

Liquidity, Volume, and Volatility

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Motivation

We investigate the relation between the **liquidity**, **volume**, and **volatility** of individual U.S. stocks since 2002 (post-decimalization)

- ▶ What drives stock market liquidity?
 - ▶ Adverse selection
 - ▶ Inventory risk
- ▶ Dynamics of liquidity, volume, and volatility important for:
 - ▶ Dynamic portfolio allocation
(Collin-Dufresne, Daniel, and Sağlam (2018))
 - ▶ Costs associated with exiting a position

Liquidity and Trading Volume: Theory

Theoretically, high trading volume is generally associated with high liquidity (\sim low spreads)

- ▶ Adverse selection and market breakdown
 - ▶ More uninformed trading alleviates the adverse selection problem ([Kyle \(1985\)](#))
- ▶ Higher volume implies less risk for market makers who can more easily find off-setting trades ([Demsetz \(1968\)](#))
 - ▶ Lower cost of trading leads to more trading
- ▶ Invariance of Transaction Costs Hypothesis
([Kyle and Obizhaeva \(2016\)](#))

$$\text{▶ \%spread}_{i,t} \propto \left[\frac{\sigma_{i,t}^2}{P_{i,t} V_{i,t}} \right]^{\frac{1}{3}}$$

Liquidity and Trading Volume: Empirical Evidence

- ▶ Positive volume-liquidity relation supported mostly by cross-sectional evidence (Stoll (2000))
- ▶ Only limited (and contradicting) evidence about the time-series relation
 - ▶ Spreads widen in response to higher volume (Lee, Mucklow, and Ready (1993))
 - ▶ Positive correlation between changes in spread and volume at the market level (Chordia et al. (2001))
 - ▶ No relation at market level (Johnson (2008))
 - ▶ Few studies control for volatility

Key Findings

1. Positive association between volume and spread for large stocks, mostly driven by the common component of volume
2. Volatility of high-frequency order imbalances explains (1) and is an important variable for the dynamics of liquidity
3. Volatility of high-frequency order imbalances seems to reflect inventory risk and is priced in the cross-section of weekly returns

Related Literature

- ▶ Volume and volatility (Clark (1973); Epps and Epps (1976); Tauchen and Pitts (1983); Gallant, Rossi, and Tauchen (1992); Andersen (1996))
- ▶ Spreads (Glosten and Harris (1988); Hasbrouck (1991); Foster and Viswanathan (1993); Bollen, Smith, and Whaley (2004))
- ▶ Liquidity and volume (Lee, Mucklow, and Ready (1993); Chordia, Roll, and Subrahmaniam (2000); Johnson (2008); Barinov (2010))
- ▶ Order imbalance (Chordia, Roll, and Subrahmanyam (2002); Chordia, Hu, Subrahmanyam, and Tong (2018))

Data and Variables

Sample:

- ▶ U.S. common stocks; 2002-2017
 - ▶ Price > \$5 and market capitalization > \$100 million

Main variables:

- ▶ **Effective spread:** $2|\ln P_{i,t} - \ln M_{i,t}|$ dollar/share-weighted over the trading day (Holden and Jacobsen (2014))
 - ▶ Similar results with dollar effective spread
- ▶ **Volume:** share turnover (during trading hours)
 - ▶ Similar results with CRSP turnover
- ▶ **Volatility:** average absolute return over the past five trading days or realized volatility
 - ▶ Similar results with $|r_t|$, $|r_{t,\text{intraday}}|$

Methodology

Volume and volatility elasticities of spread:

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \epsilon_{i,t}$$

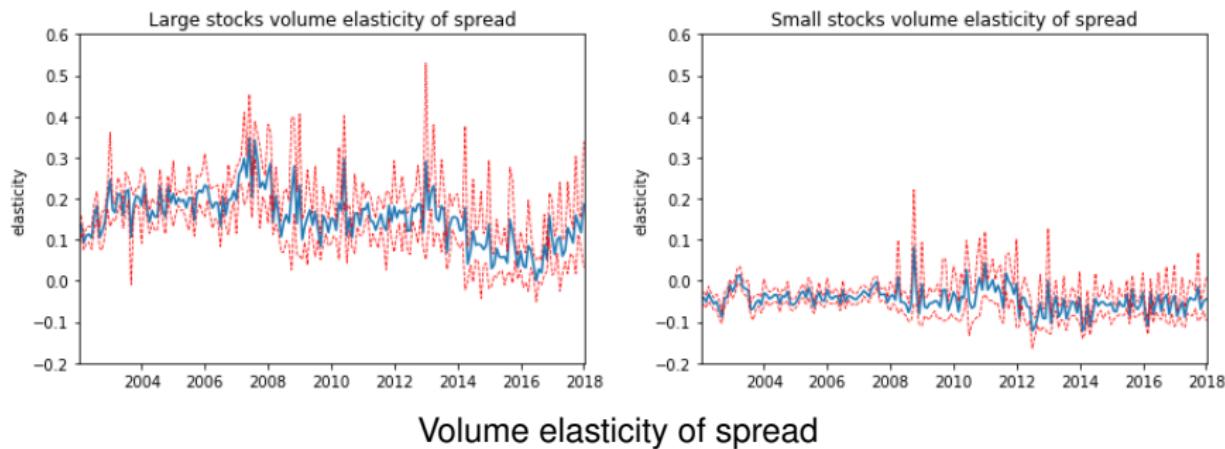
$$\log s_{i,t} = \alpha_i + \beta_\sigma \log \sigma_{i,t} + \epsilon_{i,t}$$

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$

- ▶ Levels, changes, and vector autoregressions
- ▶ Invariance (Kyle and Obizhaeva (2016)): $s_{i,t} \propto \left[\frac{\sigma_{i,t}^2}{P_{i,t} V_{i,t}} \right]^{\frac{1}{3}}$,
where V is the share volume and P is the share price
- ▶ Controls: daily price and market capitalization;
day-of-the-week and month-of-the-year indicators
- ▶ Estimated each month/year on stocks sorted into market capitalization quintiles

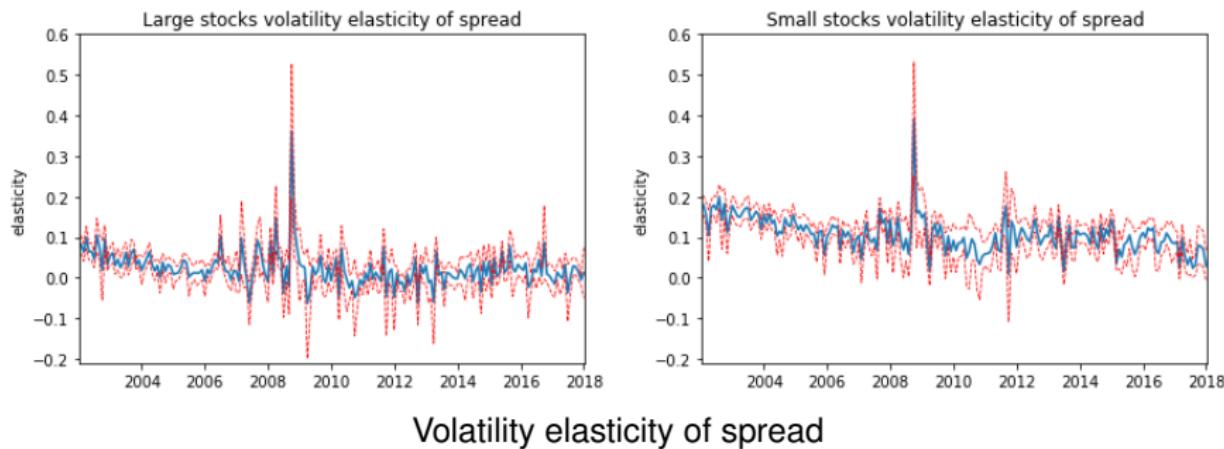
Results for Large vs. Small Stocks Volume

$$\log s_{i,t} = \alpha_i + \beta_{\tau} \log \tau_{i,t} + \beta_{\sigma} \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$



Results for Large vs. Small Stocks Volatility

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$



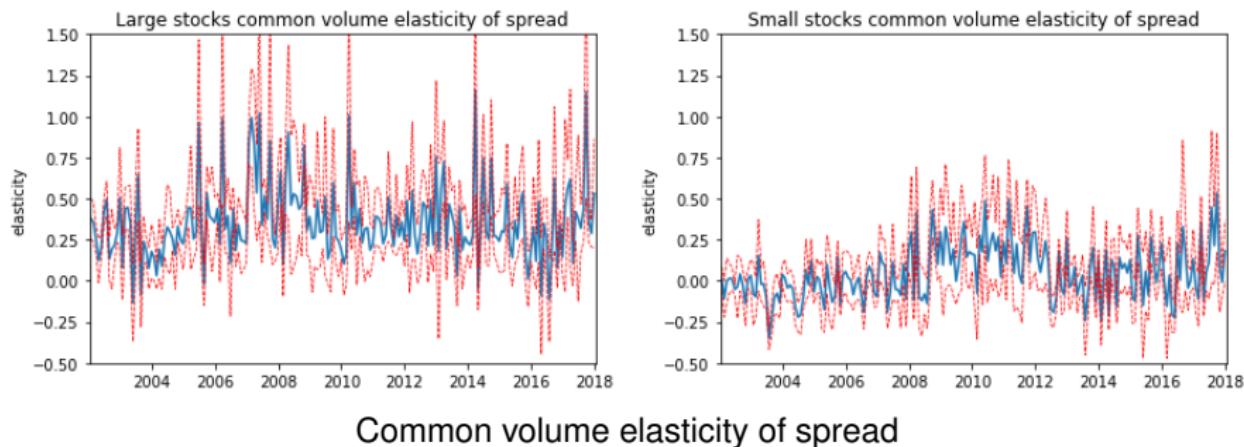
Decomposing Volume and Volatility

Systematic vs. idiosyncratic volume and volatility

- ▶ *Adverse selection channel:*
 - ▶ Idiosyncratic volatility is naturally linked to 'insider information' and adverse selection
 - ▶ Idiosyncratic volume is more linked to 'information events' that trigger more informed trading
 - ▶ Systematic component can be relevant if adverse-selection due to differential interpretation of public news
- ▶ *Inventory risk channel:*
 - ▶ Systematic volume shock consumes liquidity everywhere

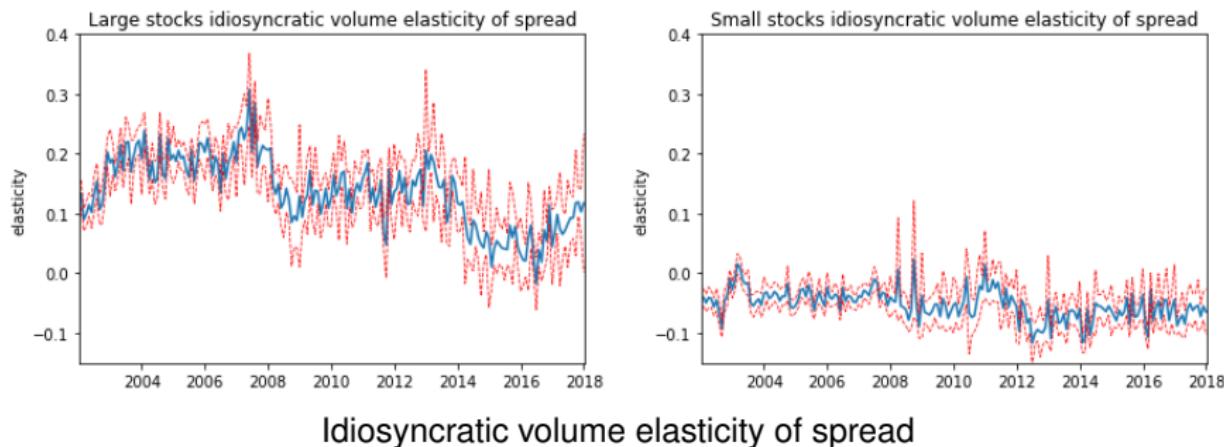
Large vs Small Stocks: Common Volume Component

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C} \tau_{i,t}^C + \beta_{\tau,I} \tau_{i,t}^I + \beta_{\sigma,C} \sigma_{i,t}^C + \beta_{\sigma,I} \sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



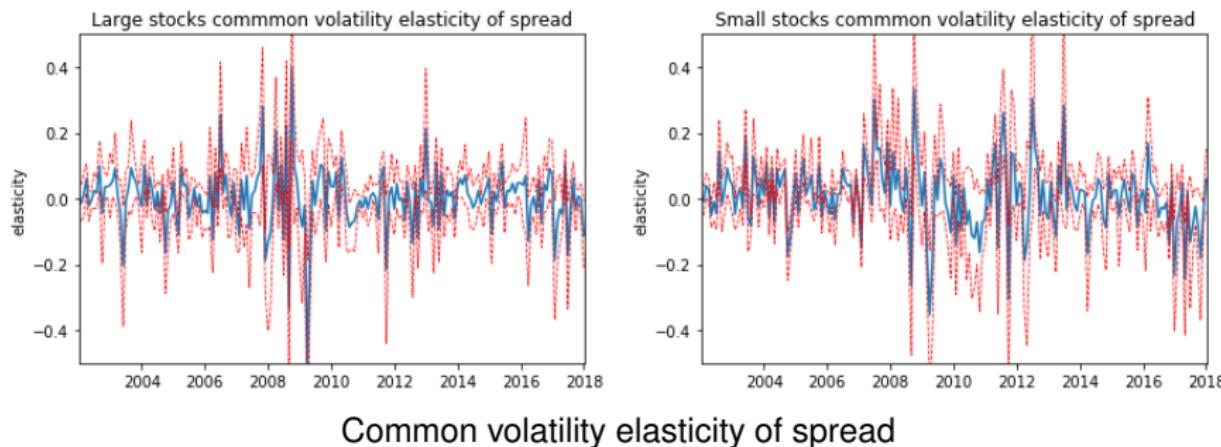
Large vs Small Stocks: Idiosyncratic Volume Comp.

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C} \tau_{i,t}^C + \beta_{\tau,I} \tau_{i,t}^I + \beta_{\sigma,C} \sigma_{i,t}^C + \beta_{\sigma,I} \sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



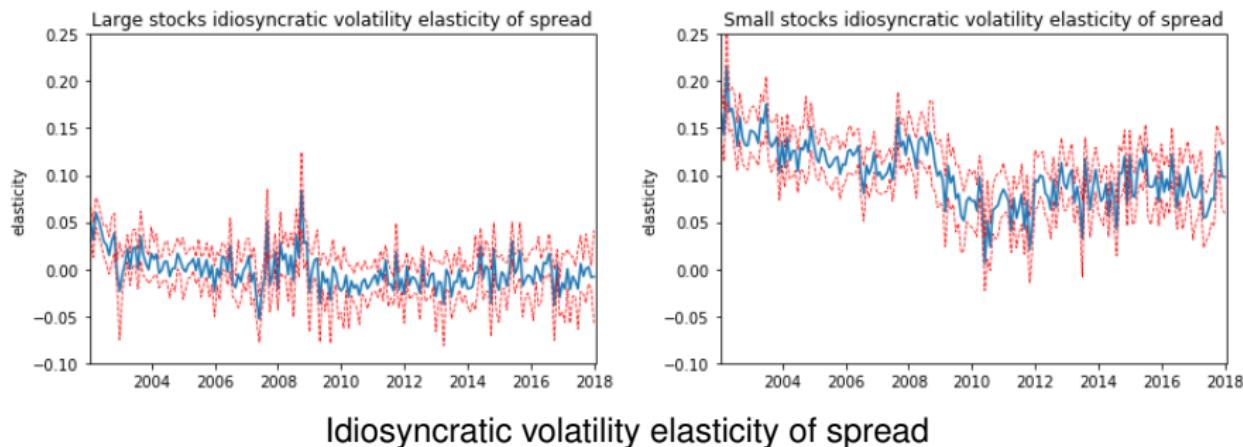
Large vs Small Stocks **Common Volatility Comp.**

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C} \tau_{i,t}^C + \beta_{\tau,I} \tau_{i,t}^I + \beta_{\sigma,C} \sigma_{i,t}^C + \beta_{\sigma,I} \sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



Large vs Small Stocks: Idiosyncratic Volatility Comp.

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C} \tau_{i,t}^C + \beta_{\tau,I} \tau_{i,t}^I + \beta_{\sigma,C} \sigma_{i,t}^C + \beta_{\sigma,I} \sigma_{i,t}^I + \text{controls} + \epsilon_{i,t}$$



Inventory Model

Natural to distinguish between volume and order imbalance
(one-sided volume) (e.g., Chordia et al. (2002))

- ▶ Long-lived liquidity provider with CARA

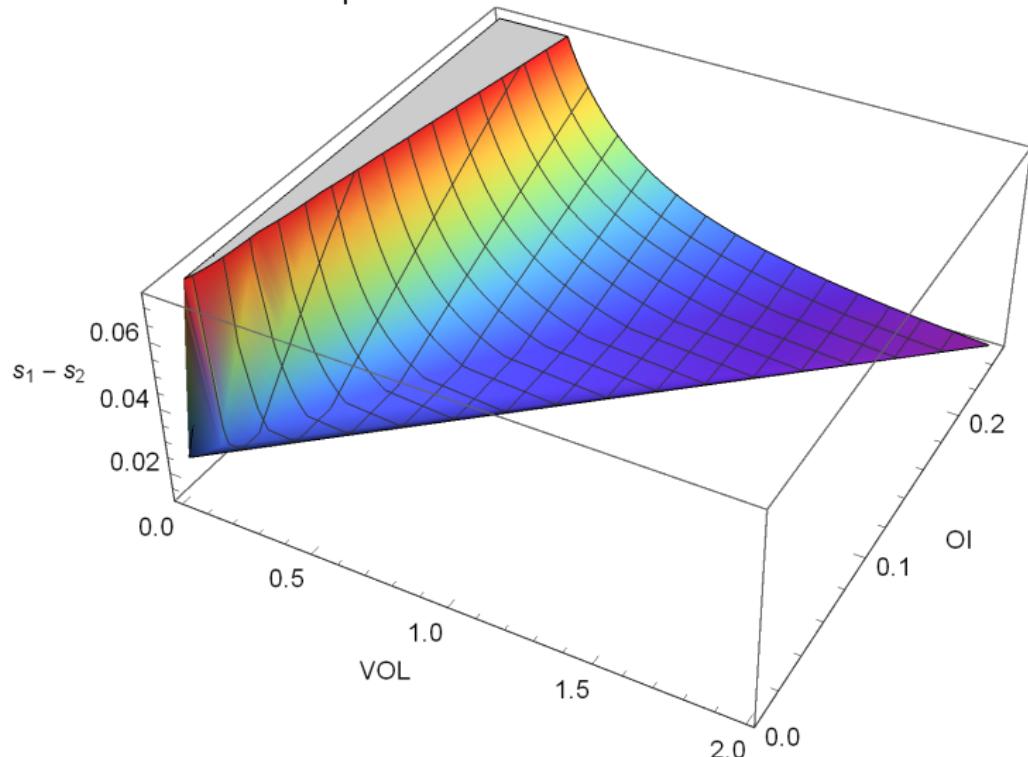
$$\max_{c_t, n_t} \mathbb{E} \left[\int_0^{\infty} -e^{-\beta t - \alpha c_t} \right]$$

- ▶ One dividend-paying asset and one risk-free asset
- ▶ The liquidity providers absorbs supply shocks from buyers and sellers that arrive asynchronously (price impact)
- ▶ Her inventory follows a Markov chain with transition intensities $\lambda_{i,j}$
- ▶ What is the effect of higher volume on the spread?

Inventory Model

Bid-Ask spread as a function of Volume and Variance of Order Imbalance

Bid and Ask spread as function of Volume and Order Imbalance



Volatility of Order Imbalances

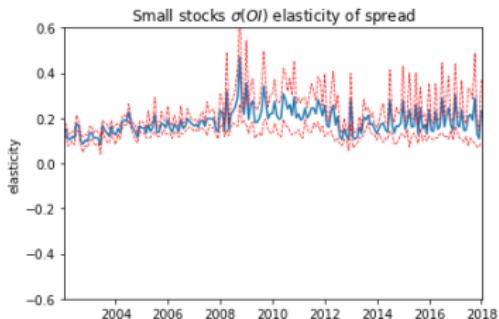
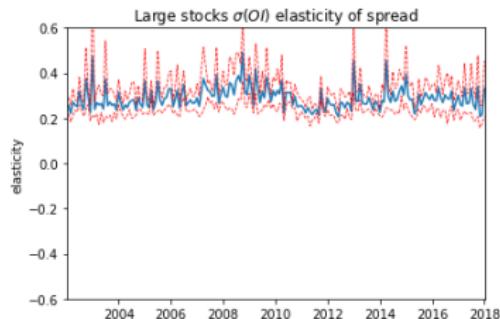
Simple inventory model suggests to distinguish **volume** from **order imbalance** (to capture 'one-sided' volume)

- ▶ Compute order imbalance as a proportion of shares outstanding over every 5mn interval of the trading day
 - ▶ High frequency market making
- ▶ $\sigma(OI)$ is the standard deviation of the 5mn imbalance, computed each day
 - ▶ Control: realized volatility ▶ details

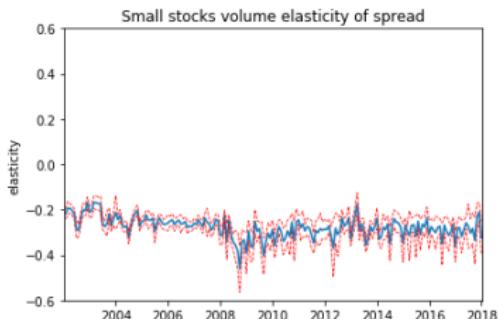
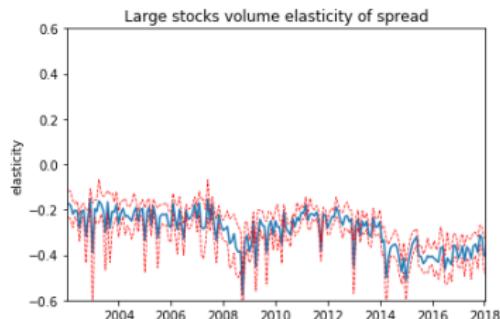
Volatility of Order Imbalances

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \beta_{\sigma(OI)} \log \sigma(OI)_{i,t} + \text{controls} + \epsilon_{i,t}$$

$\sigma(OI)$
elastic-
ity of
spread



volume
elastic-
ity of
spread



average R^2 increases from 11.48% (14.12%) to 22.82% (19.26%)

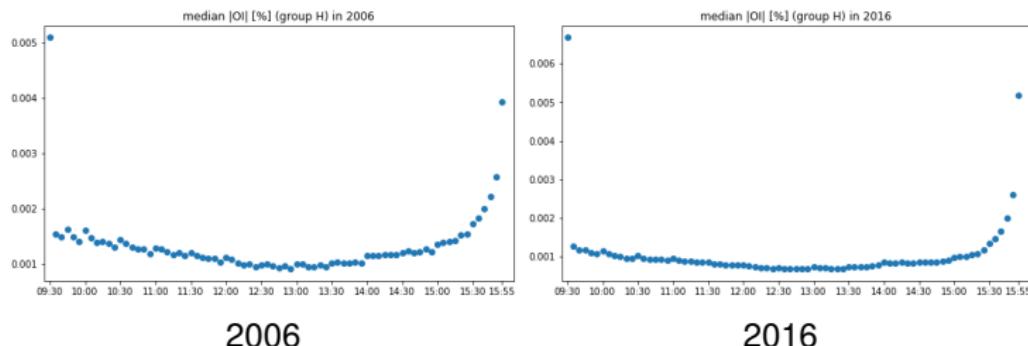
Interpretation of Order Imbalance Volatility $\sigma(OI)$

► Relation with other liquidity measures [► details](#)

- $\sigma(OI)$ is positively associated with [price impact \(Amihud\)](#)
- $\sigma(OI)$ is negatively associated with [depth](#)

► Intraday patterns

- Inventory effects should be stronger at the end of the day



Spread Decomposition

Large stocks in 2018

$$\text{effective spread} = \underbrace{\text{realized spread}}_{\text{trsign}*(p_t - m_{t+5})} + \underbrace{\text{adverse selection}}_{\text{trsign}*(m_{t+5} - m_t)}$$

| Month | (a) Adverse selection | | | (b) Realized spread | | |
|-------|-----------------------|-----------------------|-----------------------------|---------------------|-----------------------|-----------------------------|
| | β_{τ} | β_{RVol} | $\beta_{\sigma(\text{OI})}$ | β_{τ} | β_{RVol} | $\beta_{\sigma(\text{OI})}$ |
| 1 | -1.03** (-2.51) | 1.85*** (9.00) | 0.31 (0.95) | -1.26*** (-9.18) | 0.38** (2.16) | 1.47*** (11.51) |
| 2 | -0.74*** (-3.91) | 1.60*** (10.22) | 0.19 (1.26) | -1.19*** (-7.06) | 0.40** (2.29) | 1.37*** (10.98) |
| 3 | -1.29*** (-3.57) | 1.20*** (4.77) | 1.11* (1.78) | -1.31*** (-7.86) | 0.22 (0.94) | 1.69*** (7.45) |
| 4 | -0.73*** (-4.03) | 1.45*** (16.55) | 0.14 (0.91) | -0.92*** (-9.73) | 0.14 (0.69) | 1.49*** (8.78) |
| 5 | -0.68*** (-3.45) | 1.51*** (10.02) | 0.15 (0.73) | -1.08*** (-8.62) | 0.28** (2.36) | 1.30*** (10.26) |
| 6 | -1.53*** (-2.72) | 1.35*** (4.72) | 1.30 (1.49) | -1.33*** (-4.45) | 0.31 (1.53) | 1.79*** (8.16) |
| 7 | -0.77*** (-3.89) | 1.53*** (11.51) | 0.13 (0.73) | -1.06*** (-7.68) | 0.28* (1.91) | 1.48*** (9.57) |
| 8 | -0.84*** (-3.86) | 1.77*** (17.53) | 0.22 (1.09) | -1.05*** (-8.98) | 0.50*** (3.33) | 1.29*** (10.42) |
| 9 | -1.24*** (-3.28) | 1.31*** (7.14) | 1.06 (1.59) | -1.19*** (-6.84) | 0.06 (0.51) | 1.82*** (6.74) |
| 10 | -0.71*** (-4.76) | 1.64*** (12.93) | 0.13 (0.77) | -0.92*** (-7.99) | 0.53*** (3.88) | 1.15*** (11.65) |
| 11 | 0.17 (0.19) | 1.35* (1.81) | -0.78** (-2.56) | -1.84** (-2.05) | 0.87 (1.25) | 2.26*** (6.00) |
| 12 | -1.11 (-1.20) | 0.75 (0.82) | 0.42 (0.91) | -2.21* (-1.90) | 1.26 (1.22) | 2.74** (2.49) |

⇒ Order imbalance volatility mostly associated with realized spread

Pricing: sequential portfolio sorts

NYSE, Amex, and NASDAQ common stocks over 2002-2017 (797 weekly observations); NYSE breakpoints

| | | α_{FF4}^{VW} (turnover then order imbalance volatility) | | | | | |
|------------|----------|--|---------|---------|---------|-------------------|-----|
| | | low $\sigma(OI)$ | 2 | 3 | 4 | high $\sigma(OI)$ | H-L |
| low turn. | -0.02 | 0.02 | 0.02 | 0.02 | 0.08*** | 0.10** | |
| | (-0.66) | (0.55) | (0.51) | (0.50) | (2.78) | (2.56) | |
| 2 | -0.01 | 0.05* | -0.00 | 0.01 | 0.06* | 0.06 | |
| | (-0.30) | (1.72) | (-0.05) | (0.39) | (1.66) | (1.56) | |
| 3 | 0.00 | 0.03 | 0.06** | 0.09*** | 0.11*** | 0.11*** | |
| | (0.09) | (0.88) | (2.02) | (3.23) | (3.65) | (2.65) | |
| 4 | -0.09*** | 0.00 | 0.01 | -0.04 | 0.12*** | 0.20*** | |
| | (-2.91) | (0.13) | (0.24) | (-1.15) | (4.03) | (4.59) | |
| high turn. | -0.05 | -0.07 | 0.04 | -0.05 | 0.08* | 0.13** | |
| | (-0.94) | (-1.28) | (0.68) | (-0.98) | (1.68) | (1.98) | |

Pricing: value-weighted Fama-MacBeth regressions

NYSE, Amex, and NASDAQ common stocks over 2002-2017 (797 weeks)

dependent variable: r_t (weekly return in percent)

| | coeff. (t-stat) | coeff. (t-stat) | coeff. (t-stat) |
|---|-----------------|-----------------|-------------------|
| $\sigma(\text{OI})_{t-1}$ | 0.064** (2.35) | 0.086*** (3.02) | 0.083*** (3.40) |
| turn_{t-1} | | -0.037 (-1.00) | -0.026 (-0.67) |
| ME_{t-1} | | | -0.012 (-0.31) |
| r_{t-1} | | | -1.652*** (-3.91) |
| ILLIQ_{t-1} | | | -0.009 (-0.25) |
| RVol_{t-1} | | | -0.023 (-0.32) |
| ES_{t-1} | | | -0.023 (-0.63) |
| $\sigma(\text{OI}/\text{VOL})_{t-1}^{\text{month}}$ | | | 0.056 (1.42) |
| \bar{N} | 2,628 | 2,628 | 2,591 |
| \bar{R}^2 | 0.020 | 0.036 | 0.104 |

Conclusion

- ▶ New evidence about the time-series (and cross-sectional) relation between liquidity, volume, and volatility
 - ▶ Adverse selection theories fit well the day-to-day variation in spread, volume, and volatility of small stocks
 - ▶ Inventory risk seems more important for the day-to-day variation in spread, volume, and volatility of large stocks
- ▶ Controlling for volatility of (high-frequency) order imbalances reconciles evidence between large and small stocks
 - ⇒ is consistent with simple inventory risk model, and
 - ⇒ adds substantial explanatory power
- ▶ Order imbalance volatility seems to reflect inventory risk and is priced in the cross-section of weekly returns

Appendix

Descriptive Statistics (Small Stocks)

| | | 2004 | 2008 | 2012 | 2016 |
|----------------|-------------------|---------|---------|---------|---------|
| Small caps | | | | | |
| spread [bp] | mean | 70.18 | 96.68 | 62.69 | 70.32 |
| | median | 51.33 | 50.35 | 40.85 | 44.66 |
| | σ (within) | 48.62 | 103.16 | 49.98 | 63.32 |
| turnover [%] | mean | 0.50 | 0.52 | 0.42 | 0.48 |
| | median | 0.19 | 0.27 | 0.23 | 0.25 |
| | σ (within) | 1.38 | 0.83 | 0.89 | 1.28 |
| volatility [%] | mean | 1.83 | 3.06 | 1.72 | 1.87 |
| | median | 1.53 | 2.44 | 1.50 | 1.51 |
| | σ (within) | 1.06 | 2.13 | 1.02 | 1.58 |
| obs. | | 146,897 | 132,182 | 119,480 | 126,515 |

Descriptive Statistics (Large Stocks)

back

| | | 2004 | 2008 | 2012 | 2016 |
|----------------|-------------------|---------|---------|---------|---------|
| Large caps | | | | | |
| spread [bp] | mean | 8.27 | 8.29 | 4.65 | 4.77 |
| | median | 6.59 | 6.20 | 3.65 | 3.63 |
| | σ (within) | 5.95 | 10.23 | 3.04 | 4.31 |
| turnover [%] | mean | 0.67 | 1.42 | 0.90 | 0.82 |
| | median | 0.46 | 1.03 | 0.67 | 0.61 |
| | σ (within) | 0.58 | 1.22 | 0.74 | 0.63 |
| volatility [%] | mean | 1.17 | 2.70 | 1.16 | 1.23 |
| | median | 1.01 | 2.03 | 1.01 | 1.01 |
| | σ (within) | 0.57 | 1.99 | 0.58 | 0.72 |
| obs. | | 151,157 | 137,730 | 121,479 | 129,411 |

Correlations

cross-sectional averages of the stocks' time-series correlations [back](#)

| | Small caps | | | | | |
|---------------|------------|----------|-------|------|---------------|---------------------|
| | τ | σ | $ r $ | RVol | $ \text{OI} $ | $\sigma(\text{OI})$ |
| s | -0.17 | 0.22 | 0.18 | 0.40 | -0.06 | -0.00 |
| τ | | 0.24 | 0.23 | 0.32 | 0.59 | 0.78 |
| σ | | | 0.49 | 0.47 | 0.10 | 0.12 |
| $ r $ | | | | 0.41 | 0.13 | 0.14 |
| RVol | | | | | 0.12 | 0.17 |
| $ \text{OI} $ | | | | | | 0.60 |

| | Large caps | | | | | |
|---------------|------------|----------|-------|------|---------------|---------------------|
| | τ | σ | $ r $ | RVol | $ \text{OI} $ | $\sigma(\text{OI})$ |
| s | 0.15 | 0.34 | 0.22 | 0.51 | 0.15 | 0.30 |
| τ | | 0.41 | 0.32 | 0.48 | 0.40 | 0.72 |
| σ | | | 0.50 | 0.61 | 0.14 | 0.22 |
| $ r $ | | | | 0.41 | 0.13 | 0.19 |
| RVol | | | | | 0.14 | 0.26 |
| $ \text{OI} $ | | | | | | 0.48 |

How Does Order Imbalance Volatility Affect Other Liquidity Measures?

► Price impact

- In the line of Amihud (2002):

$$\text{ILLIQ}_{it} = \frac{1}{\#\text{traded intervals}} \sum_{k \in \{j | \text{DVOL}_{itj} > 0\}} \frac{|\text{r}_{itk}|}{\text{DVOL}_{itk}}$$

- Alternative: $r_{itk} = \delta_{it} + \lambda_{it} \sqrt{|\text{OI}_{itk}^{\$}|} \text{sign}(\text{OI}_{itk}^{\$}) + e_{it}$
(Hasbrouck (2009))

► Depth

- Time-weighted share depth at the best bid and best ask (as a fraction of shares outstanding)

Price Impact (Amihud)

| Year | β_{τ} | β_{RVol} | $\beta_{\sigma(\text{OI})}$ |
|------|--------------------|-----------------------|-----------------------------|
| 2002 | -1.10*** (-54.89) | 0.90*** (40.95) | 0.24*** (18.72) |
| 2003 | -1.21*** (-56.32) | 0.88*** (81.51) | 0.29*** (27.03) |
| 2004 | -1.20*** (-100.27) | 0.88*** (65.68) | 0.27*** (37.15) |
| 2005 | -1.17*** (-103.29) | 0.90*** (44.62) | 0.24*** (39.38) |
| 2006 | -1.15*** (-98.54) | 0.92*** (87.78) | 0.21*** (35.00) |
| 2007 | -1.12*** (-101.99) | 0.98*** (72.96) | 0.16*** (29.75) |
| 2008 | -1.10*** (-82.55) | 0.96*** (58.25) | 0.10*** (18.51) |
| 2009 | -1.07*** (-141.59) | 0.92*** (41.61) | 0.10*** (15.21) |
| 2010 | -1.10*** (-72.77) | 0.92*** (26.01) | 0.11*** (21.17) |
| 2011 | -1.12*** (-107.44) | 0.96*** (45.96) | 0.11*** (23.25) |
| 2012 | -1.09*** (-101.16) | 0.83*** (72.48) | 0.12*** (19.40) |
| 2013 | -1.14*** (-67.34) | 0.89*** (30.81) | 0.14*** (23.18) |
| 2014 | -1.14*** (-148.76) | 0.88*** (86.04) | 0.15*** (36.93) |
| 2015 | -1.15*** (-123.23) | 0.89*** (66.42) | 0.15*** (32.22) |
| 2016 | -1.14*** (-108.79) | 0.88*** (52.77) | 0.14*** (32.52) |
| 2017 | -1.11*** (-135.75) | 0.79*** (67.02) | 0.15*** (43.56) |

$\bar{R}^2(\%)$

77.05

Depth

▶ back

| Year | β_{τ} | β_{RVol} | $\beta_{\sigma(\text{OI})}$ | β_s |
|------|-----------------|-----------------------|-----------------------------|-------------------|
| 2002 | 0.35*** (20.58) | -0.22*** (-9.94) | -0.00 (-0.32) | -0.19*** (-15.34) |
| 2003 | 0.43*** (22.47) | -0.31*** (-30.46) | -0.04*** (-5.18) | -0.09*** (-16.94) |
| 2004 | 0.47*** (31.15) | -0.42*** (-14.13) | -0.04*** (-7.23) | -0.10*** (-15.55) |
| 2005 | 0.46*** (30.23) | -0.44*** (-15.49) | -0.05*** (-10.80) | -0.07*** (-13.95) |
| 2006 | 0.44*** (30.65) | -0.51*** (-18.67) | -0.06*** (-11.93) | -0.07*** (-12.71) |
| 2007 | 0.41*** (25.41) | -0.56*** (-22.82) | -0.02*** (-4.58) | -0.04*** (-6.93) |
| 2008 | 0.40*** (18.33) | -0.69*** (-17.47) | -0.01*** (-2.70) | 0.02*** (2.78) |
| 2009 | 0.38*** (23.72) | -0.66*** (-22.94) | -0.00 (-0.12) | -0.03*** (-3.54) |
| 2010 | 0.39*** (16.04) | -0.66*** (-14.65) | -0.01 (-1.07) | -0.02** (-2.39) |
| 2011 | 0.38*** (19.13) | -0.65*** (-17.34) | -0.02*** (-3.06) | 0.03*** (4.03) |
| 2012 | 0.35*** (29.22) | -0.40*** (-22.19) | -0.03*** (-6.01) | -0.00 (-0.22) |
| 2013 | 0.40*** (18.79) | -0.48*** (-10.24) | -0.05*** (-9.43) | 0.02** (2.47) |
| 2014 | 0.31*** (34.06) | -0.39*** (-23.40) | -0.01 (-1.56) | -0.01*** (-2.92) |
| 2015 | 0.30*** (21.97) | -0.34*** (-15.81) | -0.02*** (-4.05) | 0.01*** (2.77) |
| 2016 | 0.30*** (15.37) | -0.37*** (-11.26) | -0.02*** (-4.91) | 0.03*** (4.61) |
| 2017 | 0.28*** (26.71) | -0.27*** (-14.35) | -0.03*** (-10.16) | 0.02*** (4.23) |

\bar{R}^2 (%)

41.70

Evidence from Intraday Patterns

The degree of informed trading and liquidity trading is likely not constant over the day

1. Informational advantage of trading on overnight information is likely short-lived (Foster and Viswanathan (1990))
2. Liquidity traders cluster their trades to reduce adverse selection (Admati-Pfleiderer (1980))

Evidence from Intraday Patterns

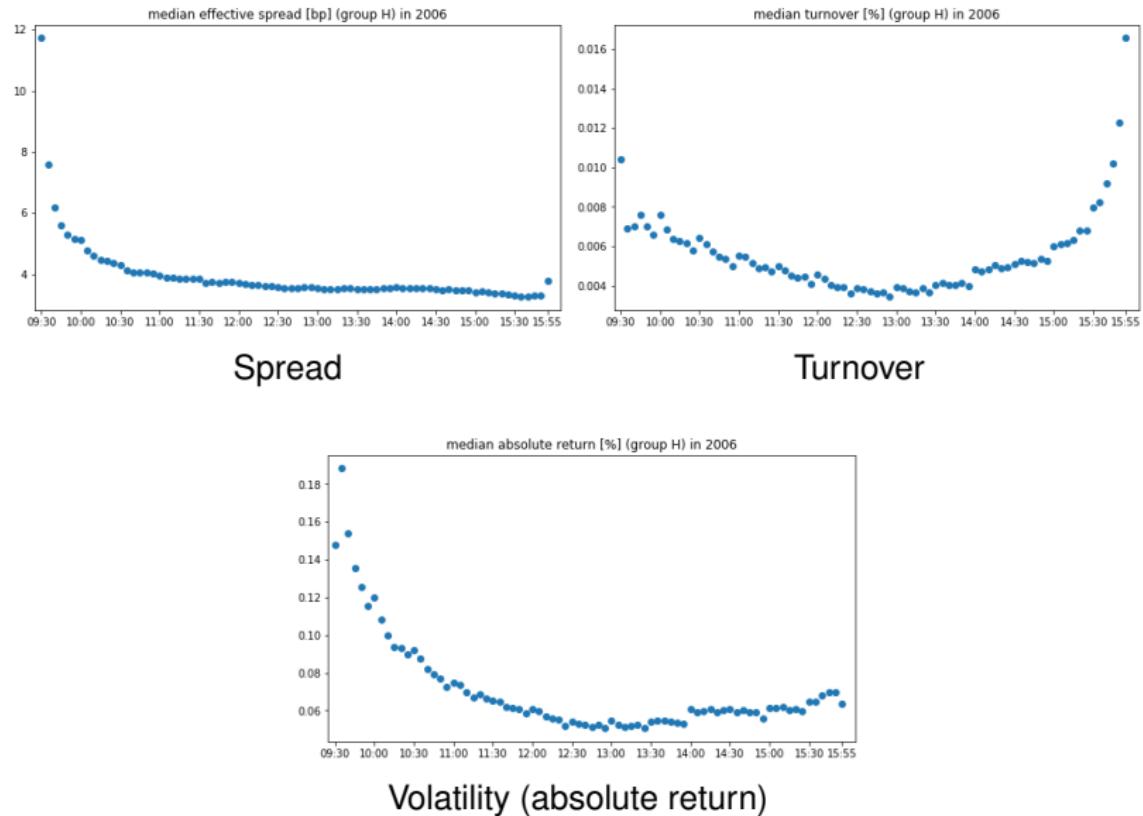
The degree of informed trading and liquidity trading is likely not constant over the day

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Informative to examine intraday patterns of elasticities

- ▶ Split the day into five-minute intervals and focus on large stocks
- ▶ We are *not* looking at levels but at sensitivities
 - ▶ Control for interval-stock fixed effects

Intraday Median Values - 2006



Intraday Evidence

- ▶ Volume elasticity of spread is higher at the end of the day, when inventory risk or market power may be high
 - ▶ Consistent with evidence from intraday order imbalances
 - ▶ Appendix
- ▶ The intraday elasticity pattern does not 'mechanically' reflect intraday variations in spread, volume, and volatility
 - ▶ Spreads may be lower around the close but are more sensitive to trading volume

This evidence supports adverse selection effects and competition/inventory effects

- ▶ More competitive liquidity provision in recent years?

Volume in the continuous-time Kyle model

- ▶ $VOL = \frac{1}{2}(|dX_t^i| + |dX_t^u| + |dX_t^i + dX_t^u|)$
- ▶ Insider trade in absolutely continuous fashion: $dX_t^i = \mu_i dt$
- ▶ Whereas $dX_t^u = \sigma_u dZ_t$ for some Brownian motion Z_t
- ▶ $E[VOL]^2 = 2/\pi\sigma_u^2 dt$
- ▶ Total cumulative order flow is $Y_t = X_t^u + X_t^i$ and $Var[dY_t] = \sigma_u^2 dt$

Inventory Shocks and Endogenous Entry

Allow for entry of liquidity providers at a fixed cost in the model of [Campbell, Grossman, and Wang \(1993\)](#)

- ▶ Stationary OLG economy with exogenous risk-free rate and a risky asset that pays dividends every date
- ▶ Liquidity providers with exponential utility absorb volatile supply shocks every date
- ▶ In equilibrium, we show that an increase in the volatility of supply shocks *decreases* price impact, in contrast to the original model
- ▶ The inventory explanation requires some barriers to entry

Gallant-Rossi-Tauchen (1992) Methodology ▶ back

For each stock regress the spread and turnover series on a set of control variables x :

$$y = x'\beta + u.$$

The residuals are used to construct the following variance equation:

$$\log(u^2) = x'\gamma + v.$$

The adjusted y series is then given by:

$$y_{\text{adj}} = a + b(\hat{u}/\exp(x'\gamma/2)),$$

where the parameters a and b are chosen such that the mean and standard deviation of y_{adj} are the same as that of y .

Control variables x : day-of-the-week dummies; month-of-the-year dummies; a dummy for trading days around holidays when the stock market is closed; a dummy for trading days on federal holidays when the stock market is open; linear and quadratic trend variables. For the turnover series, we also include a cubic trend variable.

Measure of Volatility: Realized Volatility

What about a more sophisticated measure of volatility?

- ▶ *Realized variance*: $\text{RVol}(K)_t^2 = \sqrt{\sum_{k=1}^K r_{t,k}^2}$, where $r_{t,k}$ is the intraday return over interval k
- ▶ But what should we expect?

Using log returns, it can be shown that:

$$\text{RVol}(k)_t^2 = r_t^2 + \Pi_t,$$

where $\Pi_t = \sum_{k=2}^K (-2 \sum_{j=1}^{k-1} r_{t,j}) r_{t,k} \Rightarrow$ intraday reversal strategy
 $\text{corr}(s_t, \Pi_t) > 0$?

Large Stocks' Elasticities with Realized Volatility

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C} \tau_{i,t}^C + \beta_{\tau,I} \tau_{i,t}^I + \beta_{\text{RVol}} \text{RVol}_{i,t} + \text{controls} + \epsilon_{i,t}$$

| Year | $\beta_{\tau,C}$ | $\beta_{\tau,I}$ | β_{RVol} |
|------|------------------|------------------|-----------------------|
| 2002 | 0.12** (2.46) | 0.02** (2.47) | 0.42*** (13.22) |
| 2003 | -0.05 (-1.05) | 0.08*** (11.45) | 0.45*** (42.76) |
| 2004 | 0.01 (0.29) | 0.07*** (11.23) | 0.38*** (39.58) |
| 2005 | 0.16*** (3.18) | 0.07*** (11.79) | 0.34*** (28.41) |
| 2006 | 0.11*** (2.77) | 0.08*** (11.05) | 0.30*** (29.47) |
| 2007 | 0.25*** (5.45) | 0.09*** (8.98) | 0.33*** (16.58) |
| 2008 | 0.12*** (2.64) | 0.00 (0.18) | 0.42*** (17.93) |
| 2009 | 0.09** (1.99) | 0.03*** (3.28) | 0.24*** (11.07) |
| 2010 | 0.10*** (2.70) | 0.03*** (3.40) | 0.27*** (11.77) |
| 2011 | 0.06** (1.96) | 0.02* (1.73) | 0.30*** (17.04) |
| 2012 | 0.27*** (3.12) | 0.03*** (3.23) | 0.27*** (16.69) |
| 2013 | 0.13*** (2.68) | 0.02 (1.52) | 0.31*** (16.16) |
| 2014 | 0.08 (1.19) | -0.06*** (-4.30) | 0.34*** (17.54) |
| 2015 | -0.00 (-0.01) | -0.11*** (-9.76) | 0.41*** (19.61) |
| 2016 | -0.07* (-1.96) | -0.11*** (-8.53) | 0.39*** (18.30) |
| 2017 | 0.11 (1.52) | -0.10*** (-7.36) | 0.40*** (20.47) |

\bar{R}^2 (%)

20.59

Large Stocks' Elasticities with Realized Volatility

▶ back

$$\Delta s_{i,t} = \alpha_i + \beta_{\tau,C} \Delta \tau_{i,t}^C + \beta_{\tau,I} \Delta \tau_{i,t}^I + \beta_{\text{RVol}} \Delta \text{RVol}_{i,t} + \text{controls} + \epsilon_{i,t}$$

| Year | $\beta_{\tau,C}$ | $\beta_{\tau,I}$ | β_{RVol} |
|------|------------------|------------------|-----------------------|
| 2002 | 0.18*** (2.68) | 0.06*** (6.10) | 0.35*** (10.06) |
| 2003 | 0.00 (0.02) | 0.14*** (16.32) | 0.41*** (35.94) |
| 2004 | 0.15*** (3.10) | 0.13*** (18.16) | 0.36*** (39.17) |
| 2005 | 0.29*** (4.30) | 0.16*** (19.68) | 0.31*** (27.00) |
| 2006 | 0.23*** (4.96) | 0.16*** (18.66) | 0.26*** (25.61) |
| 2007 | 0.52*** (6.81) | 0.22*** (14.90) | 0.25*** (13.52) |
| 2008 | 0.37*** (4.75) | 0.10*** (9.23) | 0.31*** (15.01) |
| 2009 | 0.28*** (3.64) | 0.12*** (9.35) | 0.19*** (9.09) |
| 2010 | 0.29*** (5.14) | 0.13*** (10.37) | 0.21*** (9.33) |
| 2011 | 0.19*** (4.51) | 0.10*** (9.62) | 0.23*** (16.73) |
| 2012 | 0.43*** (3.28) | 0.13*** (9.07) | 0.19*** (10.59) |
| 2013 | 0.24*** (3.94) | 0.11*** (8.10) | 0.25*** (16.15) |
| 2014 | 0.32*** (3.30) | 0.03* (1.79) | 0.28*** (15.30) |
| 2015 | 0.20*** (3.22) | -0.02 (-1.24) | 0.32*** (14.16) |
| 2016 | 0.16** (2.39) | -0.03*** (-2.64) | 0.34*** (20.53) |
| 2017 | 0.39*** (3.58) | -0.01 (-0.44) | 0.32*** (18.53) |

\bar{R}^2 (%)

8.84