EPFL & SFI

Information and Asset Prices

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Assignment 2

1. Glosten Milgrom model with normally distributed asset value

Suppose the payoff of a stock at time 1 is $v \sim (v_0, \sigma_v^2)$. Suppose that a set of informed investors observe a signal $z = v + \epsilon$ where $\epsilon \sim (0, \sigma_\epsilon^2)$ independent of v. Competitive market makers have to quote a bid-ask spread $\{A, B\}$ to an arriving trader who wants to buy or sell one unit so as to earn zero profits on average. That is they quote $A = E[v \,|\, buy]$ and $B = E[v \,|\, sell]$. Define $m(z) = E[v \,|\, z]$ the informed traders' estimate of the asset value.

1. What information does a buy (respectively a sell) order from an informed client convey to the market maker about $m(z)$?
2. Prove that $E[m(z) \,|\, m(z) > K] = E[v \,|\, m(z) > K]$. Use this result to show that if it is common-knowledge that there are only informed agents in the market then there can be no equilibrium (i.e., the ask-bid spread would tend to $(-\infty, +\infty)$).
3. Prove that if $X \sim N(\mu_x, \sigma_x^2)$ then $E[X|X > K] = \mu_X + \sigma_X \frac{n(\frac{K-\mu}{\sigma})}{1-N(\frac{K-\mu}{\sigma})}$ where $n(x)$ is the standard normal density function and $N(x)$ its cumulative distribution.
4. Suppose now that there is a fraction α of traders that are informed and a fraction $1-\alpha$ of traders that are uninformed and that will buy (for non-informational reasons) with probability γ^b and sell with probability $\gamma^s = 1 - \gamma^b$.

   Show that the competitive ask and bid prices posted by the market makers each solve a non-linear equation that depends on the parameters of the model. Can you prove (under what conditions) there exists an equilibrium? Analyze the solution (numerially), to see how bid-ask spreads depend on $\alpha, \sigma_\epsilon, \sigma_v$.
5. Will market makers change their bid-ask spread after they observe a first buy (respectively sell) order?

2. Kyle (1985) with risk-aversion and residual risk

Consider a Kyle (1985) model where the risk-averse 'insider' is endowed at date 0 with $X_0 \sim N(\mu_x, \sigma_x^2)$ shares in a firm whose terminal payoff is $v \sim N(v_0, \sigma_v^2)$. The payoff v and the initial position X_0 are known to the 'insider' at date 0 and will become public at date 1 to all market participants. Suppose further that X_0 is correlated with the asset value v with some correlation coefficient ρ. The insider maximizes $E[-e^{-\gamma W_1}]$ by sending market orders for ΔX shares to the market-maker who will execute the trade at price P. The market maker sets the price P competitively, so as to have zero expected profit across all her trades. In addition to orders from the insider, the market maker receives market orders from noise traders $U \sim N(0, \sigma_u^2)$. Thus the market maker observes only the total order flow $Y = \Delta X + U$. Assume that investors can also invest at a risk-free rate that we set to zero for simplicity.

1. Solve for the equilibrium trading strategy and the equilibrium price.
2. Analyze your solution as a function of risk-aversion. How does an increase in γ change (i) the market liquidity, i.e., Kyle's lambda, (ii) the equilibrium price, (iii) information efficiency (i.e., posterior variance of the fundamental conditional on order-flow) relative to the original Kyle (1985) model with $\gamma = 0$?
3. Does the (uncertainty about the) initial position $(X_0, \mu_X, \rho, \sigma_x^2)$ of the insider affect the equilibrium price and market liquidity? Give some intuition.
4. Instead, assume now that terminal value is $v + \epsilon$, where ϵ $N(0, \sigma_\epsilon^2)$ is some residual risk faced by the insider that is uncorrelated with v, X_0. Otherwise the structure of the model is identical. Derive the equilibrium in this market. Does residual risk change the predictions of the model? In particular, how does uncertainty about the initial position of the insider affect equilibrium prices and market liquidity in the csae with residual risk? Can you derive equilibria with negative price impact? Give some intuition.

   Hint: you may resort to numerical analysis to illustrate the properties of the equilibrium (in particular in the case with residual risk this could be helpful).

3. Kyle (1985) with many informed agents

1. Solve the Kyle (1985) one-period model with N informed risk-neutral agents who get to observe the draw of the asset value $v \sim N(v, \sigma_v^2)$ before sending their market orders X_i $i = 1, \ldots, N$ to the market maker. Market Makers are risk-neutral, observe the total order flow $Y = \sum_{i=1}^{N} X_i + U$ and set prices so as to clear the market and break-even. How does this affect
   - the optimal demand of each of the insiders,
   - the equilibrium price impact of their trades (λ),
   - their unconditional expected average profits?
2. What happens to price discovery (market efficiency) when $N \to \infty$?

4. Kyle (1985) with different information structure

1. Solve the Kyle (1985) one-period model, but assume that the informed agent gets to observe the draw of the asset value $v \sim N(v, \sigma_v^2)$ as well as the draw of the uninformed traders' demand $U \sim N(0, \sigma^2)$, before sending her market order X to the market maker. Market Makers are risk-neutral, observe the total order flow $Y = X + U$ and set prices so as to clear the market and break-even. How does this affect
   - the optimal demand of the insider,
   - the equilibrium price impact of her trades (λ),
   - her unconditional expected average profits?
2. Now, suppose that the market maker can observe X and U separately (e.g., the order flows arrive from different computer IP numbers, but she does not know which one is informed and which one is not).
   - Can you solve the Kyle (1985) equilibrium in that case? What changes?