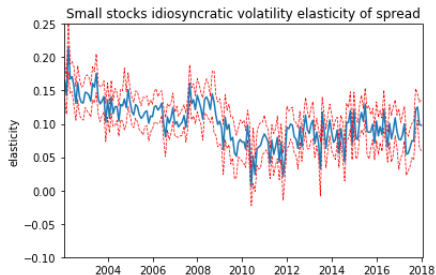
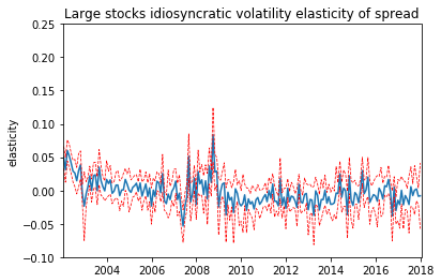
$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau_{i,t}^C + \beta_{\tau,I}\tau_{i,t}^I + \beta_{\sigma,C}\sigma_{i,t}^C + \beta_{\sigma,I}\sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



Common volatility elasticity of spread

# Large vs Small Stocks: Idiosyncratic Volatility Comp.

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau_{i,t}^C + \beta_{\tau,I}\tau_{i,t}^I + \beta_{\sigma,C}\sigma_{i,t}^C + \beta_{\sigma,I}\sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



Idiosyncratic volatility elasticity of spread

# Inventory Model

Natural to distinguish between volume and order imbalance (one-sided volume) (e.g., Chordia et al. (2002))

- ▶ Long-lived liquidity provider with CARA

$$\max_{c_t, n_t} E\left[\int_0^\infty -e^{-\beta t - \alpha c_t}\right]$$

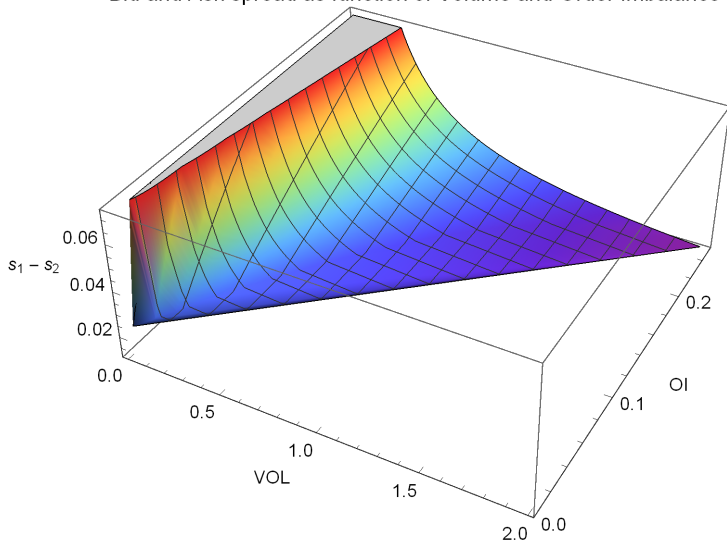
- ▶ One dividend-paying asset and one risk-free asset
- ▶ The liquidity providers absorbs supply shocks from buyers and sellers that arrive asynchronously (price impact)
- ▶ Her inventory follows a Markov chain with transition intensities  $\lambda_{i,j}$
- ▶ What is the effect of higher volume on the spread?



# Inventory Model

## Bid-Ask spread as a function of Volume and Variance of Order Imbalance

Bid and Ask spread as function of Volume and Order Imbalance



# Volatility of Order Imbalances

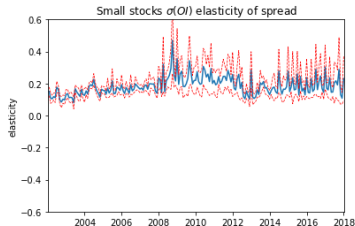
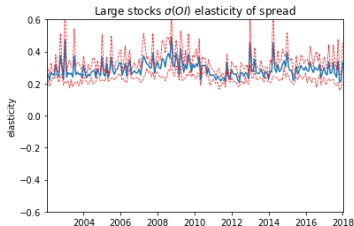
Simple inventory model suggests to distinguish **volume** from **order imbalance** (to capture 'one-sided' volume)

- ▶ Compute order imbalance as a proportion of shares outstanding over every 5mn interval of the trading day
  - ▶ High frequency market making
- ▶  $\sigma(\text{OI})$  is the standard deviation of the 5mn imbalance, computed each day
  - ▶ Control: realized volatility [▶ details](#)

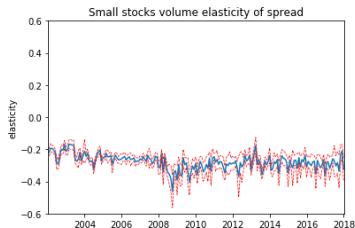
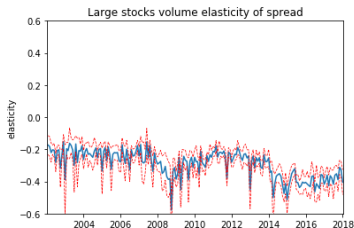
# Volatility of Order Imbalances

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \beta_{\sigma(\text{OI})} \log \sigma(\text{OI})_{i,t} + \text{controls} + \epsilon_{i,t}$$

$\sigma(\text{OI})$   
elastic-  
ity of  
spread



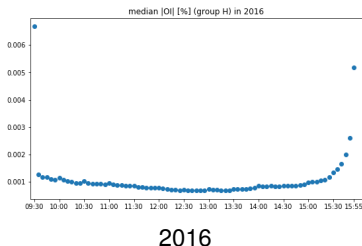
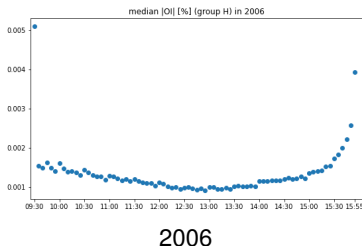
volume  
elastic-  
ity of  
spread



average  $R^2$  increases from 11.48% (14.12%) to 22.82% (19.26%)

# Interpretation of Order Imbalance Volatility $\sigma(\text{OI})$

- ▶ **Relation with other liquidity measures** [▶ details](#)
  - ▶  $\sigma(\text{OI})$  is positively associated with **price impact** (Amihud)
  - ▶  $\sigma(\text{OI})$  is negatively associated with **depth**
- ▶ **Intraday patterns**
  - ▶ Inventory effects should be stronger at the end of the day



# Spread Decomposition

Large stocks in 2018

$$\text{effective spread} = \underbrace{\text{realized spread}}_{\text{trsign}*(p_t - m_{t+5})} + \underbrace{\text{adverse selection}}_{\text{trsign}*(m_{t+5} - m_t)}$$

Month	(a) Adverse selection			(b) Realized spread		
	$\beta_\tau$	$\beta_{\text{RVol}}$	$\beta_{\sigma(\text{OI})}$	$\beta_\tau$	$\beta_{\text{RVol}}$	$\beta_{\sigma(\text{OI})}$
1	-1.03** (-2.51)	1.85*** (9.00)	0.31 (0.95)	-1.26*** (-9.18)	0.38** (2.16)	1.47*** (11.51)
2	-0.74*** (-3.91)	1.60*** (10.22)	0.19 (1.26)	-1.19*** (-7.06)	0.40** (2.29)	1.37*** (10.98)
3	-1.29*** (-3.57)	1.20*** (4.77)	1.11* (1.78)	-1.31*** (-7.86)	0.22 (0.94)	1.69*** (7.45)
4	-0.73*** (-4.03)	1.45*** (16.55)	0.14 (0.91)	-0.92*** (-9.73)	0.14 (0.69)	1.49*** (8.78)
5	-0.68*** (-3.45)	1.51*** (10.02)	0.15 (0.73)	-1.08*** (-8.62)	0.28** (2.36)	1.30*** (10.26)
6	-1.53*** (-2.72)	1.35*** (4.72)	1.30 (1.49)	-1.33*** (-4.45)	0.31 (1.53)	1.79*** (8.16)
7	-0.77*** (-3.89)	1.53*** (11.51)	0.13 (0.73)	-1.06*** (-7.68)	0.28* (1.91)	1.48*** (9.57)
8	-0.84*** (-3.86)	1.77*** (17.53)	0.22 (1.09)	-1.05*** (-8.98)	0.50*** (3.33)	1.29*** (10.42)
9	-1.24*** (-3.28)	1.31*** (7.14)	1.06 (1.59)	-1.19*** (-6.84)	0.06 (0.51)	1.82*** (6.74)
10	-0.71*** (-4.76)	1.64*** (12.93)	0.13 (0.77)	-0.92*** (-7.99)	0.53*** (3.88)	1.15*** (11.65)
11	0.17 (0.19)	1.35* (1.81)	-0.78** (-2.56)	-1.84** (-2.05)	0.87 (1.25)	2.26*** (6.00)
12	-1.11 (-1.20)	0.75 (0.82)	0.42 (0.91)	-2.21* (-1.90)	1.26 (1.22)	2.74** (2.49)

⇒ Order imbalance volatility mostly associated with realized spread

# Pricing: sequential portfolio sorts

NYSE, Amex, and NASDAQ common stocks over 2002-2017 (797 weekly observations); NYSE breakpoints

	$\alpha_{FF4}^{VW}$ (turnover then order imbalance volatility)					
	low $\sigma(OI)$	2	3	4	high $\sigma(OI)$	H-L
low turn.	-0.02 (-0.66)	0.02 (0.55)	0.02 (0.51)	0.02 (0.50)	0.08*** (2.78)	0.10** (2.56)
2	-0.01 (-0.30)	0.05* (1.72)	-0.00 (-0.05)	0.01 (0.39)	0.06* (1.66)	0.06 (1.56)
3	0.00 (0.09)	0.03 (0.88)	0.06** (2.02)	0.09*** (3.23)	0.11*** (3.65)	0.11*** (2.65)
4	-0.09*** (-2.91)	0.00 (0.13)	0.01 (0.24)	-0.04 (-1.15)	0.12*** (4.03)	0.20*** (4.59)
high turn.	-0.05 (-0.94)	-0.07 (-1.28)	0.04 (0.68)	-0.05 (-0.98)	0.08* (1.68)	0.13** (1.98)

# Pricing: value-weighted Fama-MacBeth regressions

NYSE, Amex, and NASDAQ common stocks over 2002-2017 (797 weeks)

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dependent variable:  $r_t$  (weekly return in percent)

	coeff. (t-stat)	coeff. (t-stat)	coeff. (t-stat)
$\sigma(OI)_{t-1}$	0.064** (2.35)	0.086*** (3.02)	0.083*** (3.40)
$turn_{t-1}$		-0.037 (-1.00)	-0.026 (-0.67)
$ME_{t-1}$			-0.012 (-0.31)
$r_{t-1}$			-1.652*** (-3.91)
$ILLIQ_{t-1}$			-0.009 (-0.25)
$RVol_{t-1}$			-0.023 (-0.32)
$ES_{t-1}$			-0.023 (-0.63)
$\sigma(OI/VOL)_{t-1}^{month}$			0.056 (1.42)
$\bar{N}$	2,628	2,628	2,591
$\bar{R}^2$	0.020	0.036	0.104

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# Conclusion

- ▶ New evidence about the time-series (and cross-sectional) relation between liquidity, volume, and volatility
  - ▶ Adverse selection theories fit well the day-to-day variation in spread, volume, and volatility of small stocks
  - ▶ Inventory risk seems more important for the day-to-day variation in spread, volume, and volatility of large stocks
- ▶ Controlling for volatility of (high-frequency) order imbalances reconciles evidence between large and small stocks
  - ⇒ is consistent with simple inventory risk model, and
  - ⇒ adds substantial explanatory power
- ▶ Order imbalance volatility seems to reflect inventory risk and is priced in the cross-section of weekly returns



# Appendix

# Descriptive Statistics (Small Stocks)

		2004	2008	2012	2016
Small caps spread [bp]	mean	70.18	96.68	62.69	70.32
	median	51.33	50.35	40.85	44.66
	$\sigma$ (within)	48.62	103.16	49.98	63.32
turnover [%]	mean	0.50	0.52	0.42	0.48
	median	0.19	0.27	0.23	0.25
	$\sigma$ (within)	1.38	0.83	0.89	1.28
volatility [%]	mean	1.83	3.06	1.72	1.87
	median	1.53	2.44	1.50	1.51
	$\sigma$ (within)	1.06	2.13	1.02	1.58
obs.		146,897	132,182	119,480	126,515

# Descriptive Statistics (Large Stocks) [▶ back](#)

		2004	2008	2012	2016
Large caps spread [bp]	mean	8.27	8.29	4.65	4.77
	median	6.59	6.20	3.65	3.63
	$\sigma$ (within)	5.95	10.23	3.04	4.31
turnover [%]	mean	0.67	1.42	0.90	0.82
	median	0.46	1.03	0.67	0.61
	$\sigma$ (within)	0.58	1.22	0.74	0.63
volatility [%]	mean	1.17	2.70	1.16	1.23
	median	1.01	2.03	1.01	1.01
	$\sigma$ (within)	0.57	1.99	0.58	0.72
obs.		151,157	137,730	121,479	129,411

# Correlations

cross-sectional averages of the stocks' time-series correlations

[▶ back](#)

Small caps						
	$\tau$	$\sigma$	$ r $	RVol	$ OI $	$\sigma(OI)$
$s$	-0.17	0.22	0.18	0.40	-0.06	-0.00
$\tau$		0.24	0.23	0.32	0.59	0.78
$\sigma$			0.49	0.47	0.10	0.12
$ r $				0.41	0.13	0.14
RVol					0.12	0.17
$ OI $						0.60

Large caps						
	$\tau$	$\sigma$	$ r $	RVol	$ OI $	$\sigma(OI)$
$s$	0.15	0.34	0.22	0.51	0.15	0.30
$\tau$		0.41	0.32	0.48	0.40	0.72
$\sigma$			0.50	0.61	0.14	0.22
$ r $				0.41	0.13	0.19
RVol					0.14	0.26
$ OI $						0.48

# How Does Order Imbalance Volatility Affect Other Liquidity Measures?

## ► Price impact

- In the line of Amihud (2002):

$$ILLIQ_{it} = \frac{1}{\# \text{traded intervals}} \sum_{k \in \{j | DVOL_{itj} > 0\}} \frac{|r_{itk}|}{DVOL_{itk}}$$

- Alternative:  $r_{itk} = \delta_{it} + \lambda_{it} \sqrt{|OI_{itk}^{\$}|} \text{sign}(OI_{itk}^{\$}) + e_{it}$   
(Hasbrouck (2009))

## ► Depth

- Time-weighted share depth at the best bid and best ask (as a fraction of shares outstanding)

# Price Impact (Amihud)

Year	$\beta_{\tau}$	$\beta_{RVol}$	$\beta_{\sigma(OI)}$
2002	-1.10*** (-54.89)	0.90*** (40.95)	0.24*** (18.72)
2003	-1.21*** (-56.32)	0.88*** (81.51)	0.29*** (27.03)
2004	-1.20*** (-100.27)	0.88*** (65.68)	0.27*** (37.15)
2005	-1.17*** (-103.29)	0.90*** (44.62)	0.24*** (39.38)
2006	-1.15*** (-98.54)	0.92*** (87.78)	0.21*** (35.00)
2007	-1.12*** (-101.99)	0.98*** (72.96)	0.16*** (29.75)
2008	-1.10*** (-82.55)	0.96*** (58.25)	0.10*** (18.51)
2009	-1.07*** (-141.59)	0.92*** (41.61)	0.10*** (15.21)
2010	-1.10*** (-72.77)	0.92*** (26.01)	0.11*** (21.17)
2011	-1.12*** (-107.44)	0.96*** (45.96)	0.11*** (23.25)
2012	-1.09*** (-101.16)	0.83*** (72.48)	0.12*** (19.40)
2013	-1.14*** (-67.34)	0.89*** (30.81)	0.14*** (23.18)
2014	-1.14*** (-148.76)	0.88*** (86.04)	0.15*** (36.93)
2015	-1.15*** (-123.23)	0.89*** (66.42)	0.15*** (32.22)
2016	-1.14*** (-108.79)	0.88*** (52.77)	0.14*** (32.52)
2017	-1.11*** (-135.75)	0.79*** (67.02)	0.15*** (43.56)

$\bar{R}^2(\%)$

77.05

Year	$\beta_{\tau}$	$\beta_{\text{RVol}}$	$\beta_{\sigma(\text{OI})}$	$\beta_s$
2002	0.35*** (20.58)	-0.22*** (-9.94)	-0.00 (-0.32)	-0.19*** (-15.34)
2003	0.43*** (22.47)	-0.31*** (-30.46)	-0.04*** (-5.18)	-0.09*** (-16.94)
2004	0.47*** (31.15)	-0.42*** (-14.13)	-0.04*** (-7.23)	-0.10*** (-15.55)
2005	0.46*** (30.23)	-0.44*** (-15.49)	-0.05*** (-10.80)	-0.07*** (-13.95)
2006	0.44*** (30.65)	-0.51*** (-18.67)	-0.06*** (-11.93)	-0.07*** (-12.71)
2007	0.41*** (25.41)	-0.56*** (-22.82)	-0.02*** (-4.58)	-0.04*** (-6.93)
2008	0.40*** (18.33)	-0.69*** (-17.47)	-0.01*** (-2.70)	0.02*** (2.78)
2009	0.38*** (23.72)	-0.66*** (-22.94)	-0.00 (-0.12)	-0.03*** (-3.54)
2010	0.39*** (16.04)	-0.66*** (-14.65)	-0.01 (-1.07)	-0.02** (-2.39)
2011	0.38*** (19.13)	-0.65*** (-17.34)	-0.02*** (-3.06)	0.03*** (4.03)
2012	0.35*** (29.22)	-0.40*** (-22.19)	-0.03*** (-6.01)	-0.00 (-0.22)
2013	0.40*** (18.79)	-0.48*** (-10.24)	-0.05*** (-9.43)	0.02** (2.47)
2014	0.31*** (34.06)	-0.39*** (-23.40)	-0.01 (-1.56)	-0.01*** (-2.92)
2015	0.30*** (21.97)	-0.34*** (-15.81)	-0.02*** (-4.05)	0.01*** (2.77)
2016	0.30*** (15.37)	-0.37*** (-11.26)	-0.02*** (-4.91)	0.03*** (4.61)
2017	0.28*** (26.71)	-0.27*** (-14.35)	-0.03*** (-10.16)	0.02*** (4.23)

 $\bar{R}^2(\%)$ 

41.70

# Evidence from Intraday Patterns

The degree of informed trading and liquidity trading is likely not constant over the day

1. Informational advantage of trading on overnight information is likely short-lived (Foster and Viswanathan (1990))
2. Liquidity traders cluster their trades to reduce adverse selection (Admati-Pfleiderer (1980))



# Evidence from Intraday Patterns

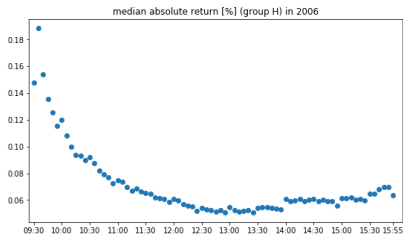
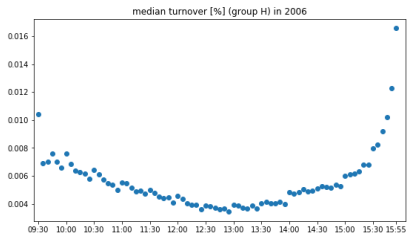
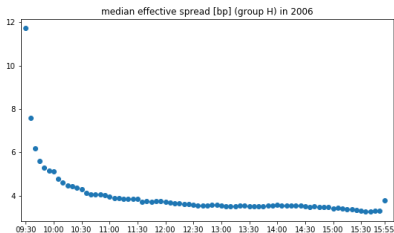
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Informative to examine intraday patterns of elasticities

- ▶ Split the day into five-minute intervals and focus on large stocks
- ▶ We are *not* looking at levels but at sensitivities
  - ▶ Control for interval-stock fixed effects

# Intraday Median Values - 2006



# Intraday Evidence

- ▶ Volume elasticity of spread is higher at the end of the day, when inventory risk or market power may be high
  - ▶ Consistent with evidence from intraday order imbalances
    - ▶ Appendix
- ▶ The intraday elasticity pattern does not ‘mechanically’ reflect intraday variations in spread, volume, and volatility
  - ▶ Spreads may be lower around the close but are more sensitive to trading volume

This evidence supports adverse selection effects and competition/inventory effects

- ▶ More competitive liquidity provision in recent years?

# Volume in the continuous-time Kyle model

- ▶  $VOL = \frac{1}{2}(|dX_t^i| + |dX_t^u| + |dX_t^i + dX_t^u|)$
- ▶ Insider trade in absolutely continuous fashion:  $dX_t^i = \mu_i dt$
- ▶ Whereas  $dX_t^u = \sigma_u dZ_t$  for some Brownian motion  $Z_t$
- ▶  $E[VOL]^2 = 2/\pi \sigma_u^2 dt$
- ▶ Total cumulative order flow is  $Y_t = X_t^u + X_t^i$  and  $Var[dY_t] = \sigma_u^2 dt$

# Inventory Shocks and Endogenous Entry

Allow for entry of liquidity providers at a fixed cost in the model of [Campbell, Grossman, and Wang \(1993\)](#)

- ▶ Stationary OLG economy with exogenous risk-free rate and a risky asset that pays dividends every date
- ▶ Liquidity providers with exponential utility absorb volatile supply shocks every date
- ▶ In equilibrium, we show that an increase in the volatility of supply shocks *decreases* price impact, in contrast to the original model
- ▶ The inventory explanation requires some barriers to entry

## Gallant-Rossi-Tauchen (1992) Methodology [▶ back](#)

For each stock regress the spread and turnover series on a set of control variables  $x$ :

$$y = x'\beta + u.$$

The residuals are used to construct the following variance equation:

$$\log(u^2) = x'\gamma + v.$$

The adjusted  $y$  series is then given by:

$$y_{\text{adj}} = a + b(\hat{u} / \exp(x'\gamma/2)),$$

where the parameters  $a$  and  $b$  are chosen such that the mean and standard deviation of  $y_{\text{adj}}$  are the same as that of  $y$ .

Control variables  $x$ : day-of-the-week dummies; month-of-the-year dummies; a dummy for trading days around holidays when the stock market is closed; a dummy for trading days on federal holidays when the stock market is open; linear and quadratic trend variables. For the turnover series, we also include a cubic trend variable.

# Measure of Volatility: Realized Volatility

What about a more sophisticated measure of volatility?

- ▶ *Realized variance*:  $\text{RVol}(K)_t^2 = \sqrt{\sum_{k=1}^K r_{t,k}^2}$ , where  $r_{t,k}$  is the intraday return over interval  $k$
- ▶ But what should we expect?

Using log returns, it can be shown that:

$$\text{RVol}(k)_t^2 = r_t^2 + \Pi_t,$$

where  $\Pi_t = \sum_{k=2}^K (-2 \sum_{j=1}^{k-1} r_{t,j}) r_{t,k} \Rightarrow$  intraday reversal strategy  
 $\text{corr}(s_t, \Pi_t) > 0?$

# Large Stocks' Elasticities with Realized Volatility

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau_{i,t}^C + \beta_{\tau,I}\tau_{i,t}^I + \beta_{\text{RVol}}\text{RVol}_{i,t} + \text{controls} + \epsilon_{i,t}$$

Year	$\beta_{\tau,C}$	$\beta_{\tau,I}$	$\beta_{\text{RVol}}$
2002	0.12** (2.46)	0.02** (2.47)	0.42*** (13.22)
2003	-0.05 (-1.05)	0.08*** (11.45)	0.45*** (42.76)
2004	0.01 (0.29)	0.07*** (11.23)	0.38*** (39.58)
2005	0.16*** (3.18)	0.07*** (11.79)	0.34*** (28.41)
2006	0.11*** (2.77)	0.08*** (11.05)	0.30*** (29.47)
2007	0.25*** (5.45)	0.09*** (8.98)	0.33*** (16.58)
2008	0.12*** (2.64)	0.00 (0.18)	0.42*** (17.93)
2009	0.09** (1.99)	0.03*** (3.28)	0.24*** (11.07)
2010	0.10*** (2.70)	0.03*** (3.40)	0.27*** (11.77)
2011	0.06** (1.96)	0.02* (1.73)	0.30*** (17.04)
2012	0.27*** (3.12)	0.03*** (3.23)	0.27*** (16.69)
2013	0.13*** (2.68)	0.02 (1.52)	0.31*** (16.16)
2014	0.08 (1.19)	-0.06*** (-4.30)	0.34*** (17.54)
2015	-0.00 (-0.01)	-0.11*** (-9.76)	0.41*** (19.61)
2016	-0.07* (-1.96)	-0.11*** (-8.53)	0.39*** (18.30)
2017	0.11 (1.52)	-0.10*** (-7.36)	0.40*** (20.47)

$\bar{R}^2(\%)$

20.59



# Large Stocks' Elasticities with Realized Volatility [▶ back](#)

$$\Delta s_{i,t} = \alpha_i + \beta_{\tau,C} \Delta \tau_{i,t}^C + \beta_{\tau,I} \Delta \tau_{i,t}^I + \beta_{\text{RVol}} \Delta \text{RVol}_{i,t} + \text{controls} + \epsilon_{i,t}$$

Year	$\beta_{\tau,C}$	$\beta_{\tau,I}$	$\beta_{\text{RVol}}$
2002	0.18*** (2.68)	0.06*** (6.10)	0.35*** (10.06)
2003	0.00 (0.02)	0.14*** (16.32)	0.41*** (35.94)
2004	0.15*** (3.10)	0.13*** (18.16)	0.36*** (39.17)
2005	0.29*** (4.30)	0.16*** (19.68)	0.31*** (27.00)
2006	0.23*** (4.96)	0.16*** (18.66)	0.26*** (25.61)
2007	0.52*** (6.81)	0.22*** (14.90)	0.25*** (13.52)
2008	0.37*** (4.75)	0.10*** (9.23)	0.31*** (15.01)
2009	0.28*** (3.64)	0.12*** (9.35)	0.19*** (9.09)
2010	0.29*** (5.14)	0.13*** (10.37)	0.21*** (9.33)
2011	0.19*** (4.51)	0.10*** (9.62)	0.23*** (16.73)
2012	0.43*** (3.28)	0.13*** (9.07)	0.19*** (10.59)
2013	0.24*** (3.94)	0.11*** (8.10)	0.25*** (16.15)
2014	0.32*** (3.30)	0.03* (1.79)	0.28*** (15.30)
2015	0.20*** (3.22)	-0.02 (-1.24)	0.32*** (14.16)
2016	0.16** (2.39)	-0.03*** (-2.64)	0.34*** (20.53)
2017	0.39*** (3.58)	-0.01 (-0.44)	0.32*** (18.53)

$\bar{R}^2(\%)$

8.84