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Benchmarks in Search Markets

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ABSTRACT

We characterize the role of benchmarks in price transparency of over-the-counter markets. A benchmark can raise social surplus by increasing the volume of beneficial trade, facilitating more efficient matching between dealers and customers, and reducing search costs. Although the market transparency promoted by benchmarks reduces dealers' profit margins, dealers may nonetheless introduce a benchmark to encourage greater market participation by investors. Low-cost dealers may also introduce a benchmark to increase their market share relative to high-cost dealers. We construct a revelation mechanism that maximizes welfare subject to search frictions, and show conditions under which it coincides with announcing the benchmark.

AN ENORMOUS QUANTITY OF OVER-THE-COUNTER (OTC) trades are negotiated by counterparties who rely on the observation of benchmark prices. In this paper we explain how benchmarks affect pricing and trading behavior by reducing market opacity, we characterize the welfare impact of benchmarks, and we show how the incentives of regulators and dealers to support benchmarks depend on market structure.

Trillions of dollars in loans are negotiated at a spread over LIBOR or EURIBOR, benchmark interbank borrowing rates. LIBOR is the London Interbank Offered Rate. EURIBOR is the Euro Interbank Offered Rate. For U.S. dollar LIBOR alone, the Market Participants Group (MPG) on Reference Rate Reform (2014) (chaired by one of the authors of this paper) reports that over

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3 trillion dollars in syndicated loans and over 1 trillion dollars in variable-rate bonds are negotiated relative to LIBOR. The report of the Market Participants Group lists many other fixed-income products that are negotiated at a spread over the "interbank offered rates" known as LIBOR, EURIBOR, and TIBOR, across five major currencies. As of the end of 2013, the Bank for International Settlements (2014) reports a total notional outstanding of interest rate derivatives of 583 trillion U.S. dollars, the vast majority of which reference LIBOR or EURIBOR. These swap contracts and many other derivatives reference benchmarks but are not themselves benchmark products. Other extremely popular benchmarks for overnight interest rates include SONIA, the Sterling OverNight Index Average, and EONIA, the Euro OverNight Index Average. The WM/Reuters daily fixings are the dominant benchmarks in the foreign exchange market, which covers over $5 trillion per day in transactions.[1] There are also popular benchmarks for a range of commodities including silver, gold, oil, and natural gas, among others.[2] Benchmarks are additionally used to provide price transparency for manufactured products such as pharmaceuticals and automobiles.[3]

Among other roles, benchmarks mitigate search frictions by lowering the informational asymmetry between dealers and their "buy-side" customers. We consider a market for an asset in which dealers offer price quotes to customers who are relatively uninformed about the typical cost to dealers of providing the asset. We provide conditions under which adding a benchmark to an opaque OTC market can improve efficiency by encouraging entry by customers, improving matching efficiency, and reducing total search costs.

Recent major scandals over the manipulation of benchmarks for interest rates, foreign currencies, commodities, and other assets have made the robustness of benchmarks a major concern of international investigators and policy makers. This paper offers a theoretical foundation for the public policy support of transparent financial benchmarks. In Section IV we discuss the manipulation of benchmarks in more detail.

Our model works roughly as follows. In an OTC market with a finite number of dealers and a continuum of investors that we call "traders," the cost to a dealer of providing the asset to a trader is the sum of a dealer-specific (idiosyncratic) component and a component that is common to all dealers. (In practice the clients of financial intermediaries may be buying or selling the asset. We consider the case in which traders wish to buy. The opposite case is effectively the same, up to sign changes.) The existence of a benchmark is taken to mean

[1] See Foreign Exchange Benchmark Group (2014), which reports that 160 currencies are covered by the WM/Reuters benchmarks. These benchmarks are fixed at least daily and by currency pair within the 21 major "trade" currencies.

[2] The London Bullion Market Association provides benchmarks for gold and silver. Platts provides benchmarks for oil, refined fuels, and iron ore (IODEX). Another major oil price benchmark is ICE Brent. ICIS Heren provides a widely used price benchmark for natural gas.

[3] For a discussion of the Average Wholesale Price (AWP) drug price benchmarks, see Gencarelli (2005). The Kelly Blue Book publishes the "Fair Purchase Price" of automobiles based on the average transaction price by model and location.

that the common cost component is publicly announced. Each trader privately observes whether her search cost is high or low. Traders are searching for a good price, and dealers offer them price quotes that depend endogenously on the presence of a benchmark. Each dealer posts an offer price, which is available for execution by any trader, anonymously. Traders, who have a commonly known value for acquiring the asset, contact the dealers sequentially, expending a costly search effort or costly delay with each successive dealer contacted. At each point in time, the trader, given all of the information available to her at that time (including past price offers and, if published, the benchmark), decides whether to buy, continue searching, or exit the market. All market participants maximize their conditional expected net payoffs, at all times, in a perfect Bayesian equilibrium.

Under natural parameter assumptions, which vary with the specific result, we show that publishing the benchmark is socially efficient because of three types of effects.

First, publication of the benchmark encourages efficient entry by traders, thus increasing the realized gains from trade. The benchmark improves the information available to traders about the likely price terms they will face. This assists traders in deciding whether to participate in the market, based on whether there is a sufficiently large conditional expected gain from trade. The increased transparency of prices created by the benchmark induces dealers to compete more aggressively in their quotes. In this sense, publication of the benchmark mitigates the hold-up problem caused by dealers' incentives to quote less attractive prices once the search costs of traders have been sunk.

Second, benchmarks improve matching efficiency, which leads to a higher market share for low-cost dealers. When the benchmark is not observed by traders, high-cost dealers exploit the ignorance of traders about the cost of providing the asset and may conduct sales despite the presence of more efficient competitors. The benchmark allows traders to decompose a price offer into a common-cost component and a dealer-specific component for cost and profit margin. As a result, if search costs are sufficiently small, customers trade with the most efficient dealers.

Finally, benchmarks reduce wasteful search by *(i)* alerting traders that gains from trade are too small to justify entry, and *(ii)* helping traders infer whether they should stop searching because they have likely encountered a low-cost dealer.

We also characterize cases in which the introduction of a benchmark *lowers* welfare. This can happen when the market is already relatively efficient without the benchmark.[4]

We embed the price transparency problem—add a benchmark or not—into a broader design framework by characterizing a socially optimal revelation mechanism. Here, we take the case in which dealers have the same costs. We show that whenever the gain from trade between a dealer and a trader

[4] This finding is consistent with the insight of Asriyan, Fuchs, and Green (2015) (in a very different model) that welfare can be nonmonotone in the degree of transparency.

is lower than an endogenous threshold, an optimal mechanism reveals the benchmark. However, when the gain from trade is above the threshold, the optimal mechanism reveals only this fact, without informing traders about the exact level of the gain. Going further, we derive conditions under which the optimal mechanism coincides with disclosing the benchmark. In broad terms, publishing the benchmark approximates the optimal mechanism whenever an opaque market (with no information about costs available to investors) would generate low participation by traders.

A related question is: Who implements a benchmark? Perhaps surprisingly, dealers often have sufficiently strong incentives to add a benchmark. What matters is whether the resulting reduction in dealer profit margin is more than offset by the increased volume of trade. This helps explain why almost all existing benchmarks have been introduced by dealers without regulatory pressure. On the other hand, there are cases in which benchmarks would enhance welfare but dealers lack the incentives to introduce them. Thus, there is scope for regulators to improve market efficiency by promoting benchmarks or other forms of price transparency. The introduction by the Financial Industry Regulatory Authority (FINRA) of post-trade transparency in the U.S. corporate bond markets is a case in point. Recently, in succession, the United Kingdom, Japan, and the European Union have introduced legislation in support of financial benchmarks. As of this writing, the United States has no benchmark legislation.

When dealers have heterogeneous costs for providing the asset, we show that the most efficient dealers can use a benchmark as a "price transparency weapon" that drives inefficient competitors out of the market and draws trades to dealers in the "benchmark club." This may help explain why benchmarks such as LIBOR were first introduced into the Eurodollar loan market by large London-based banks.[5]

Our results are consistent with a significant body of empirical literature on the impact of adding post-trade transparency to the U.S. corporate bond market with the introduction of TRACE in 2003. Bid-ask spreads were usually (although not always) lowered by TRACE, as shown by Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Goldstein, Hotchkiss, and Sirri (2007), and Asquith, Covert, and Pathak (2013). However, Asquith, Covert, and Pathak (2013) also show that TRACE lowered transaction volumes in some less liquid segments of the market. They speculate that some dealers may have reduced their commitment of capital to the market because the additional price transparency reduced their intermediation rents.[6] Consistent with this view, we show that improved price transparency squeezes the market share and profit of less efficient dealers.

Our analysis draws upon techniques first used in search-based models of labor markets, in a literature surveyed by Rogerson, Shimer, and Wright

[5] See Hou and Skeie (2013).

[6] Additional arguments for and against greater price transparency in the corporate bond market are discussed by Bessembinder and Maxwell (2008).

(2005). The framework that we consider features mixed strategies in pricing, as modeled by Varian (1980), Burdett and Judd (1983), and Stahl (1989), among others, and uncertainty about the distribution of prices, as in Rothschild (1974). Our model builds on that of Janssen, Pichler, and Weidenholzer (2011), with two important differences that allow us to study welfare implications. First, we introduce endogenous entry to study efficient participation in the market.[7] With endogenous entry, we show that the result of Janssen, Pichler, and Weidenholzer (2011)—that sellers never wish to disclose their costs to the market—may fail. Indeed, in our model setting, the fact that dealers often wish to publish a benchmark is consistent with the historical emergence of dealer-supported financial benchmarks. Second, we permit heterogeneity in dealers' costs. We show that benchmarks promote trade with more efficient dealers.

Our analysis of matching efficiency is related to Benabou and Gertner (1993), who examine the influence of inflationary uncertainty (similar in spirit to the effect of cost uncertainty in our model) on welfare and on the split of surplus between consumers and firms. The relationship between their approach and ours with regard to uncertainty can be described as "local" versus "global." Benabou and Gertner (1993) examine the marginal effect on welfare when uncertainty is reduced slightly, while the introduction of a benchmark in our setting significantly reduces this source of uncertainty. A limitation of their model is its restriction to only two sellers.

The rest of the paper is organized as follows. Section I describes the transparency role of benchmarks in OTC financial markets. Section II solves a search model in which dealers have homogeneous costs of providing an asset to customers, with a focus on how adding a benchmark affects entry efficiency. Section III extends the model to heterogeneous dealers' costs, with a focus on how adding a benchmark affects matching efficiency. Section IV briefly discusses benchmark manipulation and implementation. Section V concludes.

I. The Role of Benchmarks in Over-the-Counter Markets

A benchmark price is a measure of "the going price" of a standardized asset at a specified time.[8] Benchmarks are usually published at a daily or sometimes higher frequency, and are used for at least three main purposes:[9]

[7] Janssen, Moraga-González, and Wildenbeest (2005) model the entry of buyers when sellers' cost is common knowledge but they do not focus on the effect of information disclosure about dealers' costs.

[8] The standardized asset may actually be a composite of several closely related assets, as for the case of the Brent oil benchmark, which is a "basket of physical oil cargoes in the North Sea—Brent, Forties, Oseberg, and Ekofisk (BFOE)." See Bank of England (2014), which states that "a 'benchmark' means an index, rate or price that: (a) is determined from time to time by reference to the state of the market; (b) is made available to the public (whether free of charge or on payment); and (c) is used for reference for purposes that include one or more of the following: (i) determining the interest payable, or other sums due, under loan agreements or under other contracts relating to investments; (ii) determining the price at which investments may be bought or sold or the value of investments; (iii) measuring the performance of investments."

[9] For more discussion of these and other roles of benchmarks, see Duffie and Stein (2015).

(i) The settlement of contracts, such as forwards or options, whose payoffs depend formulaically on the benchmark price of the referenced asset.
(ii) Ex post monitoring by nondealer market participants of the quality of trade execution that they have received.
(iii) Price transparency in comparison shopping, that is, for the purpose of comparing a quoted price to the benchmark price, which is a signal of prices that might be available elsewhere in the market. Comparison shoppers can then decide whether to accept the quoted price or to look for a better one.

When discussing the benefit of regulating benchmarks, the Bank of England (2014) refers to the price transparency role of benchmarks as one of "determining the price at which investments may be bought or sold."

All of these roles are important. In this paper, we focus on the third role, comparison shopping in otherwise opaque OTC markets. This role also has some connection with the second role of execution monitoring. Suppose, for example, that a firm normally relies on its main relationship bank to convert its foreign currency receivables into its own currency. With the benefit of the published WM/Reuters daily foreign exchange benchmarks, the firm can monitor whether its bank has actually achieved good execution prices on the firm's behalf. If not, the firm would eventually ask a different bank to perform the same service. Although this suggests a different model from that offered in this paper, the economic impact of a benchmark on market efficiency through its execution monitoring role is obviously closely related to its impact through its price transparency role in a search-based market.

Benchmarks would be almost redundant, from the viewpoint of pre-trade price transparency, if the best executable price quotes were published and accessible to all market participants, for example, on an open central limit order book. Markets with this high level of pre-trade transparency include those for exchange-traded equities and derivatives. Our model is instead more relevant to the case of an opaque OTC market, in which a high level of pre-trade price transparency is not available. In opaque OTC markets, buy-side investors are generally not aware of recent transaction prices, the range of quotes that dealers might provide to them, or which dealers are providing better quotes at a given time. These OTC markets cover standardized loans, foreign exchange, repurchase agreements, certain OTC derivatives, and many types of commodities.

Price transparency in some OTC markets is increased through benchmarks, multidealer electronic trading platforms, or various forms of post-trade transaction reporting. For example, some types of U.S. bond markets have post-trade transaction reporting through TRACE.[10] The Dodd–Frank Act, Japanese regulations, and the European Union's revised Markets in Financial Instruments Directive (MiFID II) mandate post-trade transaction reporting for some

[10] See Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Goldstein, Hotchkiss, and Sirri (2007), and Asquith, Covert, and Pathak (2013).

classes of OTC financial products. OTC markets that have less comprehensive transaction reporting and rely more heavily on benchmarks for price transparency include those for large short-term bank loans, foreign exchange, and commodities.

II. A Model of Benchmarks as a Transparency Tool

This section describes a search-based model of an OTC market, the equilibrium behavior of market participants, and its efficiency properties. The main results compare the social surplus generated by a market that includes a benchmark with that of a market that does not include a benchmark but is otherwise identical. This section addresses the case of homogeneous dealers' costs. Section III introduces heterogeneity in dealers' costs, allowing for an effect of benchmarks on matching efficiency.

A. Model Setup

This subsection introduces model primitives, which include market participants, the trading protocol, and the definition of market equilibrium. Interpretation and further motivation of the model primitives are found toward the end of this subsection.

Market participants consist of a finite number $N \geq 2$ of dealers and an infinite set of traders distributed uniformly on $[0, 1]$. All trades are for a unit amount of a given asset. For concreteness, we model trader–dealer encounters in which a dealer sells and a trader buys. The model can be equivalently formulated with the buying and selling roles reversed.[11] The important distinction between the two types of agents is that dealers make markets by offering executable price quotes, whereas traders contact dealers sequentially and accept their quotes or not, in a manner to be described.

Any dealer can supply the asset at a per-unit cost of c, a random variable with a cumulative distribution function G whose support is $[\underline{c}, \bar{c}]$, for some $\underline{c} \geq 0$ and $\underline{c} < \bar{c} < \infty$.

All traders have a known constant value $v > 0$ for acquiring the asset. We consider the case $v \leq \bar{c}$, so that the gain from trade, $\max\{v - c,\ 0\}$, is zero for sufficiently high cost outcomes. Trader $j \in [0,\ 1]$ incurs a search cost of s_j for making contact with a new dealer. For tractability, we suppose that $s_j = 0$ with some probability μ in $(0, 1)$, and that $s_j = s$ with probability $1 - \mu$, for some constant $s > 0$. Search costs are independent across almost every pair of traders. By the exact law of large numbers of Sun (2006), μ is also the fraction of traders with zero search cost, almost surely.[12] The presence of some traders

[11] In financial markets, "buy" and "sell" should not be interpreted literally as buying or selling a good, like a car. For instance, in the market for credit default swaps, a dealer can sell protection and the trader buys protection, with zero market value of the contract. Similarly, in a loan market, the dealer who sells the asset may be interpreted as a bank that provides a loan.

[12] We adopt Sun's construction of the agent space and probability space, and the measurable subsets of the product of these two spaces, so as to allow without further comment various

with zero search cost overcomes the usual Diamond paradox.[13] Because search costs in practice often arise from delay costs, for simplicity and concreteness we refer to traders with zero search cost as "fast traders" and to those with nonzero search cost as "slow traders."

The presence of a benchmark is taken to mean the publication of the dealers' cost c. We compare two market designs: the benchmark case and the no-benchmark case.

The game proceeds as follows. If there is a published benchmark, then c is first revealed to all traders. Each dealer i then posts a price p_i that constitutes a binding offer to sell one unit of the asset at this price to any trader. This offer price is observed only by those traders who contact the dealer. The dealer is aware of whether a benchmark is published when quoting the price p_i. The dealer's offer p_i does not lapse if a trader initially declines it and later returns to take the offer. This assumption is made for tractability.[14]

Traders, without yet having observed the quotes of any dealers, make entry decisions. Entry means contacting one of the dealers, chosen with equal likelihood across the N dealers, and observing that dealer's offer. The entry of a slow trader is equivalent to (first-time) search and incurs the cost s. Failure to enter the market ends the game for the trader. If a trader enters, he may accept the first offer or continue searching by contacting another randomly selected dealer, again with the uniform distribution over the yet-to-be-visited dealers. The order of dealer contacts is independent across traders. At any point, a trader may choose to accept the offer from any previously contacted dealer, in which case a transaction is made at the chosen dealer's offer and the trader leaves the market. A trader may exit the market at any point without trading, even after having contacted all N dealers.

Dealers observe neither the price offers posted by other dealers nor the order in which traders contact dealers. Traders observe nothing about the searches or transactions of other traders.

A (mixed) strategy for dealer i is a measurable function mapping the dealer's cost c to a probability distribution over price offers. In the absence of a benchmark, a strategy for trader j maps the trader's search cost s_j and any prior history of observed offers to a choice from: (i) accept one of the observed offers, (ii) continue searching, or (iii) exit. (If the trader has not yet visited any dealer, the decision to continue searching is equivalent to the decision to enter the market.) In the presence of a benchmark, the strategy of a trader may also

applications of the exact law of large numbers for a continuum of essentially pairwise-independent random variables.

[13] The Diamond paradox (Diamond (1971)) refers to cases in which all dealers charge the monopoly price in a unique equilibrium with no search.

[14] Relaxing this "recall" assumption in Section II makes no difference to equilibrium behavior because, as we will show, a slow trader accepts the first quote on the equilibrium path. (See also Janssen and Parakhonyak (2013).) In Section III the recall assumption substantially simplifies the analysis. Zhu (2012) shows that, without recall, revisiting a dealer is taken as a negative signal of a trader's outside option and leads to a worse quote.

depend on the published benchmark c. The payoff of dealer i is $(p_i - c)Q_i$, where Q_i is the total quantity of sales[15] by dealer i. If trader j successfully conducts a purchase, say from dealer i, then her payoff is $v - p_i - s_j K_j$, where K_j is the number of dealers that she contacted. If she does not purchase the asset, then her payoff is $-s_j K_j$.

An equilibrium is a collection of strategies for the respective agents, possibly mixed (allowing randomization), with the property that each agent's strategy maximizes at each time that agent's expected payoff conditional on the information available to the agent at that time and taking as given the strategies of the other agents. We focus on symmetric perfect Bayesian equilibria. We also assume, essentially without loss of generality, that fast traders play their weakly dominant strategy of always entering the market and contacting all dealers.[16] As is conventional in the literature covering search-based markets, we restrict attention to reservation-price equilibria unless otherwise indicated. These are equilibria in which a trader's decision to continue searching can be based at any time on a cutoff for the best offer observed up to that point.

Our definition of the benchmark as the dealer cost c captures the essence of benchmark practice for a range of OTC markets in which the benchmark is the interdealer price. For example, LIBOR is the average lending rate in the interbank market. (In our model, all banks have the same cost.) Banks then offer loans to their customers at spreads over LIBOR. The LBMA Gold Price, the gold benchmark of the London Bullion Market Association, is the market-clearing price set in an interdealer auction that is run every day for the express purpose of determining the daily gold benchmark. Dealers then quote gold prices to their customers, who are aware of the previously published fixing. (The LBMA Silver Price has a similar daily fixing.) The WM/Reuters foreign exchange benchmark for each major currency pair is the average of transaction prices on two leading electronic trading platforms that occur over a five-minute fixing window.[17] As with LIBOR, this implies that publishing the foreign exchange benchmark still leaves some residual noise in customer assessments of dealer costs that does not apply in our basic model. We consider this effect later in the paper. In general, we avoid more complicated models of the benchmark simply for reasons of tractability and conciseness. Section II.F provides conditions under which publishing the dealer cost c provides the socially optimal level of pre-trade transparency.

[15] That is, $Q_i = \int_0^1 1_{(i,j)}\, dj$, where $1_{(i,j)}$ has the outcome of one if trader j accepts the offer of dealer i, and of zero otherwise. This integral is always well defined and, under our equilibrium strategies, satisfies the exact law of large numbers, using the Fubini property of Sun (2006).

[16] This assumption is without loss of generality in that, for every equilibrium in which fast traders do not play this strategy, there exists a payoff-equivalent equilibrium in which they do. The only exception is the degenerate Diamond-paradox equilibrium, in which all dealers quote the price v, fast traders contact no more than one dealer, and slow traders do not enter.

[17] The sampling window used to be one minute, but it was widened to five minutes following the recommendation of the Foreign Exchange Benchmark Group (2014) in September 2014.

B. The Benchmark Case

We first characterize equilibrium in the benchmark case, where c is published before trade begins. A considerable part of the analysis here draws upon the work of Janssen, Moraga-González, and Wildenbeest (2005) and Janssen, Pichler, and Weidenholzer (2011).

In the event that $c > v$, there are no gains from trade, and in light of the benchmark information, slow traders do not enter. There can be no trade in equilibrium. If $v - s \leq c \leq v$, because dealers never quote prices below their costs, slow traders still do not enter. Fast traders, however, enter and buy from the dealer that offers the lowest price. It is easy to show that the only equilibrium is one in which all dealers quote a price of c, amounting to Bertrand competition among dealers. We therefore concentrate on the interesting case, the event in which $c < v - s$.

We fix some candidate probability λ_c of entry by slow traders, to be determined in equilibrium. Conditional on entry, the optimal policy of a slow trader is characterized by Weitzman (1979): search until she contacts a dealer whose offer is no higher than some cutoff r_c, which depends neither on the history of received offers nor on the number of dealers that have not yet been visited.

A standard search-theoretic argument—found, for example, in Varian (1980) and elaborated in Appendix A—implies that the only possible equilibrium response of dealers is a mixed strategy in which offers are drawn from a continuous (nonatomic) distribution whose support has r_c as its maximum. Because, in equilibrium, a dealer's price is never worse than a slow trader's reservation price, a slow trader buys from the first dealer that she contacts.

Let $F_c(\cdot)$ be the equilibrium cumulative distribution function of a dealer's price offer. Given the traders' strategies, a contacted dealer assigns the posterior probability

$$q(\lambda_c) = \frac{\mu}{\mu + \frac{1}{N}\lambda_c(1-\mu)} \tag{1}$$

that the visiting trader is fast. Here, we use the property that a slow trader enters with probability λ_c and visits this particular dealer with probability $1/N$. Because, in equilibrium, dealers must be indifferent between all price offers in the support $[\underline{p}_c, r_c]$ of the distribution, we have

$$\left[\underbrace{(1-q(\lambda_c))}_{P(\text{Sell to slow trader})} + \underbrace{q(\lambda_c)\left(1-F_c(p)\right)^{N-1}}_{P(\text{Sell to fast trader})}\right](p-c) = \underbrace{(1-q(\lambda_c))}_{P(\text{Sell to slow trader})}(r_c - c). \tag{2}$$

We use the fact that a slow trader accepts a price $p \leq r_c$ for sure, but a fast trader accepts p if and only if all other dealers offer worse prices. Thus, the equilibrium cumulative distribution function F_c of price offers is given by

$$F_c(p) = 1 - \left[\frac{\lambda_c(1-\mu)}{N\mu}\frac{r_c - p}{p-c}\right]^{\frac{1}{N-1}}. \tag{3}$$

The lowest price $\underline{p}_c$ in the support is determined by the boundary condition $F_c(\underline{p}_c) = 0$.

We can now calculate the optimal reservation price $r_c^\star$ of slow traders. Because traders value the asset at v, we must have $r_c^\star \leq v$. The definition of reservation price implies that, after observing a quote of $p = r_c^\star$, a trader must be indifferent between immediately accepting the offer and continuing to search, so that

$$v - r_c^\star = -s + v - \int_{\underline{p}_c}^{r_c^\star} p \, dF_c(p). \tag{4}$$

Substituting the solution for $F_c(p)$ and conducting a change of variables yields[18]

$$r_c^\star = c + \frac{1}{1 - \alpha(\lambda_c)} s, \tag{5}$$

where

$$\alpha(\lambda_c) = \int_0^1 \left(1 + \frac{N\mu}{\lambda_c(1-\mu)} z^{N-1}\right)^{-1} dz < 1. \tag{6}$$

By direct calculation, the expected offer conditional on c is

$$\int_{\underline{p}_c}^{r_c^\star} p \, dF_c(p) = (1 - \alpha(\lambda_c))c + \alpha(\lambda_c) r_c^\star.$$

Equation (5) states that the maximum price that a slow trader is willing to accept is the cost of the asset plus a dealer profit margin equal to the trader's search cost s multiplied by a proportionality factor that reflects an entry externality, represented through the function α. This entry externality arises as follows. If the slow-trader entry probability λ_c is low, the market consists mainly of fast traders, and competition among dealers pushes the expected profit margins of dealers to zero, in that $\lim_{\lambda \to 0} \alpha(\lambda) = 0$. That is, the trading protocol converges to an auction run by fast traders. In contrast, if λ_c is close to one, then slow traders constitute a considerable part of the market, and the existence of search frictions allows dealers to exert their local monopoly power and sell at prices bounded away from their costs.

To complete the description of equilibrium, we must specify the optimal entry decisions of slow traders. Holding the entry probability λ_c fixed, the expected payoff of a slow trader conditional on c and on entry is

$$\pi(\lambda_c) = v - s - \int_{\underline{p}_c}^{r_c^\star} p \, dF_c(p) = v - \frac{1}{1 - \alpha(\lambda_c)} s - c.$$

It can be verified that $\pi(\lambda_c)$ is strictly decreasing in λ_c through the role of $\alpha(\lambda_c)$.

[18] The change of variables is $z = 1 - F_c(p)$. See Janssen, Pichler, and Weidenholzer (2011) for details.

If $\pi(\lambda_c)$ is strictly positive at $\lambda_c = 1$, then the equilibrium slow-trader entry probability $\lambda_c^\star$ must be one. Because α is maximized at $\lambda_c = 1$, this happens if and only if

$$c \le v - \frac{1}{1-\bar{\alpha}} s,$$

where

$$\bar{\alpha} = \alpha(1) = \int_0^1 \left(1 + \frac{N\mu}{1-\mu} z^{N-1}\right)^{-1} dz. \tag{7}$$

If the profit $\pi(\lambda_c)$ is negative at $\lambda_c = 0$, then there is no entry by slow traders, that is, $\lambda_c^\star = 0$. Since $\alpha(0) = 0$, this happens whenever $c > v - s$.

Finally, if $c \in (v - s/(1-\bar{\alpha}),\ v - s)$, then we have "interior entry," in that $\lambda_c^\star \in (0,\ 1)$ is uniquely determined by the equation

$$s = \left(1 - \alpha\left(\lambda_c^\star\right)\right)(v - c). \tag{8}$$

We summarize these results with the following proposition.

PROPOSITION 1: *In the benchmark case, the equilibrium payoffs are unique and there exists a reservation-price equilibrium in which the following properties hold.*

1. *Entry. In the event that $c \ge v - s$, no slow traders enter. If*

$$v - \frac{s}{1-\bar{\alpha}} < c < v - s,$$

then slow traders enter with the conditional probability $\lambda_c^\star \in (0,\ 1)$ determined by equation (8). If $c \le v - s/(1-\bar{\alpha})$, then slow traders enter with conditional probability equal to one.

2. *Prices. In the event that $c > v$, dealers quote arbitrary offers no lower than c. If $c \in [v - s,\ v]$, then dealers quote offers equal to c. If $c < v - s$, then every dealer quotes offers drawn with the conditional probability distribution function F_c given by (3).*
3. *Traders' reservation prices. In the event that $c < v - s$, conditional on entry, a slow trader's reservation price $r_c^\star$ is given by (5).*
4. *Social surplus. The conditional expected total social surplus given c is*

$$\lambda_c^\star(1-\mu)(v - c - s) + \mu(v - c)^+,$$

where $(v - c)^+ \equiv \max\{v - c, 0\}$. The conditional expected profit of each dealer is

$$\frac{\lambda_c^\star(1-\mu)}{N} \frac{s}{1 - \alpha\left(\lambda_c^\star\right)}.$$

An immediate implication of Proposition 1 is that entry by slow traders is inefficient. In equilibrium, if $c \in (v - s/(1-\bar{\alpha}),\ v - s)$, the gain from trade for any slow traders is larger than the search cost, but we do not observe full entry. This inefficiency can be understood as a hold-up problem. Once traders enter, search costs are sunk and dealers make higher-than-efficient price offers. Taking into account this hold-up problem, slow traders enter only if gains from trade $v - c$ are significantly higher.

C. The No-Benchmark Case

The absence of a benchmark prevents traders from observing the common component c. In this case, traders face complicated Bayesian inferences based on the observed price offers in assessing the attractiveness of these offers. To keep the model tractable, we restrict attention to equilibria in which traders, when on the equilibrium path, follow a reservation-price strategy.[19] That is, in the k^{th} round of search a slow trader has a reservation price of the form $r_{k-1}(p_1, p_2, \ldots, p_{k-1})$, where $(p_1, p_2, \ldots, p_{k-1})$ is the history of prior price offers. According to this reservation-price strategy, any offer $p_k < r_{k-1}(p_1, p_2, \ldots, p_{k-1})$ is immediately accepted and any offer $p_k > r_{k-1}(p_1, p_2, \ldots, p_{k-1})$ is not immediately accepted. An offer $p_k = r_{k-1}(p_1, p_2, \ldots, p_{k-1})$ is accepted with some (mixing) probability that is determined in equilibrium. For simplicity, here forward we describe an offer that is not immediately accepted as "rejected," bearing in mind that the trader retains the option to later accept the offer.

We first characterize reservation-price equilibria, assuming one exists. We provide conditions under which a reservation-price equilibrium exists. The following lemma is an important step in characterizing a reservation-price equilibrium.

LEMMA 1: *In every reservation-price equilibrium in which slow traders enter with strictly positive probability, (i) the first-round reservation price $r_0^\star$ is equal to v and (ii) for each outcome of c strictly below v, the upper limit of the support of the conditional distribution of price offers is v.*

Without the benchmark, a trader's ignorance of the common component c of dealers' costs makes it more difficult for her to evaluate the attractiveness of price offers. Lemma 1 states that this information asymmetry causes a slow trader to accept any price offer below her value v for the asset, in a reservation-price equilibrium. Thus, only two things can happen if a positive mass of slow traders enter. If $c \le v$, a slow trader buys from the first dealer that she contacts. If $c > v$, then a slow trader will observe a price offer above

[19] Although this restriction is standard in the literature, Janssen, Parakhonyak, and Parakhonyak (2014) analyze non-reservation-price equilibria in a consumer search model with two firms. They assume that the customer's value is sufficiently high relative to firms' cost that there is no issue of entry efficiency, a key focus of our model. They also assume that the two firms have identical costs, drawn with the same outcome from a binomial distribution. This shuts down the matching efficiency on which we focus in the next section. Because of these assumptions and the technical difficulties in solving non-reservation-price equilibria in our setting, we follow the more usual convention in the literature of focusing on reservation-price equilibrium.

her value for the asset, conclude that there is no gain from trade, and exit the market. This outcome—slow traders entering only to discover that there is no gain from trade—is a waste of costly search that would be avoided if there were a benchmark. With a benchmark, as seen in Proposition 1, slow traders do not enter unless the conditional expected gain from trade exceeds the cost s of entering the market and making contact with a dealer.

Using Lemma 1, we can describe the reservation-price equilibrium without the benchmark, analogously with Proposition 1. We define the expected gain from trade as

$$X = G(v)\,[v - \mathbb{E}(c \,|\, c \le v)]\,, \tag{9}$$

that is, the probability of a positive gain from trade multiplied by the expected gain given that it is positive. Let $\lambda^\star$ denote the equilibrium probability of entry by slow traders.

PROPOSITION 2: *In the no-benchmark case, if a reservation-price equilibrium exists, it must satisfy the following properties:*

1. *Entry. If $s \ge X$, no slow traders enter, that is, $\lambda^\star = 0$. If $s \in ((1-\bar{\alpha})X,\, X)$, the fraction $\lambda^\star$ of entering slow traders solves*

$$s = (1 - \alpha(\lambda^\star))X. \tag{10}$$

   If $s \le (1-\bar{\alpha})X$, all slow traders enter with probability $\lambda^\star = 1$.
2. *Prices. In the event that $c > v$, dealers quote an arbitrary price offer no lower than c. If $c \le v$, dealers quote prices drawn from the cumulative distribution*

$$F_c(p) = 1 - \left[\frac{\lambda^*(1-\mu)}{N\mu}\,\frac{v-p}{p-c}\right]^{\frac{1}{N-1}}. \tag{11}$$

3. *Traders' reservation prices. Conditional on entry, a slow trader has a reservation price of v at her first dealer contact. If this first dealer's price offer is no more than v, the slow trader accepts it. Otherwise the slow trader rejects it and exits the market.*
4. *Surplus. The expected total social surplus is $\lambda^\star(1-\mu)(X-s)+\mu X$, and the expected profit of each dealer is $\lambda^\star(1-\mu)X/N$.*

The markets with and without benchmarks, characterized by Propositions 1 and 2, respectively, share some common features. In both, dealers' strategies depend on the realization of the benchmark c, and slow traders never contact more than one dealer on the equilibrium path. The distribution of quoted prices and the entry probability of slow traders are characterized by functions whose forms, with and without a benchmark, are similar.

That said, there are two crucial differences. First, slow traders' entry decisions in the presence of the benchmark depend on the realization (through publication of the benchmark) of the gain from trade. By contrast, without a benchmark, entry depends only on the (unconditional) expected gain from trade. Second, with the benchmark, the reservation price of slow traders generally depends on the realization of the benchmark c. Absent the benchmark, however, a slow trader's reservation price is always v. As a consequence, an offer of v is in the support of price offers regardless of the outcome of c.

Existence of Reservation-Price Equilibria in the No-Benchmark Case

Before comparing welfare with and without the benchmark, it remains to characterize conditions under which a reservation-price equilibrium exists without the benchmark. Providing general conditions for existence in this setting is challenging. While significant progress on existence has been made by Janssen, Pichler, and Weidenholzer (2011), their results do not apply in our setting because they assume that a trader's value v is so large that varying its level has no effect on the equilibrium. We cannot make this assumption because the size of gains from trade plays a key role in our analysis of entry. Benabou and Gertner (1993) also provide partial existence results for the case of two dealers, but in a different setting.

Appendix A provides a necessary and sufficient condition for the existence of a reservation-price equilibrium in the case of two dealers, and an explicit sufficient condition for existence with $N > 2$ dealers. The main conclusion is summarized as follows.

PROPOSITION 3: *There exists some $\underline{s} < X$ such that, for any search cost s greater than $\underline{s}$, a reservation-price equilibrium in the no-benchmark case exists and is payoff-unique.*

Proposition 3 states that the equilibrium described in Proposition 2 exists if the search cost is sufficiently large. The condition $\underline{s} < X$ ensures that there exists an equilibrium with strictly positive probability of entry by slow traders. If $s \geq X$, there exists a trivial reservation-price equilibrium in which slow traders do not enter.

D. Welfare Comparison

We now show that if search costs are high relative to the expected gain from trade, then introducing the benchmark raises the social surplus by encouraging the entry of slow traders.

As noted above, entry may be inefficiently low under search frictions due to the hold-up problem and the negative externality in the entry decisions of slow traders. Because a search cost is sunk once a slow trader has visited a dealer, a dealer can more heavily exploit its local-monopoly pricing power. Expecting this outcome, slow traders may refrain from entry despite the positive expected gain from trade. The hold-up problem is more severe when more slow traders enter (because this raises the posterior belief of a dealer that he faces a slow trader). These effects apply both with and without the benchmark. The question is whether benchmarks alleviate or exacerbate this situation.

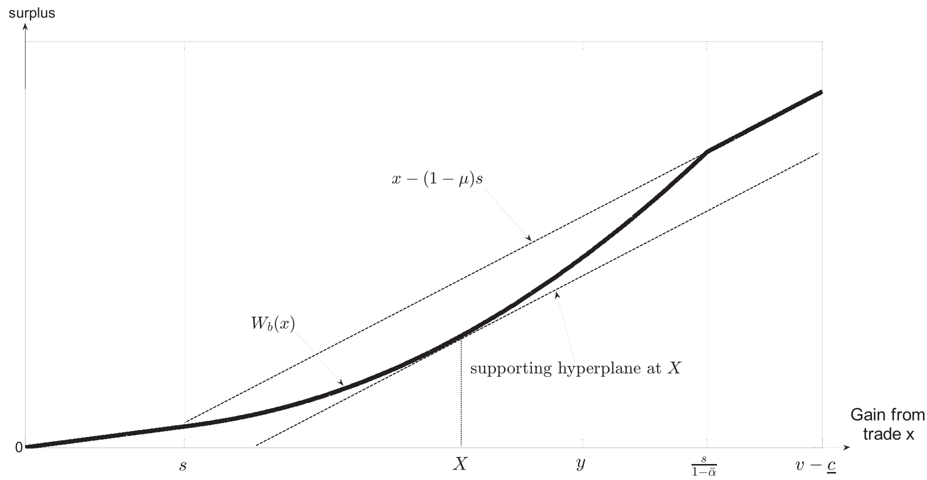

Figure 1. Conditional expected social surplus given the realized gain x from each trade.

We now state the main result of this section, giving conditions under which adding the benchmark improves welfare by encouraging entry.

THEOREM 1: *Suppose that (i) $s \geq (1-\bar{\alpha})(v-\underline{c})$ or (ii) $s \geq (1-\psi)X$ holds, where $\psi \in (0, \bar{\alpha})$ is a constant that depends only on μ and N.*[20] *Then a reservation-price equilibrium in the no-benchmark case (if it exists) yields a lower social surplus than that of the equilibrium in the benchmark case. Condition (i) holds if there are sufficiently many dealers or if the fraction μ of fast traders is small enough.*

There are two key sources of intuition behind Theorem 1. First, the presence of a benchmark allows slow traders to make their entry decisions contingent on additional information about the magnitude of gains from trade. In equilibrium with the benchmark, entry is higher precisely when gains from trade are larger. In other words, if the unconditional probability of entry were the same across the two settings, then social surplus would be higher in the benchmark case because, in the equilibrium with the benchmark, volume is positively correlated with gains from trade. Second, adding the benchmark reduces the information asymmetry between dealers and traders. Without the benchmark, a slow trader is not sure whether an unexpectedly high price offer is due to a high outcome for the common cost c of dealers or to an unlucky draw from the dealer's offer distribution. Dealers exploit this informational advantage, which exacerbates the hold-up problem. By providing additional information about dealers' costs, benchmarks give more bargaining power to slow traders.

The proof of the theorem is illustrated in Figure 1, which depicts the dependence of the benchmark-market social welfare function $W_b(x)$ on the realized

[20] We have $\psi = \frac{1}{2}[\sqrt{(1-\bar{\alpha}+\bar{\alpha}\beta)^2 + 4\bar{\alpha}(1-\bar{\alpha})} - (1-\bar{\alpha}+\bar{\alpha}\beta)]$, where $\beta = N\mu/(1-\mu)$, and $\bar{\alpha}$ is defined by equation (7).

gain from trade $x = \max\{v - c, 0\}$. The proof first shows that the expected social surplus in the no-benchmark case is actually equal to $W_b[\mathbb{E}(x)]$. We thus want to show that $\mathbb{E}[W_b(x)] \geq W_b(\mathbb{E}(x))$. Because slow traders increase their entry probability when the benchmark-implied gain from trade is large, we can prove that $W_b(\cdot)$ is convex over the set of x for which the entry probability is interior. Condition (i) ensures the convexity of $W_b(\cdot)$ on its entire domain, allowing an application of Jensen's Inequality. The alternative condition (ii) ensures that $W_b(\cdot)$ is subdifferentiable at $X = \mathbb{E}(x)$, yielding the same comparison. Both conditions require that the search cost s is sufficiently high.

We emphasize that Theorem 1 is neither mechanical nor trivial. In fact, one can find conditions under which the welfare ranking in Theorem 1 is reversed. That is, there are cases in which adding a benchmark can harm welfare. The severity of the hold-up problem decreases with the size of gains from trade. Without the benchmark, the expected size of gains from trade determines entry. When the expected gains from trade are high relative to search costs, all slow traders enter in the absence of benchmarks, overcoming the hold-up problem. With the benchmark, however, the actual size of gains from trade determines entry. Slow-trader entry is high when c is low and is low when c is high. For some parameters, it is more efficient to "pool" the entry decisions without the benchmark than to let entry depend on the realized benchmark cost.

PROPOSITION 4: *Suppose that the equilibrium described by Proposition 2 exists. If (i) $(1 - \bar{\alpha})(v - \bar{c}) < s$, (ii) $s \leq (1 - \bar{\alpha})X$, and (iii) $G(v - s)$ is sufficiently close to one, then the expected social surplus is strictly higher without the benchmark than with the benchmark.*

The assumptions needed for the benchmark to decrease efficiency are relatively restrictive. The condition $s \leq (1 - \bar{\alpha})X$ ensures that there is full entry without the benchmark. (By Theorem 1, this condition fails if μ is small enough or N is large enough.) The condition that $s > (1 - \bar{\alpha})(v - \bar{c})$ ensures that there are cost realizations for which we do not have full entry with the benchmark. Hence, search costs can be neither too high nor too low. Finally, the condition that $G(v - s)$ is close to one ensures that the entry of slow traders is indeed socially desirable for *nearly all* cost realizations.

The conditions of Proposition 4 are easily interpreted in Figure 1. If $X > s/(1 - \bar{\alpha})$ (condition (ii)) and if the region $[0, s]$ has negligible impact on welfare (condition (iii)), then we can place a hyperplane above the graph of $W_b(\cdot)$ that is tangent to it at X. That is, we get superdifferentiability rather than subdifferentiability, reversing the welfare inequality. Condition (i) guarantees that the inequality is strict.

The reverse welfare ranking of Proposition 4 relies on the fact that there is a bounded mass of slow traders. In an alternative model in which the potential mass of slow traders is unbounded, "full entry" is impossible, and the function $W_b(\cdot)$ in Figure 1 is globally convex. In this unbounded-entry model, a reservation-price equilibrium in the no-benchmark case (if one exists) yields a lower social surplus than the equilibrium in the benchmark case. A formal proof of this claim is omitted as it follows directly from the proof of Theorem 1.

E. Dealers' Incentives to Introduce a Benchmark

As we have seen so far, the introduction of a benchmark reduces the informational advantage of dealers relative to traders. It might seem that dealers have no incentive to introduce the benchmark. In this subsection we show that the contrary can be true. Under certain conditions dealers want to introduce a benchmark in order to increase their volume of trade. We assume that dealers are able to commit to a mechanism leading to truthful revelation of c, so the question of whether they prefer to have the benchmark boils down to comparing dealers' profits with and without the benchmark. We address the implementability of adding a benchmark in Section IV.

THEOREM 2: *Suppose that (i) $s \geq (1-\bar{\alpha})(v-\underline{c})$ or (ii) $s \geq (1-\eta)X$, where $\eta \in (0, \bar{\alpha})$ is a constant that depends only on N and μ. Then, a reservation-price equilibrium in the no-benchmark case (whenever it exists) yields a lower expected profit for dealers than in the setting with the benchmark. Condition (i) holds if there are sufficiently many dealers or if the fraction μ of fast traders is small enough.*

The benchmark raises the profits of dealers by encouraging the entry of slow traders. If search costs are large relative to gains from trade (assumption *(i)* or *(ii)* of Theorem 2), dealers benefit from the increased volume of trade arising from the introduction of the benchmark. For dealers' total profits to rise with the introduction of a benchmark, entry by slow traders must be sufficiently low without the benchmark, as otherwise the benchmark-induced gain in trade volume does not compensate dealers for the reduction in profit margin on each trade.

A benchmark can be viewed as a commitment device, through which dealers promise higher expected payoffs to traders in order to encourage entry. In particular, a benchmark partially solves the hold-up problem by reducing market opaqueness and giving traders a better bargaining position.

An interesting property of benchmarks is that whenever they are added voluntarily by dealers, they are also guaranteed to increase efficiency in the market.

PROPOSITION 5: *If introducing the benchmark raises the expected profit of dealers, then it also raises the expected social welfare.*

Proposition 5 has an important policy implication. It is *never* optimal for a market regulator to try to suppress a benchmark if one is introduced by dealers. The opposite is not true. There generally exists a range of search costs over which the benchmark raises social surplus but dealers would have no incentive to commit to it. This is intuitive. Whenever the gain from trade $v-c$ exceeds the search cost s, *any* increase in entry probability is welfare-enhancing. If, however, this increase is too small to compensate for the reduction in dealers' profit margins, dealers would not opt to introduce the benchmark.

F. Under Conditions, the Benchmark Is an Optimal Mechanism

We now consider socially optimal mechanisms for reporting information about dealer costs.[21] Under conditions, among a wide class of mechanisms, social welfare cannot be improved by doing something other than simply publishing the benchmark. Throughout this section, we assume that the mechanism designer knows the dealer cost c. We also assume in this subsection that a reservation-price equilibrium exists whenever we discuss equilibrium behavior.

Up to this point, our analysis has shown that the outcome of the market equilibrium, with or without the benchmark, is fully efficient, conditional on entry by slow traders. The entry decision itself, however, could be inefficient. Thus, the mechanism design should focus on providing information to traders *before* they make their entry decisions.

Formally, a revelation mechanism $(\Pi, \mathcal{S})$ consists of a signal space $\mathcal{S}$ and a measurable mapping Π from $[\underline{c}, \bar{c}]$ to the set $\Delta(\mathcal{S})$ of probability measures on $\mathcal{S}$. The mechanism sends traders a signal $s \in \mathcal{S}$ drawn from the conditional probability distribution $\Pi(c)$. Traders observe the signal and make their entry decisions. The game then proceeds according to the protocol described in Section II.A.

We impose no restrictions on the class of signals to be sent by the mechanism designer. Announcing the benchmark is equivalent to a revelation mechanism given by $\mathcal{S} = [\underline{c}, \bar{c}]$ and $\Pi(c) = \delta_{\{c\}}$, the dirac delta at c, meaning full revelation. Providing no information before traders make their entry decisions is equivalent to a mechanism with a singleton signal space $\mathcal{S} = \{0\}$. Using techniques from the literature on Bayesian persuasion,[22] we provide the following general characterization. Recall that $x = \max\{v - c,\ 0\}$ denotes the gain from trade.

THEOREM 3: *Let*

$$y = \underset{\tilde{y} \in [0,\, v-\underline{c}]}{\operatorname{argmin}} \left| \mathbb{E}\left[x \,|\, x \geq \tilde{y}\right] - \frac{s}{1-\bar{\alpha}} \right|.$$

That is, y solves the equation $\mathbb{E}[x \,|\, x \geq y] = s/(1-\bar{\alpha})$ whenever a solution exists, and otherwise takes the boundary value of zero if $\mathbb{E}[x] > s/(1-\bar{\alpha})$, and the boundary value of $v - \underline{c}$ if $v - \underline{c} < s/(1-\bar{\alpha})$. The following revelation mechanism maximizes the expected social surplus.

1. *When $x < \max\{s,\ y\}$, announce the realization of c.*
2. *When $x \geq \max\{s,\ y\}$, announce that $v - c \geq y$ (but nothing else).*

To gain intuition for Theorem 3, consider the case in which y is an interior solution. Suppose that $y \geq s$. We first explain why it is optimal to garble information about the gain from trade when its realization is high (point 2 in

[21] A related analysis of optimal mechanisms for trade transparency in OTC markets (in a different model) is considered by Dworczak (2016).

[22] See Kamenica and Gentzkow (2011) for the formulation of the Bayesian persuasion problem, and Dworczak and Martini (2017) for the technique that we use in the proof of our theorem.

the theorem). By announcing that $v - c \geq y$, the mechanism induces the posterior belief $s/(1-\bar{\alpha})$ for the expected gain from trade among slow traders. The equilibrium of the subsequent game is that specified by Proposition 2, but with the unconditional expected gain X from trade replaced by the conditional expected gain from trade given by $s/(1-\bar{\alpha})$. Thus, there is full entry by slow traders whenever the realization x of the gain from trade is above y. If, instead, c were to be fully revealed, for the entry probability of a slow trader to reach one, the realization x of the gain from trade must exceed $s/(1-\bar{\alpha})$. (See Proposition 1). We conclude that conflating realizations of x above y into one message raises the entry probability whenever the realization of x is between y and $s/(1-\bar{\alpha})$. The garbling region $x \geq y$ is the largest possible set of realizations of x that yields full entry by slow traders. If any additional outcomes of x below y were conflated into one message m, together with the event $x \geq y$, then the conditional gain from trade would fall below $s/(1-\bar{\alpha})$ and the entry probability would decrease for *all* realizations of x leading to the message m.

To understand why it is optimal to fully disclose the cost c whenever x is to the left of the garbling region (point *1* of Theorem 3), we note that the welfare function $W_b(x)$ is convex in that domain (see Figure 1). Therefore, the optimality of full disclosure of c follows from the same forces that give rise to Theorem 1, that is, by disclosing the cost c, the mechanism introduces a beneficial positive correlation between gains from trade and entry probability.

Overall, the welfare-maximizing mechanism optimally trades off the benefits associated with the two extreme revelation schemes that are compared in Section II.D. For high cost realizations (those with $x < y$), it is optimal to fully disclose the cost c. For low-cost realizations (those with $x \geq y$), it is optimal to "pool" slow traders' incentives to enter by disclosing only that $v - c \geq y$.

The following result is a useful special case of Theorem 3.

PROPOSITION 6: *Suppose* $s \geq (1-\bar{\alpha})(v - \underline{c})$ *(Assumption (i) of Theorem 1). Fully disclosing the cost c is a social-surplus-maximizing revelation mechanism. Any optimal revelation mechanism fully discloses the cost c (almost surely) whenever* $x \in [s,\ v - \underline{c}]$.

Assumption (i) of Theorem 1 implies that the gain from trade is never above $s/(1-\bar{\alpha})$. In this case, y as defined in Theorem 3 is equal to $v - \underline{c}$. It follows that point 2 of Theorem 3 never applies, and thus the optimal mechanism is to fully disclose the benchmark c.

Proposition 6 implies that a perfectly informative benchmark is an (essentially unique) optimal mechanism if there are sufficiently many dealers or if the fraction μ of slow traders is low enough; in these cases, $\bar{\alpha}$ is sufficiently close to one. Moreover, based on the remark at the end of Section II.D, we can show that when there is an unbounded pool of slow traders, announcing the perfectly informative benchmark c is an optimal mechanism even without Assumption (i) of Theorem 1.

G. The Socially Optimal Mechanism Is Also Optimal for Dealers

In Section II.E we show that the incentives of dealers to introduce a benchmark (or not) are only partially aligned with social preferences. It turns out, however, that the socially optimal revelation mechanism fully aligns the private preferences of dealers with social preferences. (For the purposes of this section, as in Section II.F, we assume the existence of a reservation-price equilibrium.)

THEOREM 4: *The socially optimal mechanism of Theorem 3 also maximizes the expected profit of dealers, within the set of feasible revelation mechanisms.*

Theorem 4 implies that dealers always prefer to introduce the optimal mechanism described in Theorem 3. Whenever the optimal mechanism coincides with the benchmark, an even stronger conclusion applies: there is no revelation scheme that dealers would prefer over the benchmark.

Why might dealers disagree with a benevolent regulator on the desirability of having a benchmark, but always agree on the optimal mechanism? The reason is that the optimal mechanism discloses only enough information to induce entry by slow traders. When c is fully disclosed and there is full entry, that is, when $v - c \geq s/(1-\bar{\alpha})$, slow traders use the information they are given about c to negotiate constant margins over dealers' costs (that is, the reservation price $r_c^\star$ changes one-to-one with c, according to formula (5)). Under the optimal mechanism, slow traders still enter with probability one, but they are uninformed about the exact value of c. Dealers may exploit this information asymmetry and continue to enjoy the higher profit margins that they achieve in the no-benchmark case.

In light of Theorem 4, one may wonder why functioning OTC markets do not include public reporting schemes that suppress dealer cost information precisely when those costs turn out to be low enough. A possibility is that practical settings are approximately summarized by model parameters for which the optimal mechanism coincides with a benchmark (full revelation of dealer costs). It could also be the case that calculating the threshold level y for revelation of c may be difficult in practice, and that a small error in this calculation could lead to poor performance (especially if y is set too low). By comparison, a benchmark mechanism is simple and more robust than the optimal mechanism.

Finally, we note that the amount of information revealed by the optimal mechanism is increasing (in the sense of Blackwell) in the search cost s. This implies that, as search costs decrease, dealers prefer increasingly opaque markets.

H. An Illustrative Example

We conclude this section with a numerical example. Our goal is to illustrate the magnitude and direction of the modeled effects.[23] A serious empirical calibration or structural estimation is beyond our objectives.

[23] In the numerical example, we assume, as before, the existence of a reservation-price equilibrium in the no-benchmark case.

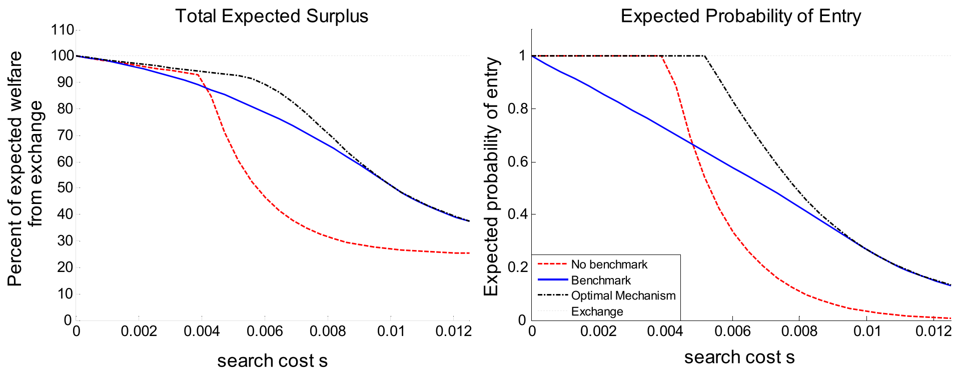

Figure 2. Total expected surplus (expressed as a percentage of expected welfare associated with a centralized-exchange market) and the probability of entry of slow traders, conditional on the event $c < v - s$, where c is dealer cost, v is value, and s is search cost. (Color figure can be viewed at wileyonlinelibrary.com)

For the sake of illustration, we assume 20 dealers. One quarter of traders are fast. Any trader's value v of the asset is normalized without loss of generality to one. The dealer cost c is uniformly distributed on $[0.9, 1.02]$. Figure 2 shows how total welfare and entry vary with the search cost s for three cases: no-benchmark, benchmark, and the optimal mechanism shown in Section II.F. Total surplus is expressed as a percentage of expected welfare in a market with a centralized exchange, or equivalently, for an OTC market with no search costs.[24] Figure 3 depicts expected execution prices and expected quotes of dealers in the no-benchmark case and in the benchmark case, for three levels of search costs.

Figure 2 shows that the no-benchmark case yields higher surplus than the benchmark case only when search costs are low, consistent with Proposition 4. However, with low search costs, the differences in expected surplus between all mechanisms are relatively small. When search costs are larger, introducing the benchmark enhances surplus, as predicted by Theorem 1. The gain can be quite significant (on the order of 30% of the expected surplus associated with a centralized exchange), especially for intermediate levels of s. The slow-trader entry probability is higher in the no-benchmark case when search costs are small. A higher probability of entry does not necessarily lead to higher surplus because the benchmark induces positive correlation between entry probability and realized gains from trade. This positive correlation is reflected by higher price volatility in the benchmark case, as shown in Figure 3. Quotes tend to be much lower (more attractive to traders) with a benchmark than without, for low-cost realizations. This is due to the associated reduction in information asymmetry, which improves the bargaining position of traders. Finally, when search costs are large, the benchmark is seen to be an optimal mechanism,

[24] This expected welfare is equal to 0.0455 under the above parameters.

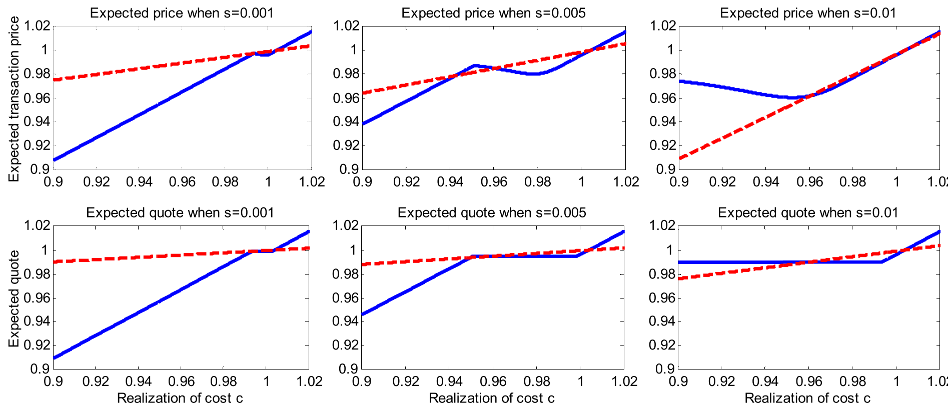

Figure 3. Expected execution prices (first row) and expected quotes of dealers (second row) as functions of dealer cost c, for search costs $s = 0.001$, $s = 0.005$, and $s = 0.01$ in the benchmark case (blue solid line) and the no-benchmark case (red dotted line). (Color figure can be viewed at wileyonlinelibrary.com)

consistent with Proposition 6. The difference between welfare with an optimal mechanism and with a benchmark is largest with intermediate search costs and is driven by the higher entry probability caused by optimal information disclosure.

III. Heterogeneous Dealer Costs and Matching Efficiency

In this section we extend the model of Section II to incorporate heterogeneous dealers' costs and address matching efficiency. We show that the pre-trade price transparency afforded by a benchmark improves the matching of traders to low-cost dealers. The beneficial impact of a benchmark on entry efficiency, shown in Section II, continues to apply in this heterogeneous-cost setting, as shown in the Internet Appendix.[25]

A. Setup

We adopt the model of Section II with one difference: dealer i has the total cost $c_i = c + \epsilon_i$ for supplying the asset to a trader, where $\epsilon_1, \ldots, \epsilon_N$ are independent binomial random variables whose outcomes are zero and Δ, with respective probabilities $\gamma \in (0, 1)$ and $1 - \gamma$. Dealer i observes c and ϵ_i, but not ϵ_j, for $j \neq i$. The published benchmark is the common dealer cost component c. As before, we can view c as the cost to dealers for acquiring the asset in the interdealer market. The new cost component ϵ_i is a private cost to dealer i for supplying the asset. For instance, a dealer's effective cost for supplying a particular asset could naturally depend on the dealer's current inventory and

[25] The Internet Appendix is available with the online version of the article on the *Journal of Finance* website.

internal risk budget. To the extent that the heterogeneity in dealers' costs arises from private information of this sort, we expect that customers are unable to distinguish, ex ante, high-cost dealers from low-cost dealers.

Throughout this section we maintain the following two assumptions.

ASSUMPTION A1: *Search is socially optimal, in that $s < \gamma\Delta$.*

ASSUMPTION A2: *Gains from trade exist with probability one. That is, $\bar{c} < v - \Delta$.*

Together, these conditions imply full entry by slow traders in equilibrium, in the presence of a benchmark. This allows us to separately identify the welfare impacts associated with matching efficiency. Assumption A1 is motivated by the observation that finding a low-cost dealer improves social welfare only if the search cost is lower than the potential improvement in matching efficiency.[26] Assumption A2 is adopted for expositional purposes only. We give generalized statements (weakening Assumption A2) of the results of this section in Appendix B. We will show that if search costs are relatively low, then adding a benchmark raises social surplus by making it easier for traders to find the efficient (that is, low-cost) dealers.

B. The Benchmark Case

In the presence of a benchmark, the key intuition for the equilibrium construction from Section II generalizes to this heterogeneous-cost setting, but the supporting arguments are more complicated and several cases need to be considered. For that reason, we focus here on parameter regions that are relevant for social surplus comparisons, and relegate a full characterization to Appendix B. Figure 4 summarizes pricing schemes that arise in equilibrium as a function of the search cost s. We begin with the following result.

PROPOSITION 7: *In the presence of a benchmark, the equilibrium is payoff-unique and slow traders use a reservation-price strategy.*

Proposition 7 is not surprising given the analysis of Section II. However, there is a subtle but important difference. Under a reservation-price strategy, a trader is indifferent between accepting an offer and continuing to search when the offer is equal to her reservation price. In the setting of Section II it does not matter whether traders accept such an offer or not because this event has zero probability. But, with idiosyncratic costs, there are parameter regions in which the only equilibrium requires traders who face an offer at their reservation price to mix between accepting and continuing to search. The mixing probabilities are important when there is an atom in the probability distribution of offers located at a trader's reservation price. In equilibrium, these atoms may arise if the reservation price is equal to the high

[26] Appendix B provides the supporting analysis when Assumption A1 fails. In that case, there will be no search in the equilibrium with the benchmark. While the absence of search is socially optimal in this case, this is not the case in which we are most interested.

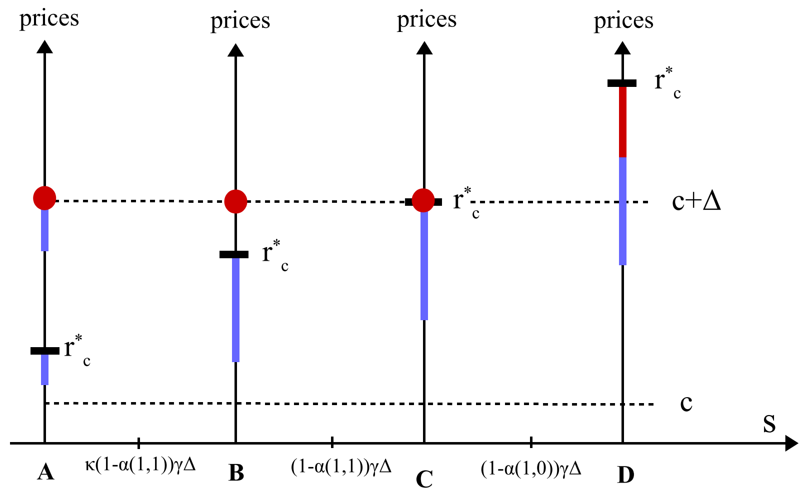

Figure 4. Price supports in different equilibrium regimes. Lines represent the nonatomic ("continuous") portions of distributions. Dots represent atoms. Low-cost dealers are shown in blue. High-cost dealers are shown in red. (Color figure can be viewed at wileyonlinelibrary.com)

outcome of dealer costs, as in Panel C of Figure 4. This affects the inference made by dealers when they calculate the probability of facing a fast trader.

To account for heterogeneous dealer costs, we need to adjust the probability that a dealer's counterparty is fast (as opposed to slow) from that given by equation (1). This probability now depends on both the entry probability λ_c and the c-conditional probability, denoted θ_c, that a slow trader rejects an offer from a high-cost dealer. As θ_c gets larger, slow traders search more, and the posterior probability that a dealer is facing a fast trader falls. We denote by $q(\lambda_c, \theta_c)$ the probability that a contacting trader is fast. Accordingly, the definition of the function $\alpha(\lambda_c)$ from equation (6) is generalized to a two-argument function $\alpha(\lambda_c, \theta_c)$ with values in $(0, 1)$. Explicit formulas are provided by equations (B5) and (B6) in Appendix B. The role of $\alpha(\lambda_c, \theta_c)$ is analogous to that of $\alpha(\lambda_c)$ in Section II. Here, $\alpha(\lambda_c, \theta_c)$ is strictly increasing in both arguments. As λ_c and θ_c increase, the probability that a counterparty is slow rises, leading dealers to quote higher prices in equilibrium. The constant $\alpha(1, 1)$ is an analogue of $\bar{\alpha}$ in Section II and bounds $\alpha(\lambda_c, \theta_c)$ from above. For the sake of simplifying upcoming expressions, we denote

$$\hat{\alpha} = \alpha(1, 1).$$

We now state the main result of this section.

PROPOSITION 8: *If $s \le (1 - \hat{\alpha})\gamma\Delta$, then the equilibrium in the benchmark case leads to efficient matching: slow traders always enter, and all traders buy from a low-cost dealer in the event that there is at least one such dealer present in the* *market. Additionally, if $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$, where $\kappa < 1$ is a constant*[27] *depending only on γ, μ, and N, the equilibrium with the benchmark achieves the second-best, in the sense that each slow trader buys from the first low-cost dealer that she contacts, minimizing search costs subject to matching efficiency.*

To understand how benchmarks lead to efficient matching and second-best performance in the above sense, consider first the case in which the search cost s is in the interval

$$(\kappa(1-\hat{\alpha})\gamma\Delta,\ (1-\hat{\alpha})\gamma\Delta).$$

This case is illustrated in Panel B of Figure 4. In equilibrium, slow traders follow a reservation-price strategy with a reservation price $r_c^\star$ that is below $c+\Delta$. Low-cost dealers quote prices according to a continuous probability distribution whose support is below this reservation price. Thus, if there are any low-cost dealers in the market, slow traders buy from the first low-cost dealer that they contact. In the event that there are only high-cost dealers in the market, which happens with probability $(1-\gamma)^N$, slow traders search the entire market and then trade with one of the high-cost dealers at the price $c+\Delta$. This second-best equilibrium outcome is therefore fully efficient at matching.

The key role of the benchmark in this case is to introduce enough transparency to permit traders to distinguish between efficient and inefficient dealers. The benchmark ensures not only that traders ultimately transact with the "right" sort of counterparty, but also that no search cost is wasted while looking for this transaction. This last conclusion is true under the weaker condition that $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$.

If $s < \kappa(1-\hat{\alpha})\gamma\Delta$, however, slow traders may search excessively. As the search cost s get smaller, the equilibrium reservation-price $r_c^\star$ also gets smaller (closer to c), and low-cost dealers are forced to quote very low prices if they want to sell at the first contact of any slow trader. Because of their cost advantage, low-cost dealers always have the "outside option" of trying head-on competition by quoting a price above the reservation price (and just below $c+\Delta$), hoping that all other dealers have high costs (in which case low-cost dealers win the resulting effective auction, making positive profits). It turns out that low-cost dealers wish to deviate to this strategy when $s < \kappa(1-\hat{\alpha})\gamma\Delta$. In the resulting equilibrium, which we illustrate in Panel A of Figure 4 and describe formally in Appendix B, matching remains efficient but we do not achieve the second best, because of the higher-than-efficient amount of search.

The intuition described above indicates that a low-cost dealer's incentive to quote a high price should disappear as the number N of dealers gets large. Indeed, as N becomes large the probability that all other dealers have high costs goes to zero quickly. We confirm in the Internet Appendix (Section I.B) that an upper bound on the potential surplus loss (compared to first-best) goes to zero exponentially fast with N when $s < \kappa(1-\hat{\alpha})\gamma\Delta$. In sharp contrast,

[27] We have $\kappa = (1-\gamma)^{N-1}/[\mu(1-\gamma)^{N-1} + (1-\mu)[1-(1-\gamma)^N]/(N\gamma)]$.

surplus losses are potentially unbounded in N when s is close to $(1-\hat{\alpha})\gamma\Delta$. Hence, for practical purposes, it is natural to focus on the case $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$.

C. The No-Benchmark Case

We now show that, without the benchmark, it is impossible to achieve the second best.

PROPOSITION 9: *In the absence of a benchmark, if $\bar{c} > \underline{c} + \Delta$ there does not exist an equilibrium that achieves the second best.*

Our proof of the proposition explores the simple idea that when there is no benchmark for traders to observe, they cannot recognize a low-cost dealer when they contact one. In the absence of a benchmark, traders can rely only on Bayesian inference based on the observed price quotes. This Bayesian inference, however, can be relatively ineffective. With low realizations of the common cost component c, high-cost dealers may make offers that "imitate" the offers that low-cost dealers would make at higher realizations of c. As a result, slow traders buy from inefficient dealers or engage in socially wasteful search. The benchmark adds enough transparency to allow traders to distinguish between high offers from low-cost dealers and low offers from high-cost dealers.

D. Welfare Comparison

As a corollary of Propositions 8 and 9, we obtain the following result, providing conditions under which adding a benchmark improves welfare.

THEOREM 5: *If (i) $\kappa(1-\hat{\alpha})\gamma\Delta \leq s \leq (1-\hat{\alpha})\gamma\Delta$ and (ii) $\bar{c} > \underline{c} + \Delta$ both hold, then the equilibrium in the benchmark case yields a strictly higher expected social surplus than that of any equilibrium in the no-benchmark case.*

The theorem does not cover the entire search-cost space. We discuss the remaining cases in the Internet Appendix (Section I.A), where we show in particular that the second best is not achieved if $s > (1-\hat{\alpha})\gamma\Delta$, even if the benchmark is present. Nonetheless, with a benchmark, if search costs are not too large, partial efficiency applies to the matching of traders to low-cost dealers. The (unique) equilibrium supporting this outcome has an interesting structure. High-cost dealers post a price $c + \Delta$ equal to the reservation price $r_c^\star$ of slow traders, as in Panel C of Figure 4. Slow traders accept that price with some nontrivial (mixing) probability that is determined in equilibrium.

When search costs are sufficiently high, as in Panel D of Figure 4, both types of dealers sell at a strictly positive profit margin, and slow traders buy from the first dealer encountered. Thus, in this case, matching is inefficient. To make welfare comparisons for this parameter region, it is necessary to explicitly characterize the no-benchmark equilibrium, which is difficult because traders can potentially search multiple times and their posterior beliefs about c are intractable.

That said, for the case of two dealers, we can provide a full characterization of reservation-price equilibria in the no-benchmark case. Under the condition $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$, we show that matching is more efficient with a benchmark than without, provided that traders use a reservation-price strategy in equilibrium. Because the details are complicated, we relegate them to the Internet Appendix (Section I.C).

E. Introduction of Benchmarks by Low-Cost Dealers

This subsection analyzes the incentives of low-cost dealers to introduce a benchmark on their own—despite opposition from high-cost dealers—as a powerful device to compete for business. We show that, under certain conditions, the collective decision of low-cost dealers to add a benchmark drives high-cost dealers' profits to zero and forces them out of the market. As a result, low-cost dealers make more profits and the market becomes more efficient overall. This may explain why emergent "benchmark clubs" are often able to quickly attract the bulk of trades in some OTC markets, as was the case with LIBOR.

To explain how "benchmark clubs" may emerge, we augment our search-market game with an earlier stage in which dealers decide whether to introduce a benchmark and, after calculating their expected profits, whether to enter the market themselves. To simplify the modeling, we suppose that there are two types of environments with respect to the cross-sectional distribution of dealer cost efficiency. With some probability $\Gamma \in (0, 1)$, there is a relatively low-cost environment in which the number L of low-cost dealers is at least two. Otherwise, there are no low-cost dealers ($L = 0$). We rule out the case in which there is exactly one low-cost dealer in the market because in that case, for a high enough cost difference Δ, the low-cost dealer would be an effective monopolist, complicating the analysis. A formal description of the game follows.

1. Pre-trade stage: the introduction of a benchmark and entry by dealers.
   (a) Nature chooses the dealer-cost environment, whose outcome is not observed. With probability $1 - \Gamma$, all dealers have high costs. With probability Γ, the number L of low-cost dealers is drawn from a truncated binomial distribution with parameters (N, γ), where the truncation restricts the support to the set $\{2, 3, \ldots, N\}$. Conditional on L, the identities of dealers with low costs are drawn independently of L and symmetrically.[28] The idiosyncratic component ϵ_i of dealer i is the private information of dealer i.
   (b) Dealers simultaneously vote, anonymously, whether to have a benchmark or not. If there are at least two votes in favor, the benchmark is introduced. (In Section IV we explain how dealers could implement a benchmark, provided that there are at least two of them.) In this

[28] This implies that $\epsilon_1, \ldots, \epsilon_N$ are no longer i.i.d. Our results would hold under more general distributions of dealer types. The only properties required of the unconditional distribution of L are (i) symmetry with respect to dealer identities, (ii) the events $L = 0$ and $L \geq 2$ both have positive probability, and (iii) the event $L = 1$ has zero probability.

case, c immediately becomes common knowledge. If the number of votes in favor is zero or one, the benchmark is not introduced.

(c) Dealers make entry decisions. For simplicity, we adopt a tie-breaking rule that dealers enter if and only if their expected trading profits are strictly positive.

(d) After dealers' entry decisions, the number of dealers that enter, denoted M, becomes common knowledge among dealers and traders.

2. Trading stage. The game proceeds as before, but with N replaced by M.

We denote by

$$X_{\Delta} = G(v - \Delta)\mathbb{E}\,(v - c - \Delta \mid c \leq v - \Delta)$$

the expected gain from trade with high-cost dealers. The following theorem establishes conditions that are sufficient to induce low-cost dealers to collectively introduce the benchmark and drive their high-cost competitors out of the market.

THEOREM 6: *Suppose that $s < (1 - \bar{\alpha})(v - \bar{c})$. Then there is a constant $\Delta^{\star}$ such that, for any dealer cost difference $\Delta \geq \Delta^{\star}$, the following are true.*

- *There exists an equilibrium of the extended game in which all low-cost dealers vote in favor of the benchmark and all high-cost dealers vote against it. There are no profitable group deviations in the voting stage.*
- *If the environment is competitive (that is, $L \geq 2$), the benchmark is introduced, all high-cost dealers stay out of the market, all low-cost dealers enter the market, and all traders enter the market.*
- *If the environment is uncompetitive ($L = 0$), the benchmark is not introduced, and high-cost dealers enter the market if and only if $X_{\Delta} > s$.*

A proof is provided in Appendix B. Here, we explain the intuition of the result.

To start, we note that the theorem makes economically significant predictions about the role of the benchmark only in the case $X_{\Delta} > s$. This case arises if s is sufficiently small. In the opposite case of $X_{\Delta} < s$, high-cost dealers earn zero profits regardless of whether the benchmark is introduced, so they are indifferent between voting in favor of or against the benchmark, and they never enter. In the discussion below, we focus on the interesting case of $X_{\Delta} > s$, in which high-cost dealers can make positive profits and strictly prefer not to introduce the benchmark.

The benchmark serves as a signaling device for low-cost dealers to announce to traders that the environment is competitive. The signal is credible because traders, expecting low prices conditional on introducing the benchmark, set a low reservation price in equilibrium. Therefore, high-cost dealers cannot imitate low-cost dealers by deviating and announcing the benchmark. Instead, they prefer to trade in opaque markets without the benchmark and with low participation by slow traders, which allows them to make positive profits.

Low-cost dealers have two distinct incentives to add the benchmark. First, adding the benchmark encourages the entry of slow traders. In addition to the intuition conveyed in Section II, in the setting of this section the benchmark plays the additional role of signaling the types of active dealers, because the benchmark is added endogenously. On the equilibrium path, once a benchmark is introduced, slow traders believe with probability one that all active dealers have low costs. If a benchmark is not introduced, slow traders believe that all dealers have high costs. As a consequence, the (correctly) perceived gain from trade by slow traders goes up considerably if a benchmark is added. This channel encourages entry. The condition $s < (1 - \bar{\alpha})(v - \bar{c})$ ensures full entry by traders if the benchmark is introduced.

Second, low-cost dealers capture additional market share by adding the benchmark. With a large enough dealer cost difference Δ, the expected gains from trade are small if the benchmark is not introduced. As a result, we show that slow traders who enter will set a reservation price $r^{\star}$ equal to v in the trading-stage subgame, and high-cost dealers inevitably capture a large proportion of trades with slow traders. If, however, the benchmark is introduced, a sufficiently large Δ makes high-cost dealers' quotes highly uncompetitive, which drives trades to low-cost dealers. Thus, although the per-trade profit of low-cost dealers may be lower with the benchmark, they capture an additional amount of trade. In fact, in equilibrium, if the environment is relatively competitive, high-cost dealers drop out completely because they cannot make any profit. Low-cost dealers handle all of the trades.

The first part of Theorem 6 asserts that in the equilibrium that we construct there are no profitable *group deviations* in the voting stage. In the usual Nash equilibrium of the voting game, if everyone is voting against or in favor, no dealer is pivotal. Each outcome may be supported in equilibrium. This arbitrariness is eliminated by allowing group deviations.

F. On Optimal Mechanisms with Heterogeneous Dealer Costs

In the heterogeneous cost setting, the efficiency of the market is driven by *(i)* matching efficiency and *(ii)* total search costs. (Recall that we have imposed parameter restrictions that guarantee full entry.) Proposition 8 shows that, when search costs are small, adding a benchmark achieves full matching efficiency. Thus, any additional benefit from an optimal mechanism must arise from reducing total search costs.

In the previous subsection we show that, under parameter conditions, with endogenous entry by dealers, high-cost dealers stay out of the market after the benchmark is published. The resulting market equilibrium is analogous to that of the model with homogeneous dealer costs. When this happens, slow traders search only once, and publication of the benchmark achieves the fully efficient outcome (subject to the institutional constraint that any trade must involve incurring the search cost s). The benchmark mechanism uses no information beyond the common cost component c. Moreover, c can be elicited from dealers in an incentive-compatible way (as will be shown in Section IV). At least under these parameter conditions, the benchmark mechanism is implementable and there is no mechanism that can improve upon it.

For arbitrary parameters, it is difficult to characterize the optimal mechanism. Even calculating the market equilibrium for any fixed mechanism (other than announcing the benchmark) is intractable in most cases. Sections II and III show a range of potential impacts of changing market transparency. Although it is not easy to formally analyze all trade-offs in one model of optimal mechanism design, announcing the benchmark seems to be a simple and robust mechanism that performs well in both versions of the model.

IV. Benchmark Manipulation and Implementation

Recent scandals involving the manipulation of interest-rate benchmarks such as LIBOR and EURIBOR, as well as currency price fixings provided by WM/Reuters, have shaken investor confidence in financial benchmarks. Serious manipulation problems or allegations have also been reported for other major benchmarks, including those for term swap rates, gold, silver, oil, and pharmaceuticals.[29] Major banks are now more reluctant to support these benchmarks in the face of potential regulatory penalties and private litigation. For example, of the 44 banks contributing to EURIBOR before the initial reports of manipulation, 18 have already dropped out of the participating panel.[30] Regulators have responded not only with sanctions,[31] but also by taking action to support more robust benchmarks. The Financial Stability Board has set up several international working groups charged with recommending reforms to interest rate and foreign exchange benchmarks that would reduce their susceptibility to manipulation while maintaining their usefulness in promoting market efficiency.[32] The United Kingdom now has a comprehensive regulatory framework for benchmarks.[33]

[29] See, respectively, Scott Patterson and Katy Burne, “CFTC Probes Potential Manipulation,” *Wall Street Journal,* April 8, 2013; Liam Vaughn, “Gold Fix Study Shows Signs of Decade of Bank Manipulation, *Bloomberg*, February 8, 2014; Patricia Hurtado, “Deutsche Bank, HSBC Accused of Silver Fix Manipulation,” *Bloomberg*, July 25, 2014; Justin Scheck and Jenny Gross, “Traders Try to Game Platts Oil Price,” *Wall Street Journal*, June 19, 2013; and Gencarelli (2002).

[30] See Jun Brundsen, “ECB Seeks Rules to Stem Bank Exodus from Benchmark Panels,” *Bloomberg*, June 19, 2014.

[31] See Gavin Finch and Nicholas Larkin, “U.K. Seeks to Criminalize Manipulation of 7 Benchmarks,” *Bloomberg*, September 25, 2014.

[32] See Official Sector Steering Group (2014), Market Participants Group (MPG) on Reference Rate Reform (2014), and Foreign Exchange Benchmark Group (2014).

[33] See Bank of England (2014). This report provides a list of OTC-market benchmarks “that should be brought into the regulatory framework originally implemented in the wake of the LIBOR misconduct scandal.” (See page 3 of the report.) A table listing the benchmarks that are recommended for regulatory treatment is found on page 15. In addition to LIBOR, which is already regulated in the United Kingdom, these are the overnight interest rate benchmarks known as SONIA and RONIA, the ISDAFix interest rate swap index, the WM/Reuters 4 pm closing foreign exchange price indices (which cover many currency pairs), the London Gold Fixing, the LBMA Silver Price, and ICE Brent (a major oil price benchmark).

So far, we have assumed that dealers can credibly commit to the truthful revelation of c. In this section we outline a simple and explicit mechanism that truthfully implements a benchmark, provided there are at least two dealers, and provided that a benchmark administrator can impose transfers in the form of fees and subsidies among them for their cost-related submissions.[34] For simplicity, we assume that $\gamma = 1$ throughout (the results can be generalized to the heterogeneous case in a straightforward way).

Suppose that there exists a benchmark administrator who can design an arbitrary "benchmark announcement" mechanism with transfers. Here, a mechanism is a pair (M, g), where $M = (M_1 \times \cdots \times M_N)$ is the product of the message spaces of the N respective dealers, and where $g : M \to [\underline{c}, \bar{c}] \times \mathbb{R}^N$. The function g maps the dealers' messages $(m_1, \ldots, m_N)$ to an announced benchmark $\tilde{c}$ and to transfers $t_1, \ldots, t_N$ from the respective dealers to the mechanism designer. Each mechanism induces a game in which dealers first submit messages. The second stage of the game is the trading game presented in Section II of this paper, in which traders assume that the announced benchmark $\tilde{c}$ is a truthful report of the actual cost c.

In this setting, "Nash implementability" means that there exists a mechanism whose associated game has a Nash equilibrium in which the announced benchmark $\tilde{c}$ is the true cost c. "Full implementability" adds the requirement that this is the unique equilibrium of the mechanism-induced game.

PROPOSITION 10: *Truthful revelation of c is Nash implementable, but is not fully Nash implementable.*

The proposition states that each dealer wants to report a message supporting the announcement of a benchmark that is the true cost c, provided that he believes that all other dealers report in this manner. However, for the mechanism that we construct, there is also an equilibrium in which all dealers report the same, but false, common cost level. The second part of Proposition 10 asserts that this cannot be avoided. That is, there exists no mechanism with a unique equilibrium in which dealers report truthfully. Informally, this means that it is impossible to elicit information about c in a way that is not susceptible to collusion.

That said, a benchmark administrator could use post-trade transaction reporting to assist with the detection of collusion. For example, if the reported cost c implies a distribution of transaction prices that differs substantially from the empirically observed distribution of transaction prices, there could be scope for further investigation by the benchmark administrator.

Specifying and solving an equilibrium model of manipulation is beyond the scope of this paper. Explicit models of benchmark manipulation in different

[34] Again, the benchmark in our setting is the common component of dealers' costs. For comparison, in a cheap-talk model, Lubensky (2016) derives conditions under which a supplier voluntarily reveals his *idiosyncratic* (or private) production cost by publishing a nonbinding price recommendation. Thus, our model and Lubensky (2016) are complementary to each other.

settings are offered by Coulter and Shapiro (2014) and Duffie and Dworczak (2014).[35]

V. Concluding Remarks

Benchmarks underlie a significant fraction of transactions in financial and nonfinancial markets, particularly those with an OTC structure that rules out a common trading venue and a publicly announced market-clearing price. This paper provides a theory of the effectiveness and endogenous introduction of benchmarks in search-based markets that are opaque in the absence of a benchmark. Our focus is the role of benchmarks in improving market transparency. That is, lowering the informational asymmetry between dealers and their customers regarding the true cost to dealers of providing the underlying asset.

In the absence of a benchmark, traders have no information other than their own search costs and what they learn individually by "shopping around" for an acceptable quote. Dealers exploit this market opaqueness in their price quotes. Adding a benchmark alleviates information asymmetry between dealers and their customers. We provide naturally motivated conditions under which the publication of a benchmark raises expected total social surplus by encouraging greater market participation by buy-side market participants, by improving the efficiency of matching, and by reducing wasteful search costs.

In some cases, dealers have an incentive to introduce benchmarks despite the associated loss of local monopoly advantage, because of a more-than-offsetting increase in the trade volume achieved through greater customer participation. When dealers have heterogeneous costs for providing the asset, those who are more cost-effective may introduce benchmarks themselves, to improve their market share by driving out higher-cost competitors.

Under homogeneous dealers' cost, disclosing the benchmark is a socially optimal mechanism if the realized dealers' cost, or the benchmark, is above an endogenous threshold. When the benchmark level is below this threshold, the optimal mechanism discloses a range of the benchmark but not its exact level. This mechanism also turns out to be optimal for dealers.

Which markets have a benchmark is not an accident of chance, but rather is likely to be an outcome of conscious decisions by dealers, case by case, when trading off the costs and benefits of the additional market transparency afforded by a benchmark. Our analysis also suggests that there may be a public

[35] Coulter and Shapiro (2014) solve a mechanism design problem with transfers in a setting that incorporates important incentives to manipulate that are absent from our model. They reach a similar conclusion in that it is possible to implement a truthful benchmark, but their mechanism can also be "rigged" for false reporting through collusion by dealers. In a different model of benchmark design and manipulation, Duffie and Dworczak (2014) show that, without transfers, an optimizing mechanism designer will generally not implement truthful reporting. Instead, considering a restricted class of mechanisms, they characterize a robust benchmark that minimizes the variance of the "garbling," which is the difference between the announced benchmark and the true cost level.

welfare role for regulators regarding which markets should have a benchmark, and also in support of the robustness of benchmarks to manipulation.

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Appendix A: Proofs for Section II

A. *Proof of Proposition 1*

Here, we fill in the gaps in the derivation of the equilibrium in the benchmark case. We focus on the nontrivial case in which $\underline{c} \leq c < v - s$.

As we argue in Section II, regardless of the price distribution that dealers use in a symmetric equilibrium, slow traders play a reservation-price strategy with some reservation price r_c. Fast traders play their weakly dominant strategy of searching the entire market. (Thus, if the trader is a fast trader, the dealers are essentially participating in a first-price auction.)

Given this strategy of traders, the following lemma establishes the properties of the dealers' equilibrium response.

LEMMA A1: *If slow traders enter with a strictly positive probability, the equilibrium price distribution cannot have atoms or gaps, and the upper limit of the distribution is equal to r_c.*

PROOF: Suppose there is an atom at some price p in the distribution of prices $F_c(\cdot)$ for some cost level $c \in (\underline{c},\ v - s)$. Suppose further that $p > c$. In this case a dealer quoting p can profitably deviate to a price $p - \epsilon$, for some small $\epsilon > 0$ (because slow traders play a reservation-price strategy, the probability of trade jumps up discontinuously). Because dealers never post prices below their costs, we must have $p = c$. But that is also impossible, because a dealer could then profitably deviate to r_c (clearly, $r_c \geq c + s$ in equilibrium). Thus, there are no atoms in the distribution.

Second, suppose that $\bar{p}_c > r_c$. In this case the dealer posting $\bar{p}_c$ makes no profits, so she could profitably deviate to r_c. In contrast, if $\bar{p}_c < r_c$, a dealer can increase profits by quoting r_c instead of $\bar{p}_c$ as this does not affect the probability of selling. Thus, $\bar{p}_c = r_c$.

Third, suppose that there is an open gap in the support of the distribution of prices conditional on some cost level c, that is, an interval $(p_1,\ p_2) \subset [\underline{p}_c,\ \bar{p}_c] \setminus \mathrm{supp}(F_c(\cdot))$. Take this interval to be maximal, that is, such that p_1 is an infimum and p_2 is a supremum, both subject to being in the support of $F_c(\cdot)$. Then we get a contradiction because the probability of selling is the same whether the dealer posts p_1 or p_2. □

The rest of the equilibrium characterization follows from the derivation in Section II.

B. Proof of Lemma 1

Let $r_0^\star$ be the equilibrium first-round reservation price for slow traders. Note that, unlike in the benchmark case, $r_0^\star$ is a number, not a function of c.

We take $c < r_0^\star$. Such a c exists because $r_0^\star \geq \underline{c} + s$. Suppose that the upper limit of the support of the distribution F_c of offer prices, $\bar{p}_c$, is (strictly) larger than $r_0^\star$. Since traders follow a reservation-price strategy, and because fast traders visit all dealers, there can be no atoms in the distribution of prices (otherwise a dealer could profitably deviate by quoting a price just below the atom). Thus, a dealer setting the price $\bar{p}_c$ never sells in equilibrium and hence makes zero profit. However, she could make positive profit by setting a price equal to $r_0^\star$. Thus, $\bar{p}_c \leq r_0^\star$. Because we took an arbitrary $c < r_0^\star$, it follows that, whenever $c < r_0^\star$, traders do not observe prices above $r_0^\star$ on the equilibrium path.

Suppose that $r_0^\star < v$. Whenever the realization of c lies above $r_0^\star$, the offer in the first round must be rejected by a slow trader (dealers cannot offer prices below their costs). In particular, a slow trader must reject the price $p^\star \in \text{supp}(F_c(\cdot))$ with $r_0^\star < p^\star \leq \inf\{p \in \text{supp}(F_c(\cdot) : c > r_0^\star\} + \delta < v$, for a sufficiently small $\delta > 0$.[36] This is a contradiction. Indeed, by the previous paragraph, conditional on observing a price $p > r_0^\star$ in the first round, the trader believes that c must lie above $r_0^\star$ with probability one. But in this case, the price $p^\star$ is within δ of the best possible price that the trader can ever be offered, so this offer should be accepted by a slow trader (if $\delta < s$), contrary to $p^\star > r_0^\star$. This shows that $r_0^\star = v$.

Finally, suppose that $\bar{p}_c < v$ for some $c < v$. Then a dealer quoting the price $\bar{p}_c$ could profitably deviate by posting a price v (the probability of trade is unaffected). This justifies the second claim.

C. Proof of Proposition 2

Fix a fraction λ of slow traders that enter. By Lemma 1 and the arguments used in the derivation of equilibrium prices in the benchmark case, the cdf of offered prices must be

$$F_c(p) = 1 - \left[\frac{\lambda(1-\mu)}{N\mu}\frac{v-p}{p-c}\right]^{\frac{1}{N-1}} \tag{A1}$$

with support $[\underline{p}_c,\ v]$, where $\underline{p}_c = \varphi(\lambda)v + (1-\varphi(\lambda))c$ and

$$\varphi(\lambda) = \frac{\lambda(1-\mu)}{N\mu + \lambda(1-\mu)}.$$

We note that the only difference with the equilibrium pricing under the benchmark is that the reservation price and probability of entry are constants, not functions of c.

[36] Such a $p^\star$ exists. As long as $c < v$, in equilibrium dealers must be posting prices below v with positive probability.

We can now calculate the expected profits of slow traders if they choose to enter:

$$\pi(\lambda) = -s + \int_{\underline{c}}^{v} \left[\int_{\underline{p}_c}^{v} (v-p)\, dF_c(p) \right] dG(c) = -s + (1-\alpha(\lambda))X,$$

where

$$X = G(v)\,[v - \mathbb{E}\,[c \,|\, c \leq v]]$$

is the expected gains from trade. By reasoning analogous to that in the benchmark case, we determine that:

- If $s \leq (1-\bar{\alpha})X$, there must be full entry by slow traders ($\lambda^\star = 1$).
- If $s \geq X$, there cannot be entry by slow traders ($\lambda^\star = 0$).
- If $s \in ((1-\bar{\alpha})X,\ X)$, then the entry of slow traders is interior, with probability $\lambda^\star$ determined uniquely by the equation (10).

D. *Proof of Proposition 3*

Given Proposition 2, to prove existence in our setting we need only show that a slow trader does not want to search after observing a price $p \leq v$ in the first round. After observing a price p, the slow trader forms a posterior probability distribution of c, given by the cdf

$$G(c \mid p) = \frac{\int_{\underline{c}}^{c} f_y(p)\, dG(y)}{\int_{\underline{c}}^{\bar{c}_p} f_y(p)\, dG(y)},$$

where $f_c(p)$ denotes the density of the distribution defined by the cdf (11), and

$$\bar{c}_p = \frac{1}{1-\varphi(\lambda^\star)} p - \frac{\varphi(\lambda^\star)}{1-\varphi(\lambda^\star)} v$$

is the upper limit of the support of the posterior distribution.

With two dealers, it is easy to provide a necessary and sufficient condition for existence. A price p is accepted in the first round if and only if

$$v - p \geq -s + \int_{\underline{c}}^{\bar{c}_p} \left[\int_{\underline{p}_c}^{p} (v-\rho) f_c(\rho)\, d\rho + (v-p)(1-F_c(p)) \right] dG(c \mid p),$$

or

$$s \geq \frac{\int_{\underline{c}}^{\bar{c}_p} \int_{\underline{p}_c}^{p} F_c(\rho) d\rho (v-c)(p-c)^{-2}\, dG(c)}{\int_{\underline{c}}^{\bar{c}_p} (v-c)(p-c)^{-2}\, dG(c)}. \tag{A2}$$

Thus, a reservation-price equilibrium with two dealers exists if and only if inequality (A2) holds for all $p \in (\underline{p}_{\underline{c}},\ v)$. The condition can be easily verified, as the expression on the right-hand side of (A2) is directly computable.

With more than two dealers, an additional difficulty arises because it is not easy to calculate the continuation value when an offer p is rejected in the first round. We can nevertheless provide a sufficient condition based on the following argument. Suppose that after observing p and forming the posterior belief about c, the slow trader is promised to find, in the next search, an offer equal to the lower limit of the price distribution. This provides an upper bound on the continuation value; thus, if the trader decides not to search in this case, she would also not search under the actual continuation value. A sufficient condition for existence is therefore that

$$s \geq (p - v) + (1 - \varphi(\lambda^\star)) \frac{\int_{\underline{c}}^{\bar{c}_p} (v - c)^2 (p - c)^{-\frac{N}{N-1}} \, dG(c)}{\int_{\underline{c}}^{\bar{c}_p} (v - c)(p - c)^{-\frac{N}{N-1}} \, dG(c)}, \tag{A3}$$

for all $p \in (\underline{p}_{\underline{c}},\ v)$. Again, inequality (A3) can be directly computed and verified.

The last step in the proof is to show that inequality (A3) holds for s in some range below X. To this end, we analyze the behavior of the posterior distribution of costs $G(c \mid p)$ after a price p is observed by a slow trader in the first round when the probability of entry $\lambda^\star$ is small. As $\lambda^\star \searrow 0$, conditional on p, the upper limit of the support of the posterior cost distribution, $\bar{c}_p$, converges to p. Thus, $G(c \mid p)$ converges pointwise to zero for $c < p$ and to one for $c > p$. By one of the (equivalent) definitions of weak* convergence of probability measures, the posterior distribution converges in distribution to an atom at p. Thus, in the limit, inequality (A3) becomes

$$s \geq (p - v) + (1 - \varphi(0))(v - p) = 0,$$

and is thus vacuously satisfied. By continuity of the right-hand side of inequality (A3), the inequality holds if $\lambda^\star$ is smaller than some $\underline{\lambda} > 0$. Recall that $\lambda^\star$ is determined uniquely by equation (10). Moreover, it is continuous and strictly decreasing in s for $s \in ((1 - \bar{\alpha})X,\ X)$, and equal to zero at $s = X$. Thus, there exists $\underline{s} < X$ such that, for all $s > \underline{s}$, $\lambda^\star$ is smaller than $\underline{\lambda}$.

E. Proof of Theorem 1

We first outline the main steps of the argument, and leave the technical details for the two lemmas that follow. To make the proof concise, we make a change of variables by defining $x = (v - c)^+ \equiv \max\{v - c, 0\}$ as the realized gain from a trade given the common cost c.

Note first that conditions (i) and (ii) both imply that $s > (1 - \bar{\alpha})X$. The case $s \geq X$ is trivial to analyze as there is no entry of slow traders without the benchmark (see Proposition 2). Thus, we focus on the range $(1 - \bar{\alpha})X < s < X$, within which Proposition 2 implies interior entry in the absence of the benchmark.

The total expected surplus in the no-benchmark case is

$$W_{nb} \equiv \left[\lambda^\star(1 - \mu) + \mu\right] X - \lambda^\star(1 - \mu)s.$$

With the benchmark, we let $\lambda(x)$ denote the probability of entry by slow traders conditional on a realized gain from trade of x. By Proposition 1,

$$\lambda(x) \quad \begin{cases} = 0, & \text{if } x \le s, \\ \text{solves } s = (1-\alpha(\lambda(x)))x, & \text{if } s < x < \frac{s}{1-\bar{\alpha}}, \\ = 1, & \text{if } x \ge \frac{s}{1-\bar{\alpha}}. \end{cases}$$

The conditional expected social surplus in the benchmark case conditional on x is

$$W_b(x) \equiv \left[\lambda(x)(1-\mu) + \mu\right] x - \lambda(x)(1-\mu)s.$$

The crucial observation, demonstrated in Lemma A2 below, is that W_b is a convex function on $[0,\ s/(1-\bar{\alpha})]$. Figure 1 depicts a typical shape of that function.

Under condition (i), W_b is convex on its entire domain. (This corresponds to cutting off the part of the domain that upsets convexity, as shown in Figure 1). We can thus apply Jensen's Inequality to obtain

$$\mathbb{E}\left[W_b(x)\right] \ge W_b\left[\mathbb{E}(x)\right] = W_b\left(\int_{\underline{c}}^{\bar{c}} (v-c)^+ \, dG(c)\right) = W_b(X) = W_{nb}.$$

To justify the last equality, one notes that $\lambda^\star$ is precisely $\lambda(X)$, by equations (8) and (10). (This inequality is actually strict because G is a nondegenerate distribution and because $\lambda(x) > 0$ with positive probability under G.)

Under condition (ii), W_b may fail to be convex on its entire domain. However, an inspection of the proof of Jensen's Inequality shows that all that is required to achieve the inequality is that the function W_b is subdifferentiable[37] at $\mathbb{E}(x)$. The slope of W_b is increasing on $[0,\ s/(1-\bar{\alpha})]$ and equal to 1 on $(s/(1-\bar{\alpha}),\ v-\underline{c}]$. Thus, a sufficient condition for existence of a supporting hyperplane of W_b at X is that $W_b'(X) \le 1$. We thus want to solve the equation $W_b'(x_0) = 1$ for $x_0 \in (s,\ s/(1-\bar{\alpha}))$ and impose $X \le x_0$. (See Figure 1.) An explicit solution is not available, so instead we show in Lemma A3 below (by approximating the functions α and λ) that this condition is implied by $s \ge (1-\psi)X$.

Finally, a simple application of the Lebesgue Dominated Convergence Theorem shows that $\bar{\alpha}$ converges (monotonically) to 1 when either $N \to \infty$ or $\mu \to 0$. Thus, condition *(i)* holds if N is large enough or if μ is small enough.

LEMMA A2: *$W_b(x)$ and $\lambda(x)$ are convex functions on* $[0,\ s/(1-\bar{\alpha})]$.

PROOF: First we prove that $\lambda(x)$ and $W_b(x)$ are convex on $(s,\ s/(1-\bar{\alpha})]$. By the Implicit Function Theorem, λ is twice differentiable on this interval and we have

$$\frac{\partial \lambda}{\partial x} = \frac{(1-\alpha(\lambda))}{\alpha'(\lambda)x} > 0$$

[37] A function $f : [a, b] \to \mathbb{R}$ is said to be subdifferentiable at x_0 if there exists a real number ξ such that, for all x in $[a, b]$, we have $f(x) - f(x_0) \ge \xi(x - x_0)$. If W_b is convex, then it is subdifferentiable on the interior of its domain by the Separating Hyperplane Theorem.

and

$$\frac{\partial^2 \lambda}{\partial x^2} = \frac{-\alpha'(\lambda)(1-\alpha(\lambda)) - (1-\alpha(\lambda))\left[\alpha'(\lambda) + \alpha''(\lambda)\frac{(1-\alpha(\lambda))}{\alpha'(\lambda)}\right]}{\left[\alpha'(\lambda)x\right]^2}.$$

Hence, $\frac{\partial^2\lambda}{\partial x^2} \geq 0$ for all $x \in (s,\ s/(1-\bar{\alpha}))$ if and only if, for all $\lambda \in (0,\ 1)$,

$$2\left[\alpha'(\lambda)\right]^2 + \alpha''(\lambda)(1-\alpha(\lambda)) \leq 0. \tag{A4}$$

Letting $\beta = N\mu/(1-\mu)$ and computing the derivatives of $\alpha(\lambda)$, we rewrite (A4) as

$$\left(\int_0^1 \frac{\beta z^{N-1}}{\left(\lambda+\beta z^{N-1}\right)^2}\,dz\right)^2 \leq \left(\int_0^1 \frac{\beta z^{N-1}}{\left(\lambda+\beta z^{N-1}\right)^3}\,dz\right)\left(\int_0^1 \frac{\beta z^{N-1}}{\lambda+\beta z^{N-1}}\,dz\right).$$

Hölder's Inequality states that, for all measurable and square-integrable functions f and g, $\|fg\|_1 \leq \|f\|_2\|g\|_2$. By letting

$$f(z) = \sqrt{\frac{\beta z^{N-1}}{\left(\lambda+\beta z^{N-1}\right)^3}} \quad \text{and} \quad g(z) = \sqrt{\frac{\beta z^{N-1}}{\lambda+\beta z^{N-1}}},$$

we have proven inequality (A4) and thus the convexity of $\lambda(x)$.

Now it becomes straightforward to check that $W_b(x)$ is convex on $[s,\ s/(1-\bar{\alpha})]$. Notice that $W_b(x)$ and $\lambda(x)$ are trivially convex on $[0,\ s]$ (because, on this interval, $\lambda(x)$ is identically zero and $W_b(x)$ is affine). Therefore, to finish the proof it is enough to make sure that $\lambda(x)$ and $W_b(x)$ are differentiable at s. We can verify this by computing the left and right derivatives: $\partial_- W_b([s]) = \mu = \partial_+ W_b([s])$ and $\partial_-\lambda([s]) = 0 = \partial_+\lambda([s])$. □

LEMMA A3: *If* $x \leq \frac{s}{1-\psi}$, *where* $\psi = \frac{1}{2}[\sqrt{(1-\bar{\alpha}+\bar{\alpha}\beta)^2 + 4\bar{\alpha}(1-\bar{\alpha})} - (1-\bar{\alpha}+\bar{\alpha}\beta)]$ *and* $\beta = \frac{N\mu}{1-\mu}$, *then* $W_b'(x) \leq 1$.

PROOF: The claim is true for $x \leq s$, and since $\psi \leq \bar{\alpha}$, we can focus on the region where $\lambda(x)$ is defined as the solution to equation (8), which can be written as

$$\alpha(\lambda(x)) = 1 - \frac{s}{x}.$$

Since $\alpha(\cdot)$ is a strictly increasing function, if we replace $\alpha(\cdot)$ in the above equation by a lower bound, any solution of the new equation will be an upper bound on $\lambda(x)$. Because $W_b(x)$ is convex in the relevant part of the domain (by Lemma A2), to make sure that $W_b'(x) \leq 1$, it's enough to require that $x \leq x_0$, where x_0 solves $W_b'(x_0) = 1$ (such x_0 exists and is unique). We have

$$W_b'(x_0) = \mu + \lambda'(x_0)(1-\mu)(x_0 - s) + \lambda(x_0)(1-\mu) = 1. \tag{A5}$$

We cannot solve this equation explicitly, so we provide a lower bound on the solution. Because $W_b'(x)$ is increasing, we need to bound $W_b'(x)$ from above. Since $\alpha(\lambda) \geq \lambda\bar{\alpha}$, by the above remark, the solution of the equation

$$\bar{\alpha}\bar{\lambda}(x) = 1 - \frac{s}{x}$$

provides an upper bound on $\lambda(x)$. That is,

$$\lambda(x) \leq \bar{\lambda}(x) = \frac{1}{\bar{\alpha}} - \frac{s}{\bar{\alpha}}\frac{1}{x}.$$

Moreover,

$$\lambda'(x) = \frac{1}{\alpha'(\lambda(x))}\frac{s}{x^2},$$

and we have, for all $\lambda \in [0,\ 1]$,

$$\alpha'(\lambda) = \int_0^1 \frac{\beta z^{N-1}}{\left(\lambda + \beta z^{N-1}\right)^2}\,dz \geq \frac{1}{\lambda+\beta}\int_0^1 \left(\frac{\lambda+\beta z^{N-1}}{\lambda+\beta z^{N-1}} - \frac{\lambda}{\lambda+\beta z^{N-1}}\right) dz$$
$$= \frac{1}{\lambda+\beta}(1-\alpha(\lambda)) \geq \frac{1-\bar{\alpha}}{\lambda+\beta}.$$

Plugging these bounds into equation (A5) and rearranging, we obtain

$$\frac{\beta + \frac{1}{\bar{\alpha}} - \frac{s}{\bar{\alpha}}\frac{1}{x_0}}{1-\bar{\alpha}}\frac{s}{x_0}\left(1 - \frac{s}{x_0}\right) + \frac{1}{\bar{\alpha}}\left[1 - \frac{s}{x_0}\right] = 1.$$

Denoting $y = 1 - s/x_0$, bounding the left-hand side from above one more time, and rearranging, we get

$$y^2 + (1 - \bar{\alpha} + \bar{\alpha}\beta)y - \bar{\alpha}(1-\bar{\alpha}) = 0.$$

The relevant solution is ψ. □

F. Proof of Proposition 4

This result follows directly from Propositions 1 and 2.

G. Proof of Theorem 2

The proof of Theorem 2 is very similar to the proof of Theorem 1, so we skip some of the details. Denote the expected profits of a dealer in the benchmark case conditional on x (where $x = (v-c)_+$) by $\chi_b(x)$ and in the case with no benchmark by χ_{nb}. Recall from Propositions 1 and 2 that

$$\chi_b(x) = \frac{\lambda(x)(1-\mu)}{N}\frac{s}{1-\alpha(\lambda(x))}$$

and $\chi_{nb} = X\lambda^\star(1-\mu)/N$.

Assume that condition (i) holds. Then, using the fact that $\lambda(x)$ is given by $s=(1-\alpha(\lambda(x)))x$ in the relevant range, we can write $\chi_b(x)=(1-\mu)\lambda(x)x/N$. By Lemma A2, $\lambda(x)$ is increasing and convex, so $\chi_b(x)$ is also convex. Therefore, applying Jensen's Inequality we get

$$\mathbb{E}\left[\chi_b(x)\right] \geq \chi_b\left(\mathbb{E}[x]\right) = \chi_b(X) = \chi_{nb}.$$

Now assume that condition (ii) holds. As in the proof of Theorem 1, we want to find a condition on X that would guarantee that the profit function χ_b is subdifferentiable at X. Using the reasoning from the proof of Theorem 1, we can establish existence of a constant $\eta \in (0, \bar{\alpha})$ that depends only on μ and N, such that $X \leq s/(1-\eta)$ guarantees existence of a supporting hyperplane at X (thus allowing us to apply Jensen's Inequality).

H. Proof of Proposition 5

For this proof only, let $\tilde{G}$ denote the distribution function of the gain from trade $\max\{v-c,\, 0\}$ (derived from the cdf G of the cost c). We also denote $\bar{s} \equiv s/(1-\bar{\alpha})$ to simplify expressions. The proof is by contradiction. Suppose that $\mathbb{E}[\chi_b(x)] \geq \chi_b(\mathbb{E}[x])$ but $\mathbb{E}[W_b(x)] < W_b(\mathbb{E}[x])$. The first inequality implies that $X=\mathbb{E}[x] < \bar{s}$ (with full entry, dealers would not want to introduce a benchmark). Using the expressions for $W_b(x)$ and $\chi_b(x)$ derived in earler proofs, and simplifying these inequalities, we obtain

$$\int_{v-\bar{c}}^{\bar{s}} \lambda(x)x\, d\tilde{G}(x) + \int_{\bar{s}}^{v-\underline{c}} \bar{s}\, d\tilde{G}(x) \geq \lambda(\mathbb{E}[x])\mathbb{E}[x], \tag{A6}$$

$$\int_{v-\bar{c}}^{\bar{s}} \lambda(x)(x-s)\, d\tilde{G}(x) + \int_{\bar{s}}^{v-\underline{c}} (x-s)\, d\tilde{G}(x) < \lambda(\mathbb{E}[x])(\mathbb{E}[x]-s). \tag{A7}$$

Combining (A6) and (A7) yields

$$\int_{v-\bar{c}}^{\bar{s}} \lambda(x)(x-s)\, d\tilde{G}(x) + \int_{\bar{s}}^{v-\underline{c}} (x-s)\, d\tilde{G}(x) < \int_{v-\bar{c}}^{\bar{s}} \lambda(x)x\, d\tilde{G}(x) + \int_{\bar{s}}^{v-\underline{c}} \bar{s}\, d\tilde{G}(x) - s\lambda(\mathbb{E}[x]),$$

or

$$\underbrace{s\left[\lambda(\mathbb{E}[x]) - \int_{v-\bar{c}}^{\bar{s}} \lambda(x)\, d\tilde{G}(x) + \int_{\bar{s}}^{v-\underline{c}} d\tilde{G}(x)\right]}_{\lambda(\mathbb{E}[x])-\mathbb{E}[\lambda(x)]} < \int_{\bar{s}}^{v-\underline{c}} (\bar{s}-x)\, d\tilde{G}(x) \leq 0.$$

Thus, we have $\lambda(\mathbb{E}[x]) < \mathbb{E}[\lambda(x)]$, that is, the probability of entry is higher with the benchmark. The last step in the proof is to show that this implies that the benchmark case yields a higher expected social surplus than the no-benchmark case (leading to a contradiction). We have

$$\mathbb{E}\left[\lambda(x)(x-s)\right] = \mathbb{E}\left[\lambda(x)(x-s)_+\right] \geq \mathbb{E}[\lambda(x)]\mathbb{E}[(x-s)_+] \geq \lambda(\mathbb{E}[x])\mathbb{E}[(x-s)_+]$$
$$\geq \lambda(\mathbb{E}[x])\mathbb{E}[x-s], \tag{A8}$$

where the first equality is true because $\lambda(x) = 0$ when $x \leq s$, the second inequality follows from the fact that $\lambda(x)$ and $(x-s)_+$ are positively correlated as random variables (their covariance is positive), the third is by the fact that $\lambda(\mathbb{E}[x]) < \mathbb{E}[\lambda(x)]$, and the last one is trivial. Because $W_b(x)$ is an affine transformation of $\lambda(x)(x-s)$, the inequality (A8) implies that $\mathbb{E}[W_b(x)] \geq W_b(\mathbb{E}[x])$, finishing the proof.

I. Proof of Theorem 3

First, we reformulate the problem as a Bayesian persuasion problem with infinite type and action spaces. A Sender (mechanism designer) who observes the state of the world (cost c) sends a signal under commitment to an uninformed Receiver (trader) who then takes an action based on the posterior belief of the state. (See Kamenica and Gentzkow (2011), along with their Online Appendix, for details.)

Given an arbitrary distribution function H of costs c, let $X_H = \mathbb{E}_H[(v-c)_+]$ be the associated expected gain from trade. In a reservation-price equilibrium (which is assumed to exist in Section II.F), the entry probability of slow traders is $\lambda(X_H)$, and the social surplus is $W_b(X_H)$. If the mechanism induces posterior belief H conditional on slow traders observing the signal, then the value of the objective function is $W_b(X_H)$. In particular, the objective function depends on the posterior belief of the cost H only through the expectation of the conditional gain from trade X_H.

We can first simplify the problem by noting that it is always optimal to reveal c whenever $x = (v-c)_+ < s$. Indeed, fixing a mechanism, suppose the probability of entry by slow traders conditional on the event $x < s$ is strictly positive. We can then construct a new mechanism that is identical to the old one, except that it discloses c whenever $x < s$. In the new mechanism, when $x < s$ and c is revealed, slow traders do not enter. This raises social surplus because not entering yields 0, whereas entering yields at most $x - s$, which is strictly negative. When $x \geq s$, any signal from the new mechanism induces a weakly higher posterior mean gain from trade than that of the original mechanism. Because social welfare $W_b(X_H)$ is increasing in the posterior mean X_H, the new mechanism does better than the old one.

Hence, we can focus on the case $x \geq s$. Because there is a one-to-one correspondence between x and c in this region, we can treat x as the primitive state. Let $\tilde{G}$ be the distribution of x conditional on $x \geq s$, which can be computed from the original distribution G of costs. Dworczak and Martini (2017) provide a method for constructing optimal signals for Bayesian persuasion problems in which the objective function depends on the posterior beliefs of the underlying random variable only through the posterior mean. Optimization can be

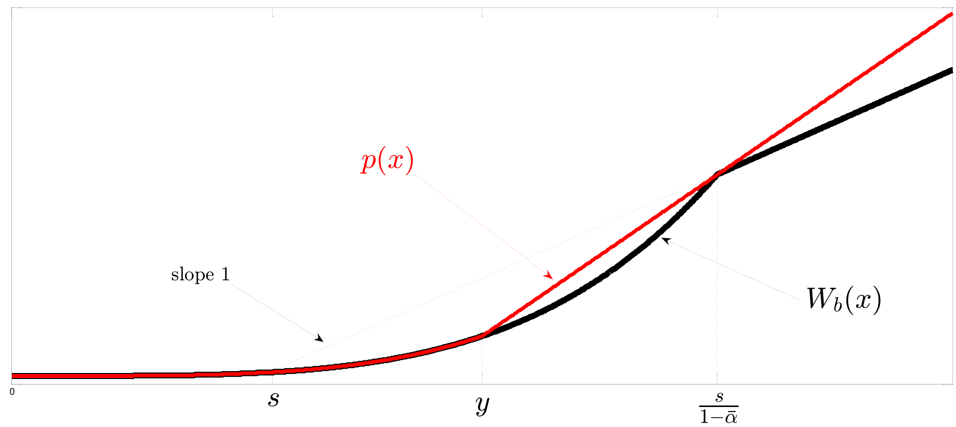

Figure A1. The functions $W_b(x)$ and $p(x)$ of the gain from trade x. (Color figure can be viewed at wileyonlinelibrary.com)

performed directly in the space of distributions over posterior means. By applying duality methods, Dworczak and Martini (2017) prove the following result, which we restate for the convenience of the reader. (The function p below can be thought of as a Lagrange multiplier.)

LEMMA A4: *Suppose that F is a cumulative distribution function on $[s,\ v-\underline{c}]$ and $p:\ [s,\ v-\underline{c}] \to \mathbb{R}$ is a convex function. If F and p satisfy*

1. $\text{supp}(F) \subseteq \text{argmax}_{x \in [s,\ v-\underline{c}]}\{W_b(x) - p(x)\}$,
2. $\mathbb{E}_F\,[p(x)] = \mathbb{E}_{\tilde{G}}\,[p(x)]$,
3. *$\tilde{G}$ is a mean-preserving spread of F,*

then F is the distribution of the posterior means of x associated with the welfare-maximizing revelation mechanism.

To exploit Lemma A4, we construct the function p as

$$p(x) = \begin{cases} W_b(x) & \text{if } x \le y \\ W_b(y) + \frac{W_b(s/(1-\bar{\alpha})) - W_b(y)}{s/(1-\bar{\alpha}) - y}(x-y) & \text{if } x \ge y, \end{cases}$$

where recall that y is defined by $\mathbb{E}[x \mid x \ge y] = s/(1-\bar{\alpha})$.

The functions W_b and p are illustrated in Figure A1. Note that $p(x)$ coincides with $W_b(x)$ for $x \le y$. To the right of y, p is linear and $p(x)$ is equal to $W_b(x)$ at exactly one point $x = s/(1-\bar{\alpha})$. By the properties of W_b analyzed in Appendix A.E, the slope of the linear component of p must be larger than one, and the slope of W_b at $x \ge s/(1-\bar{\alpha})$ is equal to one. Moreover, W_b is strictly convex on $[s,\ s/(1-\bar{\alpha})]$. It follows that

$$\text{argmax}_{x \in [s,\ v-\underline{c}]}\left\{W_b(x) - p(x)\right\} = [s,\ y] \cup \{s/(1-\bar{\alpha})\}.$$

We define the distribution F by

$$F(x) = \begin{cases} \tilde{G}(x), & x \le y, \\ \tilde{G}(y) + (1 - \tilde{G}(y))\mathbf{1}_{\{x \ge s/(1-\bar{\alpha})\}}, & x \ge y. \end{cases}$$

The distribution F coincides with $\tilde{G}$ for $x \le y$ and assigns the rest of its mass to an atom at $s/(1-\bar{\alpha})$. The distribution F satisfies condition 1 of Lemma A4 with equality. By the choice of y, F is a distribution of conditional means of $\tilde{G}$. This implies that $\tilde{G}$ is a mean-preserving spread of F, which in turn implies condition 3 of Lemma A4. To verify condition 2, we note that F and $\tilde{G}$ coincide on $[s, y]$. In the remaining interval $[y, v - \underline{c}]$, F and $\tilde{G}$ have the same mean and p is linear, leading to equal expectations of the value of function p.

Hence, we can apply Lemma A4 to conclude that F is the distribution of posterior means of x associated with a welfare-maximizing revelation mechanism. By direct calculation, the mechanism described in Theorem 3 leads to distribution of posterior means F and thus is a welfare-maximizing revelation mechanism.

J. Proof of Proposition 6

The first part of the proof follows directly from Theorem 3. To prove the necessity of disclosing x in the interval $[s, v - \underline{c}]$, note that, under Assumption (i), the function $W_b(x)$ is globally convex, and strictly convex on $[s, v - \underline{c}]$. If there were an optimal mechanism that does not disclose x fully in some set of nonzero measure in $[s, v - \underline{c}]$, then it would be dominated by a mechanism that does, by (the strict version of) Jensen's Inequality.

K. Proof of Theorem 4

The proof of Theorem 4 is analogous to the proof of Theorem 3. In a reservation-price equilibrium, as shown in Section II, the dealer's expected profit function, conditional on the gain from trade x, is given by

$$\chi_b(x) = \begin{cases} 0 & \text{if } x \le s, \\ \frac{\lambda(x)(1-\mu)}{N} x & \text{if } s/(1-\bar{\alpha}) > x > s, \\ \frac{1-\mu}{N} \frac{s}{1-\bar{\alpha}} & \text{if } x \ge s/(1-\bar{\alpha}). \end{cases}$$

Moreover, the properties of $\chi_b(x)$ coincide with the properties of the function $W_b(x)$ used in the proof of Theorem 3. Most importantly, $\chi_b(x)$ is convex for $x \le s/(1-\bar{\alpha})$ (convexity of $\lambda(x)$ was established by the proof of Theorem 1), and has a kink at $x = s/(1-\bar{\alpha})$. We can apply Lemma A4 with $W_b(x)$ replaced by $\chi_b(x)$ and use the same Lagrange multiplier p and distribution F defined in the proof of Theorem 3. (See also Figure A1.) Because the rest of the argument is identical to the proof of Theorem 3, we omit the remaining details.

Appendix B: Proofs and Supporting Content for Section III and Section IV

A. Proof of Proposition 7 and Equilibrium Characterization in the Benchmark Case

Because the distribution of costs is i.i.d. across dealers conditional on observing the benchmark, slow traders must follow a reservation-price strategy with some reservation price r_c. A stationary [38] reservation-price strategy of slow traders will now be characterized by three numbers: λ_c, the probability of entry; r_c, the reservation price; and $\hat{\theta}_c$, the probability of rejecting an offer equal to the reservation price r_c. Fixing the strategy of the dealers and the reservation price r_c, the rejection probability $\hat{\theta}_c$ determines the probability θ_c that a slow trader rejects an offer from a high-cost dealer, and vice versa. Given the one-to-one correspondence between θ_c and $\hat{\theta}_c$, for convenience we will abuse the notation for the strategy of a slow trader, denoting it by the triple $(r_c, \lambda_c, \theta_c)$. Again without loss of generality, we can assume that fast traders play their weakly dominant strategy of always entering and visiting all dealers. We ignore the issue of off-equilibrium beliefs, as it is fairly trivial to deal with.

Fixing c and a strategy $(r_c, \lambda_c, \theta_c)$, we characterize the equilibrium best-response of dealers. We start with two technical lemmas.

LEMMA B1: *In equilibrium, conditional on c (for $c < v$), if dealers of a certain type (high-cost or low-cost) make positive expected profits, then the probability distribution of price offers for that type is atomless. If high-cost dealers make zero expected profits, then in equilibrium they must quote a price equal to their cost, provided that $c + \Delta < v$.*

PROOF: The first part of the lemma can be proven using the argument from the proof of Lemma A1. To prove the second part, suppose that for some $c < v - \Delta$, a price above $c + \Delta$ is in the support of the equilibrium strategy of high-cost dealers. The probability of selling at that price (or some lower price above $c + \Delta$) must be positive since with probability $(1 - \gamma)^N$ only high-cost dealers are present in the market. Thus, we get a contradiction with the assumption that high-cost dealers make zero expected profits. □

LEMMA B2: *In equilibrium, conditional on c, if $c < v$, for any equilibrium price p_l of a low-cost dealer and any equilibrium price p_h of a high-cost dealer, we have $p_l \le p_h$.*

PROOF: The claim is true by a standard "revealed-preference" argument. Suppose that $p_l > p_h$. Fix an equilibrium, and let $\varrho(p)$ (for some fixed $c \le v$) be the

[38] Requiring stationarity, that is, the same mixing probability at every search round, simplifies the exposition and is without loss of generality. Without stationarity, there is an indeterminacy in specifying the probability of rejecting the reservation price in equilibrium. Traders can use different mixing probabilities in every search round, as long as they lead to the same posterior beliefs of dealers. This indeterminacy does not change expected equilibrium payoffs, so without loss of generality we get rid of it by requiring stationarity.

probability that a dealer sells the asset when posting the price p. Since dealers are optimizing in equilibrium, we must have

$$\varrho(p_l)(p_l - c) \geq \varrho(p_h)(p_h - c), \tag{B1}$$

$$\varrho(p_h)(p_h - c - \Delta) \geq \varrho(p_l)(p_l - c - \Delta). \tag{B2}$$

If $\varrho(p_h) \neq 0$, then

$$\varrho(p_h)(p_h - c - \Delta) < \varrho(p_h)(p_l - c - \Delta).$$

If $p_l > c + \Delta$, then $\varrho(p_h) > \varrho(p_l)$. From inequality (B1),

$$\varrho(p_l)(p_l - c) + \Delta(\varrho(p_h) - \varrho(p_l)) > \varrho(p_h)(p_h - c),$$

which contradicts inequality (B2).

We are left with two cases. First, suppose that $p_l \leq c + \Delta$. Then $p_h < c + \Delta$, which is impossible in equilibrium. Second, suppose that $\varrho(p_h) = 0$. Then it must be the case that $\varrho(p_l) = 0$ as well, which is a contradiction if $c < v$. □

Finally, we prove a lemma about the possibility of gaps in the distribution of prices. Let $\underline{p}_c^i$ and $\bar{p}_c^i$ denote the lower and upper limit of the support of the distribution of prices for dealer of type $i \in \{l, h\}$.

LEMMA B3: *In equilibrium, conditional on c (for $c < v$), there can be no gaps in the distribution of prices except for the cases in which the support of the distribution of prices of low-cost dealers consists of two intervals,* $[\underline{p}_c^l, r_c]$ *and* $[\hat{p}_c^l, \min\{c + \Delta, v\}]$*, and in which either (i) high-cost dealers post* $c + \Delta$ *or (ii)* $c > v - \Delta$.

PROOF: Suppose that there is a gap in the distribution of prices conditional on some cost level c for some type of dealers, that is, an interval $(p_1, p_2) \subset [\underline{p}_c^i, \bar{p}_c^i] \setminus \mathrm{supp}(F_c^i(\cdot))$, $i \in \{l, h\}$. We take this interval to be maximal, that is, such that p_1 and p_2 are in the support of $F_c^i(\cdot)$. It must be the case that the probability of selling is strictly larger at p_1 than at p_2, and thus, in a reservation-price equilibrium, $p_1 \leq r_c \leq p_2$ (we make use here of Lemma B2). It cannot be the case that $p_1 < r_c$ because then the dealer posting p_1 could profitably deviate to r_c. Thus $p_1 = r_c$.

By Lemma B2, $\bar{p}_c^h$ is the highest price that can be observed on the equilibrium path, and it lies above r_c. It follows, using Lemma B1, that high-cost dealers make zero expected profits (if the price distribution for high-cost dealers were atomless, the probability of selling at the price $\bar{p}_c^h > r_c$ would be zero). Moreover, either (i) high-cost dealers post $c + \Delta$ or (ii) $c > v - \Delta$. In either case we can conclude that $i = l$, that is, the gap occurs in the price distribution of low-cost dealers.

By the above, if there is a gap, then the support of the distribution for low-cost dealers consists of two intervals, the first of which must be $[\underline{p}_c^l, r_c]$. To prove that $\bar{p}_c^l = \min\{c + \Delta, v\}$, we use the fact that $\bar{p}_c^l > r_c$, and thus if $\bar{p}_c^l < \min\{c + \Delta, v\}$, the dealer quoting $\bar{p}_c^l$ would want to deviate to $\min\{c + \Delta, v\}$. □

Using the above observations, we can now show, case by case, that the equilibrium pricing strategies are uniquely pinned down when there are gains from trade. (We assume throughout that $c < v$; the opposite case is trivial.) We let $F_c^l(p)$ denote the cumulative distribution function of prices for low-cost dealers, and $F_c^h(p)$ the cumulative distribution function of prices for high-cost dealers. In most cases it is a routine exercise to rule out the possibility of a gap in the distribution, using Lemma B3. We therefore comment on this possibility explicitly only in the two cases when a gap actually occurs in equilibrium.

Case 1: $\lambda_c = 0$. When $\lambda_c = 0$, only fast traders enter. In this case, we have a standard first-price auction between dealers. There are two subcases.

When $c > v - \Delta$, high-cost dealers cannot sell in equilibrium, and the specification of their strategy is irrelevant (they can choose any price above $c + \Delta$). In this case low-cost dealers randomize according to a distribution $F_c^l(p)$ that solves the equation

$$\left[\sum_{k=0}^{N-1} \binom{N-1}{k} \gamma^k (1-\gamma)^{N-1-k} \left(1 - F_c^l(p)\right)^k\right](p-c) = (1-\gamma)^{N-1}(v-c).$$

Let us define the function

$$\Phi(z) = \frac{1}{1-(1-\gamma)^{N-1}} \sum_{k=1}^{N-1} \binom{N-1}{k} z^k \gamma^k (1-\gamma)^{N-1-k}, \tag{B3}$$

which can be viewed as a generalization of the function z^{N-1} that appears in the definition (6). It is easy to see that $\Phi(z)$ is a (strictly) increasing polynomial with $\Phi(0) = 0$, $\Phi(1) = 1$, and $\Phi(z) = z^{N-1}$ when $\gamma = 1$. Moreover, using the binomial identity, we can write $\Phi(z)$ alternatively as

$$\Phi(z) = \frac{(z\gamma + 1 - \gamma)^{N-1} - (1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}. \tag{B4}$$

Using definition (B3), we can write

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}} \frac{v-p}{p-c}\right)$$

with upper limit $\bar{p}_c^l = v$, and lower limit $\underline{p}_c^l = (1-\gamma)^{N-1}v + (1-(1-\gamma)^{N-1})c$.

When $c \leq v - \Delta$, high-cost dealers can sell in equilibrium, but a standard result from auction theory (see, for example, Fudenberg and Tirole (1991)) says that in the unique equilibrium they will make zero profit by bidding $c + \Delta$. In this case, the distribution $F_c^l(p)$ solves

$$\left[\sum_{k=0}^{N-1} \binom{N-1}{k} \left(1 - F_l(p|\, c)\right)^k \gamma^k (1-\gamma)^{N-1-k}\right](p-c) = (1-\gamma)^{N-1}\Delta,$$

and thus we get

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}} \frac{(c+\Delta)-p}{p-c}\right)$$

with upper limit $\bar{p}_c^l = c + \Delta$ and lower limit $\underline{p}_c^l = c + (1-\gamma)^{N-1}\Delta$.

Case 2: $\lambda_c > 0$. From now on, we assume $\lambda_c > 0$, that is, slow traders enter with positive probability. There are again two subcases.

When $c > v - \Delta$ (case 2.1), high-cost dealers cannot sell in equilibrium, and the specification of their strategy is irrelevant. Low-cost dealers mix according to a continuous distribution $F_c^l(p)$ on an interval with upper limit $\bar{p}_l^c = r_c$, or on a union of two intervals as in Lemma B3.

When $c \le v - \Delta$ (case 2.2), using Lemmas B1, B2, B3 and the argument from the proof of Lemma A1, we can show that only two subcases are possible:

- If $r_c \le c + \Delta$, (case 2.2.1), high-cost dealers make zero profit; they post a price $c + \Delta$ with probability one, while low-cost dealers mix according to a continuous distribution on an interval with upper limit $\bar{p}_c^l = r_c$, or on a union of two intervals as in Lemma B3.
- If $r_c > c + \Delta$ (case 2.2.2), high-cost dealers make positive profits, and in equilibrium both low-cost and high-cost dealers mix according to continuous distributions with adjacent supports ($\bar{p}_c^l = \underline{p}_c^h$), and with r_c being the upper limit of the distribution of the prices of high-cost dealers ($\bar{p}_c^h = r_c$).

Below we analyze these cases in detail and characterize the optimal search behavior of slow traders. We first define some key functions that generalize their equivalents from Section II to the case in which there is an idiosyncratic component of dealer costs. Let $q(\lambda_c,\ \theta_c)$ be the posterior probability that a customer is a fast trader, conditional on a visit, given the strategy $(r_c,\ \lambda_c,\ \theta_c)$. That is, let

$$q(\lambda_c,\ \theta_c) = \frac{N\mu}{N\mu + \frac{1-\theta_c^N(1-\gamma)^N}{1-\theta_c(1-\gamma)}\lambda_c(1-\mu)}. \tag{B5}$$

This definition generalizes formula (1). We also generalize the definition of the function α from equation (6), which now becomes a function of two arguments:

$$\alpha(\lambda_c,\ \theta_c) = \int_0^1 \left(1 + \frac{q(\lambda_c,\ \theta_c)\left(1-(1-\gamma)^{N-1}\right)}{1-q(\lambda_c,\ \theta_c)\left(1-(1-\gamma)^{N-1}\right)}\Phi(z)\right)^{-1} dz, \tag{B6}$$

where $\Phi(z)$ is defined in formula (B3). Finally, we let $\hat{\alpha} = \alpha(1,\ 1)$, which corresponds to formula (7).

To emphasize the point that we now deal with equilibrium rather than just the best response of dealers to some generic strategy of traders, we add star superscripts to symbols denoting the strategy of traders.

Case 2.1: $\lambda_c^\star > 0$, $c > v - \Delta$. In this case, we clearly have $\theta_c^\star = 1$. We first suppose that the support of the distribution for low-cost dealers is an interval. Then $F_c^l(p)$ must satisfy

$$\left[1 - q\left(\lambda_c^\star,\ 1\right) + q\left(\lambda_c^\star,\ 1\right) \sum_{k=0}^{N-1} \binom{N-1}{k} \left(1 - F_c^l(p)\right)^k \gamma^k (1-\gamma)^{N-1-k}\right](p - c)$$
$$= \left[1 - q\left(\lambda_c^\star,\ 1\right) + q\left(\lambda_c^\star,\ 1\right)(1-\gamma)^{N-1}\right]\left(r_c^\star - c\right).$$

Solving for $F_c^l(p)$, we obtain

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{1 - q\left(\lambda_c^\star,\ 1\right)\left(1 - (1-\gamma)^{N-1}\right)}{q\left(\lambda_c^\star,\ 1\right)\left(1 - (1-\gamma)^{N-1}\right)} \frac{r_c^\star - p}{p - c}\right),$$

with $\bar{p}_c^l = r_c^\star$ and lower limit

$$\underline{p}_c^l = \left[1 - q\left(\lambda_c^\star,\ 1\right)\left(1 - (1-\gamma)^{N-1}\right)\right] r_c^\star + \left[q\left(\lambda_c^\star,\ 1\right)\left(1 - (1-\gamma)^{N-1}\right)\right] c.$$

We can determine $r_c^\star$ in this case from the fact that it must solve the following equation (specifying that the trader must be indifferent at $r_c^\star$ between buying and searching), which is analogous to equation (4):

$$v - r_c^\star = -s + \gamma\left[v - \int_{\underline{p}_c^l}^{r_c^\star} p\, dF_c^l(p)\right] + (1-\gamma)\left(v - r_c^\star\right).$$

Using a change of variables, we can transform this equation into the form

$$s = \gamma\left[r_c^\star - \int_{\underline{p}_c^l}^{r_c^\star} p\, dF_c^l(p)\right] = \left(1 - \alpha\left(\lambda_c^\star,\ 1\right)\right)\gamma\left(r_c^\star - c\right).$$

We therefore have

$$r_c^\star = c + \frac{s}{\left(1 - \alpha\left(\lambda_c^\star,\ 1\right)\right)\gamma}.$$

The last thing to determine is the probability $\lambda_c^\star$ of entry by slow traders. The profit of a slow trader conditional on entry is equal to

$$\pi_c = \left(1 - (1-\gamma)^N\right)\left(v - \alpha\left(\lambda_c^\star,\ 1\right) r_c^\star - \left(1 - \alpha\left(\lambda_c^\star,\ 1\right)\right) c\right)$$
$$- \left(\sum_{k=1}^{N} (1-\gamma)^{k-1}\gamma k + (1-\gamma)^N N\right) s$$
$$= \left(1 - (1-\gamma)^N\right)\left[v - c - \frac{s}{\left(1 - \alpha\left(\lambda_c^\star,\ 1\right)\right)\gamma}\right].$$

When profit is strictly positive, we must have entry with probability one. That is, we have $\lambda_c^\star = 1$ if

$$c \leq v - \frac{s}{(1-\alpha(1,\ 1))\gamma}.$$

When profit is strictly negative, we must have entry with probability zero, meaning that $\lambda_c^\star = 0$ if

$$c \geq v - \frac{s}{(1-\alpha(0,\ 1))\gamma}.$$

This takes us back to case 1 analyzed before. Finally, if

$$v - \frac{s}{(1-\alpha(1,\ 1))\gamma} < c < v - \frac{s}{(1-\alpha(0,\ 1))\gamma},$$

then we must have interior entry $\lambda_c^\star \in (0,\ 1)$, where $\lambda_c^\star$ is the unique solution of the equation

$$s = \left(1 - \alpha\left(\lambda_c^\star,\ 1\right)\right)\gamma(v-c).$$

In this case, slow traders have zero profits and we have $r_c^\star = v$.

To check whether the above strategies constitute an equilibrium, we need to verify that the support of price offers by low-cost dealers is indeed an interval, that is, these dealers cannot profitably deviate from posting prices in the range $[\underline{p}_c^l,\ r_c^\star]$. The only deviation that we need to check is bidding v in the case $r_c^\star < v$.[39] This leads to the condition

$$\left[\mu(1-\gamma)^{N-1} + (1-\mu)\frac{1-(1-\gamma)^N}{N\gamma}\right]\frac{s}{(1-\alpha(1,\ 1))\gamma} \geq (1-\gamma)^{N-1}(v-c),$$

where the left-hand side is the expected profit from bidding $r_c^\star$ and the right-hand side is the expected profit from bidding v (a dealer quoting v can only sell if all other dealers have high costs). We define

$$\kappa = \frac{(1-\gamma)^{N-1}}{\mu(1-\gamma)^{N-1} + (1-\mu)\frac{1-(1-\gamma)^N}{N\gamma}}. \tag{B7}$$

Thus, we have an equilibrium when

$$c \geq v - \frac{s}{\kappa(1-\alpha(1,\ 1))\gamma}.$$

Note that $\kappa < 1$, and therefore

$$v - \frac{s}{\kappa(1-\alpha(1,\ 1))\gamma} < v - \frac{s}{(1-\alpha(1,\ 1))\gamma}.$$

[39] If there is a profitable deviation, this one is the most profitable.

When $c < v - s/(\kappa(1-\alpha(1,\,1))\gamma)$, by Lemma B3, we must have an equilibrium in which the support for low-cost dealers consists of two intervals: $[\underline{p}_c^l,\, r_c^\star]$ and $[\hat{p}_c^l,\, v]$. Let ζ_c be the conditional probability that a low-cost dealer posts a price in the lower interval. Then the dealer must be indifferent between $r_c^\star$ and v, which pins down ζ_c in that

$$\left[\mu(1-\gamma\zeta_c)^{N-1} + (1-\mu)\frac{1-(1-\gamma\zeta_c)^N}{N\gamma\zeta_c}\right]\left(r_c^\star - c\right) = (1-\gamma)^{N-1}(v-c). \quad \text{(B8)}$$

We define

$$\vartheta(\zeta_c) = \frac{(1-\gamma)^{N-1}}{\mu(1-\gamma\zeta_c)^{N-1} + (1-\mu)\frac{1-(1-\gamma\zeta_c)^N}{N\gamma\zeta_c}}. \quad \text{(B9)}$$

Note that $\vartheta(1) = \kappa$. Then, equation (B8) becomes

$$r_c^\star = (1-\vartheta(\zeta_c))c + \vartheta(\zeta_c)v. \quad \text{(B10)}$$

We can now determine the exact distribution of prices. In the upper interval we must have

$$\left[\sum_{k=0}^{N-1}\binom{N-1}{k}\gamma^k(1-\gamma)^{N-1-k}\left(1-F_c^l(p)\right)^k\right](p-c) = (1-\gamma)^{N-1}(v-c),$$

so we get

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}\frac{v-p}{p-c}\right).$$

In the lower interval, the distribution must satisfy

$$\left[\mu\sum_{k=0}^{N-1}\binom{N-1}{k}(\gamma\zeta_c)^k(1-\gamma\zeta_c)^{N-1-k}\left(1-\frac{F_c^l(p)}{\zeta_c}\right)^k + \frac{1-\mu}{N}\frac{1-(1-\gamma\zeta_c)^N}{\gamma\zeta_c}\right](p-c)$$
$$= \left[\mu(1-\gamma\zeta_c)^{N-1} + \frac{1-\mu}{N}\frac{1-(1-\gamma\zeta_c)^N}{\gamma\zeta_c}\right]\left(r_c^\star - c\right),$$

which gives

$$F_c^l(p) = \zeta_c - \zeta_c\Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma\zeta_c)^{N-1}}\frac{1}{\mu\vartheta(\zeta_c)}\frac{r_c^\star - p}{p-c};\,\zeta_c\right),$$

where

$$\Phi(z;\,\zeta_c) = \frac{1}{1-(1-\gamma\zeta_c)^{N-1}}\sum_{k=1}^{N-1}\binom{N-1}{k}z^k(\gamma\zeta_c)^k(1-\gamma\zeta_c)^{N-1-k}.$$

That is, $\Phi(z;\,\zeta_c)$ is the analogue to $\Phi(z)$ when replacing γ with $\gamma\zeta_c$.

Finally, the reservation price is determined by

$$v - r_c^\star = -s + \gamma \zeta_c \left[v - \int_{\underline{p}_c^l}^{r_c^\star} p \, d\left(\frac{F_c^l(p)}{\zeta_c} \right) \right] + (1 - \gamma \zeta_c) \left(v - r_c^\star \right). \tag{B11}$$

Using a change of variable $z = (\zeta_c - F_c^l(p))/\zeta_c$, we obtain

$$\int_{\underline{p}_c^l}^{r_c^\star} p \, d\left(\frac{F_c^l(p)}{\zeta_c} \right) = c + \left(r_c^\star - c \right) \tilde{\alpha}(\zeta_c),$$

where

$$\tilde{\alpha}(\zeta_c) = \int_0^1 \left(1 + \frac{1 - (1 - \gamma \zeta_c)^{N-1}}{(1 - \gamma)^{N-1}} \mu \vartheta(\zeta_c) \Phi(z;\, \zeta_c) \right)^{-1} dz.$$

Note that $\tilde{\alpha}(1) = \alpha(1,\, 1)$. From this we can calculate the optimal reservation price, determined by equation (B11), as

$$r_c^\star = c + \frac{s}{(1 - \tilde{\alpha}(\zeta_c))\gamma \zeta_c}. \tag{B12}$$

Equations (B10) and (B12) together pin down $r_c^\star$ and ζ_c. Combining them, we get a single equation that pins down ζ_c, in the form

$$s = \vartheta(\zeta_c)(1 - \tilde{\alpha}(\zeta_c))\gamma \zeta_c (v - c).$$

A unique solution $\zeta_c^\star \in (0,\, 1)$ exists if and only if $0 < s < \kappa(1 - \alpha(1,\, 1))\gamma(v - c)$, which is precisely our assumption for that case.

Note that in this range the equilibrium level $\zeta_c^\star$ will be close to one when s is close to $\kappa(1 - \alpha(1,\, 1))\gamma(v - c)$ and will converge to zero as s goes to zero.

Case 2.2.1: $c \leq v - \Delta$, $r_c^\star \leq c + \Delta$. In this case, high-cost dealers offer the price $c + \Delta$. We have two cases to consider, which we call (a) and (b).

Case (a). When $r_c^\star < c + \Delta$, we must have $\theta_c^\star = 1$. Suppose that low-cost dealers mix on an interval. Then the distribution of prices is

$$F_c^l(p) = 1 - \Phi^{-1} \left(\frac{1 - q\left(\lambda_c^\star,\, 1\right)\left(1 - (1 - \gamma)^{N-1}\right)}{q\left(\lambda_c^\star,\, 1\right)\left(1 - (1 - \gamma)^{N-1}\right)} \frac{r_c^\star - p}{p - c} \right),$$

just as in the previous case. What differs from the previous case is the profit of a slow trader conditional on entry. In the event that there are no low-cost dealers in the market, a trader buys from a high-cost dealer instead of exiting. Accordingly, the profit now becomes

$$\pi_c = v - c - (1 - \gamma)^N \Delta - \left(1 - (1 - \gamma)^N \right) \frac{s}{(1 - \alpha\left(\lambda_c^\star,\, 1\right))\gamma}.$$

We can have strictly positive entry by slow traders only if

$$v \geq c + (1 - \gamma)^N \left[\Delta - \frac{s}{(1 - \alpha\left(\lambda_c^\star,\, 1\right))\gamma} \right] + \frac{s}{(1 - \alpha\left(\lambda_c^\star,\, 1\right))\gamma}. \tag{B13}$$

Recall that we have

$$r_c^\star = c + \frac{s}{(1-\alpha\left(\lambda_c^\star,\ 1\right))\gamma}.$$

Thus, given that we assumed $r_c^\star < c + \Delta$, we have an equilibrium with positive entry if inequality (B13) holds and

$$\Delta > \frac{s}{(1-\alpha\left(\lambda_c^\star,\ 1\right))\gamma}.$$

Notice that we have

$$\begin{aligned} &v - c - (1-\gamma)^N\Delta - \left(1-(1-\gamma)^N\right)\frac{s}{(1-\alpha\left(\lambda_c^\star,\ 1\right))\gamma} \\ &\quad > v - c - (1-\gamma)^N\Delta - \left(1-(1-\gamma)^N\right)\Delta = v - c - \Delta \geq 0, \end{aligned}$$

which means that profits are always strictly positive in this case. Thus, we must have full entry, meaning $\lambda_c^\star = 1$, and this can be an equilibrium only if $s < (1-\alpha(1,\ 1))\gamma\Delta$.

Finally, we verify the supposition that low-cost dealers mix on an interval. We need to check the deviation to (just below) $c+\Delta$, which is analogous to deviation to v in the previous case. We require

$$\left[\mu(1-\gamma)^{N-1} + (1-\mu)\frac{1-(1-\gamma)^N}{N\gamma}\right]\frac{s}{(1-\alpha(1,\ 1))\gamma} \geq (1-\gamma)^{N-1}\Delta.$$

Thus, the above strategies are an equilibrium if $s \geq \kappa(1-\alpha(1,\ 1))\gamma\Delta$.

In the case $s < \kappa(1-\alpha(1,\ 1))\gamma\Delta$, we have an equilibrium with low-cost dealers mixing on two intervals $[\underline{p}_c^l,\ r_c^\star]$ and $[\hat{p}_c^l,\ c+\Delta]$. The analysis is analogous to that in the previous case 2.1 so we skip some details. First, the indifference condition between $r_c^\star$ and $c+\Delta$ is[40]

$$\left(r_c^\star - c\right) = \vartheta(\zeta_c)\Delta. \tag{B14}$$

The upper part of the distribution is given by

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}\frac{c+\Delta-p}{p-c}\right),$$

while the lower part is

$$F_c^l(p) = \zeta_c - \zeta_c\Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma\zeta_c)^{N-1}}\frac{1}{\mu\vartheta(\zeta_c)}\frac{r_c^\star-p}{p-c};\ \zeta_c\right).$$

[40] Note that $c+\Delta$ is the upper limit of the support but prices posted by a low-cost dealer are below $c+\Delta$ with probability one. Thus, when we say that the dealer must be indifferent between posting $r_c^\star$ and $c+\Delta$, we really mean $c+\Delta-\epsilon$ for arbitrarily small $\epsilon \to 0$, which leads to the formula below.

The reservation price is determined by equation (B11). Simplifying as before, we obtain

$$r_c^\star = c + \frac{s}{(1-\tilde{\alpha}(\zeta_c))\gamma\zeta_c}.$$

Combining with equation (B14), ζ_c is pinned down by the equation

$$s = \vartheta(\zeta_c)(1-\tilde{\alpha}(\zeta_c))\gamma\zeta_c\Delta.$$

This equation does not depend on c, so neither does the solution. That is, $\zeta_c^\star$ is independent of c and solves

$$s = \vartheta(\zeta)(1-\tilde{\alpha}(\zeta))\gamma\zeta\Delta.$$

This equation has a unique solution in $(0,\ 1)$ precisely when $0 < s < \kappa(1 - \alpha(1,\ 1))\gamma\Delta$, which was our assumption for this case.

Case (b). We now look at the second possibility: $r_c^\star = c + \Delta$. We can now have $\theta_c^\star \in (0,\ 1)$, and this matters for equilibrium pricing through the impact on the posterior beliefs of dealers. The probability $F_c^l(p)$ of an offer of p or less by a low-cost dealer solves

$$\left[1 - q\left(\lambda_c^\star,\ \theta_c^\star\right) + q\left(\lambda_c^\star,\ \theta_c^\star\right)\sum_{k=0}^{N-1}\binom{N-1}{k}\left(1-F_c^l(p)\right)^k\gamma^k(1-\gamma)^{N-1-k}\right](p-c)$$
$$= \left[1 - q\left(\lambda_c^\star,\ \theta_c^\star\right) + q\left(\lambda_c^\star,\ \theta_c^\star\right)(1-\gamma)^{N-1}\right]\left(r_c^\star - c\right).$$

The profit of a slow trader is the same as in the previous case. The condition $r_c^\star = c + \Delta$ means that we must have

$$\frac{s}{(1-\alpha\left(\lambda_c^\star,\ \theta_c^\star\right))\gamma} = \Delta.$$

This implies that we must again have entry with probability one. Thus, we have an equilibrium with full entry and the probability of rejecting an offer of $r_c^\star$ given by $\theta_c^\star$ that solves

$$s = (1-\alpha\left(1,\ \theta_c^\star\right))\gamma\Delta.$$

Note that $\theta_c^\star = \theta^\star$ (the equation, and hence the solution, is independent of c). An interior solution exists if and only if $(1-\alpha(1,\ 1))\gamma\Delta < s < (1-\alpha(1,\ 0))\gamma\Delta$. Notice that $\theta^\star$ is close to 1 when s is close to $(1-\alpha(1,1))\gamma\Delta$, and is close to zero when s is close to $(1-\alpha(1,\ 0))\gamma\Delta$.

Case 2.2.2: $c \le v - \Delta$, $r_c^\star > c + \Delta$. This is the case when high-cost dealers make positive profits and mix according to a continuous distribution $F_c^h(\cdot)$ with upper limit $r_c^\star$. We must have $\theta_c^\star = 0$. The cumulative distribution function F_c^h solves

$$\left[1 - q\left(\lambda_c^\star,\ 0\right) + q\left(\lambda_c^\star,\ 0\right)(1-\gamma)^{N-1}\left(1-F_c^h(p)\right)^{N-1}\right](p-c-\Delta)$$
$$= \left[1 - q\left(\lambda_c^\star,\ 0\right)\right]\left(r_c^\star - c - \Delta\right).$$

Simplifying, we obtain

$$F_c^h(p) = 1 - \left(\frac{1-q(\lambda_c^\star,\ 0)}{q(\lambda_c^\star,\ 0)(1-\gamma)^{N-1}}\frac{r_c^\star - p}{p-c-\Delta}\right)^{\frac{1}{N-1}}$$

with upper limit $\bar{p}_c^h = r_c^\star$ and lower limit

$$\underline{p}_c^h = \frac{1-q(\lambda_c^\star,\ 0)}{1-q(\lambda_c^\star,\ 0)(1-(1-\gamma)^{N-1})}r_c^\star + \frac{q(\lambda_c^\star,\ 0)(1-\gamma)^{N-1}}{1-q(\lambda_c^\star,\ 0)\left(1-(1-\gamma)^{N-1}\right)}(c+\Delta).$$

To simplify notation, let us denote

$$\phi\left(\lambda_c^\star\right) = \frac{1-q(\lambda_c^\star,\ 0)}{1-(1-(1-\gamma)^{N-1})q(\lambda_c^\star,\ 0)}. \tag{B15}$$

Next, $F_c^l(p)$ must solve

$$\left[1-q\left(\lambda_c^\star,\ 0\right)+q\left(\lambda_c^\star,\ 0\right)\sum_{k=0}^{N-1}\binom{N-1}{k}\left(1-F_c^l(p)\right)^k\gamma^k(1-\gamma)^{N-1-k}\right](p-c)$$
$$= \left[1-q\left(\lambda_c^\star,\ 0\right)+q\left(\lambda_c^\star,\ 0\right)(1-\gamma)^{N-1}\right]\left(\underline{p}_c^h - c\right).$$

Solving for $F_c^l(p)$, we get

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{1-q(\lambda_c^\star,\ 0)\left(1-(1-\gamma)^{N-1}\right)}{q(\lambda_c^\star,\ 0)\left(1-(1-\gamma)^{N-1}\right)}\frac{\underline{p}_c^h - p}{p-c}\right),$$

with $\bar{p}_c^l = \underline{p}_c^h$ and lower limit

$$\underline{p}_c^l = \left[1-q\left(\lambda_c^\star,\ 0\right)(1-(1-\gamma)^{N-1})\right]\underline{p}_c^h + \left[q\left(\lambda_c^\star,\ 0\right)(1-(1-\gamma)^{N-1})\right]c.$$

We need to define one more function, analogous to $\alpha(\lambda,\ \theta)$, and corresponding to the distribution of prices used by high-cost dealers. Let

$$\alpha_h(\lambda) = \int_0^1\left(1+\frac{q(\lambda,\ 0)(1-\gamma)^{N-1}}{1-q(\lambda,\ 0)}z^{N-1}\right)^{-1}dz.$$

Then, using a change of variables, we get

$$\int p\,dF_c^h(p) = (1-\alpha_h(\lambda_c^\star))(c+\Delta) + \alpha_h\left(\lambda_c^\star\right)r_c^\star$$

and

$$\int p\,dF_c^l(p) = \left(1-\alpha\left(\lambda_c^\star,\ 0\right)\right)c + \alpha\left(\lambda_c^\star,\ 0\right)\underline{p}_c^h.$$

As always, $r_c^\star$ is determined by the indifference condition

$$v - r_c^\star = -s + \gamma \left[v - \int_{\underline{p}_c^l}^{\bar{p}_c^l} p\, dF_c^l(p) \right] + (1-\gamma) \left[v - \int_{\underline{p}_c^h}^{\bar{p}_c^h} p\, dF_c^h(p) \right].$$

From this we can obtain

$$r_c^\star = c + \Delta + \frac{s - (1-\alpha(\lambda_c^\star,\, 0))\gamma\Delta}{\gamma(1-\phi(\lambda_c^\star)\alpha(\lambda_c^\star,\, 0)) + (1-\gamma)(1-\alpha_h(\lambda_c^\star))}.$$

Next, we consider the entry decision of slow traders. The profit conditional on entry is simply $v - r_c^\star$. Thus, we have entry with probability one if and only if

$$c < v - \Delta - \frac{s - (1-\alpha(1,\, 0))\gamma\Delta}{\gamma(1-\phi(1)\alpha(1,\, 0)) + (1-\gamma)(1-\alpha_h(1))}.$$

Since we have assumed that $r_c^\star > c + \Delta$, we additionally require $s > (1 - \alpha(1,\, 0))\gamma\Delta$.

Interior entry requires that $\lambda_c^\star$ solves

$$v = c + \Delta + \frac{s - (1-\alpha(\lambda_c^\star,\, 0))\gamma\Delta}{\gamma(1-\phi(\lambda_c^\star)\alpha(\lambda_c^\star,\, 0)) + (1-\gamma)(1-\alpha_h(\lambda_c^\star))}. \tag{B16}$$

An interior solution exists if and only if

$$\frac{s - (1-\alpha(0,\, 0))\gamma\Delta}{\gamma(1-\phi(0)\alpha(0,\, 0)) + (1-\gamma)(1-\alpha_h(0))} < v - c - \Delta$$
$$< \frac{s - (1-\alpha(1,\, 0))\gamma\Delta}{\gamma(1-\phi(1)\alpha(1,\, 0)) + (1-\gamma)(1-\alpha_h(1))}. \tag{B17}$$

Noticing that $\alpha_h(0) = 0$ and that $\phi(0) = 0$, we can simplify the inequality on the left to $s - (1-\alpha(0,\, 0))\gamma\Delta < v - c - \Delta$.

Finally, since we have assumed that $r_c^\star > c + \Delta$, we require $s > (1 - \alpha(\lambda_c^\star,\, 0))\gamma\Delta$. This condition is satisfied vacuously when equation (B16) holds.

When $s - (1-\alpha(0,\, 0))\gamma\Delta \geq v - c - \Delta$, we must have entry with probability zero, which brings us back to case 1.

This concludes the analysis of all cases. By direct inspection, we check that, for any given pair $(s,\, c)$, there is exactly one equilibrium (up to payoff-irrelevant changes in equilibrium strategies). Figure A2 summarizes our conclusions by depicting the equilibrium correspondence in the $(s,\, c)$ space. “Full entry” means that $\lambda_c^\star = 1$ in the relevant range. “Interior entry” means that $\lambda_c^\star \in (0,\, 1)$. When we say that “only low-cost dealers sell,” we mean that if there is at least one low-cost dealer in the market, then all customers trade with low-cost dealers. When we say that “all dealers sell” or that “high-cost dealers sell with probability θ,” we refer to the probability of selling to a slow trader upon a visit. Finally, the trapezoidal area denoted by “(gap)” corresponds to the case in which low-cost dealers have a gap in the support of their offer distribution.

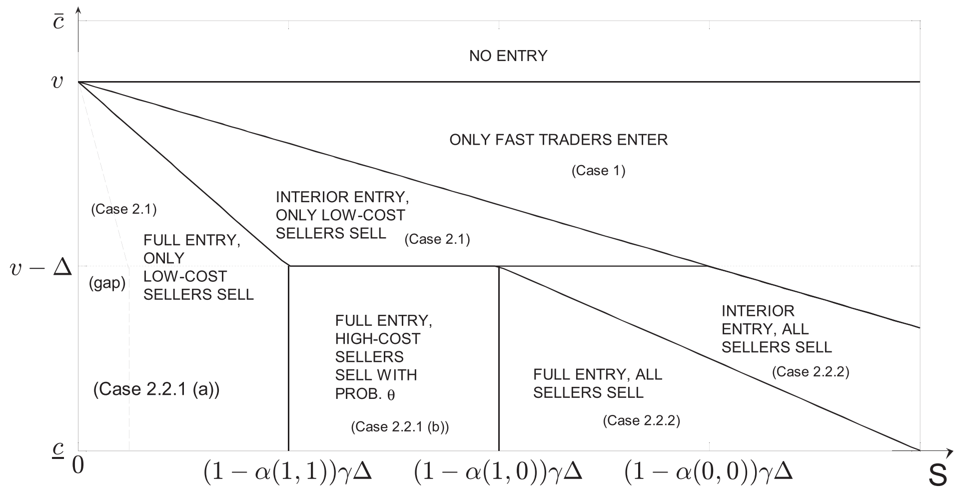

Figure A2. The benchmark case: equilibrium correspondence.

B. Proof of Proposition 8

Generalized statement (without Assumption A2): *If* $s \leq (1-\hat{\alpha})\gamma \min\{\Delta,\ v - c\}$, *then equilibrium in the benchmark case leads to efficient matching. That is, slow traders always enter, and all traders buy from a low-cost dealer, in the event that there is at least one such dealer present in the market. Additionally, if* $s \geq \kappa(1-\hat{\alpha})\gamma \min\{\Delta,\ v-c\}$, *where*[41] $\kappa < 1$, *the equilibrium with the benchmark achieves the second best, in the sense that each slow trader buys from the first low-cost dealer that she contacts, thus minimizing search costs subject to matching efficiency.*

Proof: The theorem follows directly from the derivation above (cases 2.1 and 2.2.1 (a)). When

$$\kappa(1-\hat{\alpha})\gamma \min\{\Delta,\ v-c\} \leq s \leq (1-\hat{\alpha})\gamma \min\{\Delta,\ v-c\},$$

we are in the region in which the equilibrium achieves the second-best. Slow traders always enter, and they search until they find the first low-cost dealer (low-cost dealers always post prices below the reservation price, and high-cost dealers always post prices above the reservation price). If there are no low-cost dealers in the market and $c > v - \Delta$, then traders exit without trading. When $c < v - \Delta$, they buy from a high-cost dealer. When $s < \kappa(1-\hat{\alpha})\gamma\Delta$, low-cost dealers post prices below the reservation price with probability $\zeta_c^\star \in (0,\ 1)$. Because high-cost dealers still post prices above the reservation price (and above the prices posted by low-cost dealers), the matching of traders to low-cost dealers is efficient. □

[41] We have $\kappa = (1-\gamma)^{N-1}/[\mu(1-\gamma)^{N-1} + (1-\mu)[1-(1-\gamma)^N]/(N\gamma)]$.

C. Proof of Proposition 9

Generalized statement (without Assumption A2): *In the absence of a benchmark, if* $\min\{v, \bar{c}\} > \underline{c} + \Delta$, *there does not exist an equilibrium achieving the second best.*

PROOF: In an equilibrium in which the second-best is achieved under the condition $s < \gamma\Delta$, high-cost dealers can sell only when there are no low-cost dealers in the market and the slow trader searched the entire market. Thus, if an equilibrium like this exists, high-cost dealers quote prices as if they participated in an auction with all other high-cost dealers. A standard result in auction theory says that in this case they must bid their costs, that is, they must offer to sell for $c + \Delta$.

Consider a situation in which a slow trader enters and the first dealer has low costs, for some $c < v$. If the second-best is achieved, that offer needs to be accepted by a slow trader. Under the assumption of the proposition, we can find a $c^\star$ that satisfies $v > c^\star > \underline{c} + \Delta$. By the above observation, (almost) all prices in the support of the distribution of the low-cost dealer at $c = c^\star$ must be accepted by a slow trader in the first search round. This leads to a contradiction. Since high-cost dealers post a price of $c + \Delta$ conditional on c, they make zero profits. They can profitably deviate at $c = \underline{c}$ by quoting a price in the support of the distribution of a low-cost dealer at $c = c^\star$. □

D. Generalized Statement of Theorem 5 (without Assumption A2)

THEOREM: If (i) $\kappa(1-\hat{\alpha})\gamma \min\{\Delta,\ v - \underline{c}\} \leq s \leq (1-\hat{\alpha})\gamma \min\{\Delta,\ v - \bar{c}\}$ and (ii) $\bar{c} > \underline{c} + \Delta$ both hold, then the equilibrium in the benchmark case yields a strictly higher social surplus than any equilibrium in the no-benchmark case.

PROOF: Follows directly from the generalized statements of Propositions 8 and 9. □

E. Proof of Theorem 6

To prove the theorem, we first describe the equilibrium path, and then show the optimality of dealers' strategies under a sufficiently high Δ.

If the environment is competitive, the benchmark is introduced, only low-cost dealers enter, and we have a reservation-price equilibrium in the trading-stage subgame described in Section II.B (with the exception that N is now replaced by M, which is equal to L in equilibrium). Because $s < (1-\bar{\alpha})(v - \bar{c})$, we have full entry in this case, and the reservation price of slow traders is

$$r_c^\star = c + \frac{s}{1 - \bar{\alpha}_L},$$

where the subscript L in $\bar{\alpha}_L$ indicates that N is replaced by L in the definition of $\bar{\alpha}$ given by equation (7).

If the environment is uncompetitive (all dealers have high costs), the benchmark is not introduced and high-cost dealers enter if and only if $X_\Delta > s$. To see this, note that in this case we can apply the analysis of Section II.C with the exception that c is replaced by $c + \Delta$ (correspondingly, X is replaced by X_Δ). In particular, high-cost dealers make strictly positive expected profits if and only if $X_\Delta > s$ because this condition guarantees that there is positive probability of entry by slow traders, according to Proposition 2. Existence follows from Proposition 3 for all $\Delta \geq \Delta_1^\star$ for some $\Delta_1^\star$ with $X_{\Delta_1^\star} > s$. Indeed, inspection of the proof shows that a sufficient condition is that $X_\Delta - s$ is sufficiently small, which we can achieve by taking high enough Δ.

On the equilibrium path in the pre-play stage, low-cost dealers vote in favor of the benchmark and enter if the benchmark is introduced or if the benchmark is not introduced and $X_\Delta > s$. High-cost dealers vote against the benchmark and enter if and only if the benchmark is not introduced and $X_\Delta > s$.

We now verify the optimality of these dealer strategies.

Set $\Delta_0^\star = s/(1-\bar{\alpha})$, and suppose that $\Delta \geq \Delta_0^\star$ so that $s < (1-\bar{\alpha})\Delta$.

First, we show that a high-cost dealer does not want to deviate and enter when the benchmark is introduced. Indeed, when the benchmark is observed, slow traders follow a reservation-price strategy with

$$r_c^\star = c + \frac{s}{1-\bar{\alpha}_M} \leq c + \frac{s}{1-\bar{\alpha}},$$

using the fact that $\bar{\alpha}_M$ is increasing in M.[42] Since $s \leq (1-\bar{\alpha})\Delta$ for $\Delta \geq \Delta_0^\star$, we conclude that $c + \Delta \geq r_c^\star$. Thus, using familiar arguments from previous sections, we show that a high-cost dealer cannot make positive profits after entering the market, regardless of the identities of other dealers in the market.[43]

Second, we show that a high-cost dealer does not want to deviate and stay out of the market when the benchmark is not introduced and $X_\Delta > s$. By the remark above, high-cost dealers make strictly positive profits on the equilibrium path in that case.

Third, low-cost dealers cannot deviate by changing their entry decision because, by the specification of their strategy, they enter if and only if their expected profits are strictly positive.

Fourth, we show that any coalition of high-cost dealers does not want to deviate by voting in favor of the benchmark. By what we establish above, if the benchmark is introduced, a high-cost dealer finds it optimal not to enter and hence earns no profits. Thus, this cannot be a strictly profitable deviation.

Fifth, we show that any coalition of low-cost dealers does not want to deviate by voting against the benchmark. Note that $L \geq 2$ is common knowledge among

[42] This is shown in Janssen and Moraga-González (2004).

[43] Note that off-equilibrium-path traders may observe offers above their reservation price, something that never happens on the equilibrium path. We specify off-equilibrium beliefs of traders by saying that this off-equilibrium event does not change the belief of any trader about the types of active dealers. This is consistent with a perfect Bayesian equilibrium.

low-cost dealers. In equilibrium, the benchmark is introduced, high-cost dealers stay out, and the low-cost dealers' expected profit is equal to

$$\frac{1-\mu}{L}\frac{s}{1-\bar{\alpha}_L} > 0,$$

which does not depend on Δ. If the benchmark is not introduced, slow traders believe with probability one that only high-cost dealers are present in the market. By taking Δ high enough we can make $X_\Delta - s$ arbitrarily small, so the equilibrium probability of entry by slow traders is arbitrarily small without the benchmark (see the analysis in Section II.C). Because $L \geq 2$, the expected profits of low-cost dealers in this case converge to zero as the posterior probability of meeting a slow trader approaches zero. Because the profit on the equilibrium path is bounded away from zero, low-cost dealers do not want to deviate in this way if Δ is above some cutoff level $\Delta_2^\star$.

We conclude the proof by defining $\Delta^\star = \max\{\Delta_0^\star,\ \Delta_1^\star,\ \Delta_2^\star\}$.

Note that $\Delta_1^\star$ and $\Delta_2^\star$ can be chosen so that $X_\Delta > s$ if Δ is close to $\max\{\Delta_1^\star,\ \Delta_2^\star\}$. If, in addition, s is sufficiently small, we can guarantee that $X_\Delta > s$ for all Δ in some right neighborhood of $\Delta^\star$.

F. Proof of Proposition 10

The first part of the proposition follows from the observation that the administrator can ask all dealers to report c and punish them (with a high enough transfer) if the reports disagree. The benchmark may then be made equal (for example) to the average of the reports. The second part follows from the fact that the choice rule to be implemented is not monotonic. (See Maskin (1999) for the definition of monotonicity and the relevant result.)

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.