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# From rainfed agriculture to stress-avoidance irrigation: I. A generalized irrigation scheme with stochastic soil moisture

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ABSTRACT

With vast regions already experiencing water shortages, it is becoming imperative to manage sustainably the available water resources. As agriculture is by far the most important user of freshwater and the role of irrigation is projected to increase in face of climate change and increased food requirements, it is particularly important to develop simple, widely applicable models of irrigation water needs for short- and long-term water resource management. Such models should synthetically provide the key irrigation quantities (volumes, frequencies, etc.) for different irrigation schemes as a function of the main soil, crop, and climatic features, including rainfall unpredictability. Here we consider often-employed irrigation methods (e.g., surface and sprinkler irrigation systems, as well as modern micro-irrigation techniques) and describe them under a unified conceptual and theoretical framework, which includes rainfed agriculture and stress-avoidance irrigation as extreme cases. We obtain mostly analytical solutions for the stochastic steady state of soil moisture probability density function with random rainfall timing and amount, and compute water requirements as a function of climate, crop, and soil parameters. These results provide the necessary starting point for a full assessment of irrigation strategies, with reference to sustainability, productivity, and profitability, developed in a companion paper [Vico G, Porporato A. From rainfed agriculture to stress-avoidance irrigation: II. Sustainability, crop yield, and net profit. Adv Water Resour 2011;34(2):272–81].



## 1. Introduction

While a significant part of the world population is already experiencing water limitations [1,2], some 40% of total food production is relying on irrigated agriculture [3]. Its contribution is projected to increase [4], with higher amounts of water withdrawn [5] but slower rates of irrigated land expansion [6]. Thus addressing the 'Millennium Development' goals of halving the proportion of malnourished people in the world by 2015 while ensuring environmental sustainability not only is a tremendous agricultural endeavor but represents also the world largest water-resource challenge [7]. There is no doubt that water management will play a crucial role in this sustainable development effort [4,6,8–12].

Changes in irrigation management and water delivery methods may significantly increase the overall efficiency of irrigation and water productivity (i.e., crop yield per unit applied water [13]). One of the main improvements pertaining to irrigation scheduling is switching from time-fixed water applications to demand-based irrigation because the latter reduces runoff and deep percolation water losses. A further step to increase water productivity may be the so-called deficit irrigation [14], consisting of the deliberate under-irrigation of crops possibly allowing for mild crop water stress. In fact, depending on the crop, deficit irrigation may only marginally reduce yield or even be beneficial for crop quality [15], or for controlling excessive growth and mutual shading [16,17].

Irrigation planning and water management are complicated by hydro-climatic variability, both during the growing season and inter-annually, with extensive impacts on irrigation requirements, crop productivity and profitability, as well as available water resources. Rainfall intermittency, with its random timing and amounts, is by far the most important source of uncertainty in soil moisture variability [18]. The inherent rainfall unpredictability calls for a probabilistic framework, which is necessary to fully assess the feasibility of different irrigation strategies [19]. Climate change scenarios predict an increase in such variability in the near future, with higher frequencies of dry spells and extreme events [20], providing a further motivation to include in full the statistical features of rainfall variability rather than simply using the projected trend in annual totals [21,22].

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**Nomenclature**

| | |
|---|---|
| $C$ | Pdf normalizing constant (Eq. (7)) |
| $C_m$ | Pdf normalizing constant for micro-irrigation (Eq. (A.1)) |
| $ET(s(t))$ | Soil water losses through evapotranspiration (Eq. (1)) |
| $\langle ET \rangle$ | Long-term average daily evapotranspiration |
| $ET_{\max}$ | Evapotranspiration rate under well-watered conditions |
| $ET_{seas}$ | Total evapotranspiration over the growing season |
| $I(s(t))$ | Irrigation input to soil water balance (Eq. (1)) |
| $\langle I \rangle$ | Long-term average daily irrigation |
| $LQ(s(t))$ | Soil water losses through runoff and deep percolation (Eq. (1)) |
| $\langle LQ \rangle$ | Long-term average daily losses through runoff and deep percolation |
| $n$ | Soil porosity |
| $P(s)$ | Cumulative probability function (Eq. (9)) |
| $p(s)$ | Steady state probability density function (pdf) of soil moisture for the generalized irrigation scheme (including rainfed agriculture and micro-irrigation as extreme cases) |
| $p_0$ | Atom of probability in $\tilde{s}$ for micro-irrigation (Eq. (A.2)) |
| $p_m(s)$ | Continuous part of soil moisture probability density function for micro-irrigation (Eq. (A.1)) |
| $R(t)$ | Rainfall input to the soil water balance (Eq. (1)) |
| $\langle R \rangle$ | Long-term average daily rainfall |
| $s(t)$ | Soil moisture ranging from 0 (perfectly dry soil) to 1 (soil saturation) |
| $s^*$ | Point of incipient stomatal closure, when plant transpiration is reduced |
| $\tilde{s}$ | Soil moisture 'intervention point' triggering an irrigation application |
| $\hat{s}$ | 'Target level' to which soil moisture is replenished by an irrigation application |
| $s_1$ | Soil moisture level at which deep percolation and runoff losses take place |
| $s_{fc}$ | Soil field capacity, above which deep percolation is non-negligible |
| $s_w$ | Wilting point, corresponding to irreversible damages to plants |
| $T_{seas}$ | Length of the growing season |
| $\overline{T}_\xi^\downarrow\left(\overline{T}_\xi^\uparrow\right)$ | Mean time spent by the soil moisture process below (above) the threshold $\xi$ (Eqs. (13) and (14)) |
| $V_{ideal}(\tilde{s},\hat{s})$ | Total irrigation volume per unit area over a growing season under ideal conditions (Eq. (15)) |
| $Z_r$ | Active soil depth |
| $\alpha$ | Mean depth of rainfall events |
| $\gamma = \frac{nZ_r}{\alpha}$ | Inverse of the normalized mean depth of rainfall events |
| $\eta = ET_{\max}/(nZ_r)$ | Maximum normalized evapotranspiration (Eq. (2)) |
| $\lambda$ | Mean frequency of rainfall events |
| $\nu_{\tilde{s}}$ | Mean irrigation frequency |
| $\nu_\xi^\downarrow\left(\nu_\xi^\uparrow\right)$ | Mean frequency of downcrossing (upcrossing) of the threshold $\xi$ (Eqs. (3) and (4)) |
| $\rho(s)$ | Normalized evapotranspiration loss rate |

A vast body of literature addresses the question of water resources optimization for agricultural management, with an emphasis on profit maximization. Spatial scales range from single field level, to farm level (with decisions regarding multi-crop cultivated acreage [23–25]), up to the regional scale (including optimal reservoir release policies [26,27]), while temporal horizons range from irrigation scheduling within a single growing season [28,29] to multi-year investment planning [30,31]. Some models include the effects of hydro-climatic variability (chiefly, rainfall and evapotranspiration) at different time scales, from intra-seasonal [28,32,33] to seasonal [31,34], depending on the model time scale. The role of stochastic rainfall variability is accounted for in a variety of ways, including direct simulations [24,35], soil moisture transition probabilities [28], ad hoc noise term in the soil moisture balance [32], and probability-driven, scenario-based approaches [26,30,36]. However, most of these models have been formulated as stochastic dynamic programming problems, relying on numerical solutions.

Alternatively, the approach presented here, which extends the work of Vico and Porporato [37], provides mostly analytical, closed formulae that clearly show the impact of the main soil, crop, and climate characteristics on irrigation requirements, without requiring time-consuming direct simulations. The inclusion of hydro-climatic variability through key rainfall-regime parameters (e.g., frequency and mean amount of rainfall during a growing season) allows us to easily assess the feasibility of an irrigation strategy in a probabilistic sense under different crop and soil features, as well as climate-change scenarios.

The focus of our work is on demand-based irrigation, in which irrigation applications are triggered by worsening plant or soil water status. We describe often-employed forms of demand-based irrigation, such as traditional irrigation and modern micro-irrigation, under a unified approach bridging between rainfed agriculture and stress-avoidance irrigation. This generalized irrigation scheme is presented in Section 2. Sections 3 and 4 extend the results of Vico and Porporato [37] from stress-avoidance soil saturating traditional irrigation to this generalized irrigation scheme, providing a statistical description of soil moisture under stochastic steady-state conditions and irrigation requirements in terms of average application frequencies and supplied water volumes, as a function of rainfall, soil and crop characteristics. In a companion paper [38], the present solutions are linked to crop yield and net economic return, with the inclusion of rainfall interannual variability, to assess the optimality of different irrigation schemes with reference to total water requirements, crop yields and profitability.

## 2. A unified framework for demand-based irrigation schemes

Demand-based irrigation consists of applications of water whenever soil moisture or crop water status reaches a predefined stress level. Each water application brings soil moisture back to a fixed level, which generally corresponds to field capacity. A limiting case is represented by modern micro-irrigation, consisting of very frequent but shallow water applications that prevent the root zone soil moisture from falling below an ideal soil moisture level. Demand-based irrigation strategy is generally more efficient than scheduled irrigation applications, which in contrast supply fixed amounts of water regardless of plant/soil water status. Nevertheless, demand-based irrigation requires knowledge of crop water status, either through direct plant/soil moisture monitoring or estimates based on potential evapotranspiration [39]. A simple yet efficient way to characterize crop water status is through relative soil moisture, i.e., the fraction of water-filled soil pore volumes, ranging from 0 for dry soils to 1 at soil saturation. In general, plants may be assumed to start experiencing water stress when soil moisture falls below a threshold $s^*$ which represents the point of incipient stomatal closure [40]. Damages to vegetation tend to become more irreversible the closer soil moisture gets to the wilting point $s_w$. Because soil moisture needs to be kept well above the wilting point to generate any crop yield, in what follows, we simply set the wilting point at zero to simplify our notation but without loss of generality. Describing plant water status through soil moisture levels allows us to fully characterize demand-based irrigation strategies by means of two parameters (Fig. 1). The first one is the soil moisture level that triggers an irrigation application, i.e., the 'intervention point' $\tilde{s}$, corresponding to the percent of allowable water depletion. This level is often used in crop models such as CropWat [41] and AquaCrop [42], and is also indirectly related to the allowable daily stress used by the SWAP model [43]. The second one is the soil moisture level that each irrigation application needs to restore, i.e., the soil moisture 'target level' $\hat{s}$. The difference between the target level and the intervention point, $\hat{s} - \tilde{s}$, is proportional to the depth (e.g., volume of water per unit area) of water supplied by each irrigation application.

The most common application of both traditional and micro-irrigation is the so-called stress-avoidance irrigation, whereby an irrigation application is triggered whenever soil moisture reaches the point of incipient stress, $\tilde{s} = s^*$ (Fig. 1, central thin vertical line). Irrigation schemes with intervention points below the point of incipient stomatal closure, $\tilde{s} < s^*$, perform what is known as deficit irrigation (Fig. 1, light shaded area). Here deficit irrigation specifically refers to the practice of setting an intervention point below the point of incipient stress, thus allowing a mild water stress before irrigation is applied. For the sake of clarity, it should be noted that other authors refer to deficit irrigation as application of water amounts less than those required to compensate transpiration (e.g., [15,44]) or the deliberate under-application of irrigation

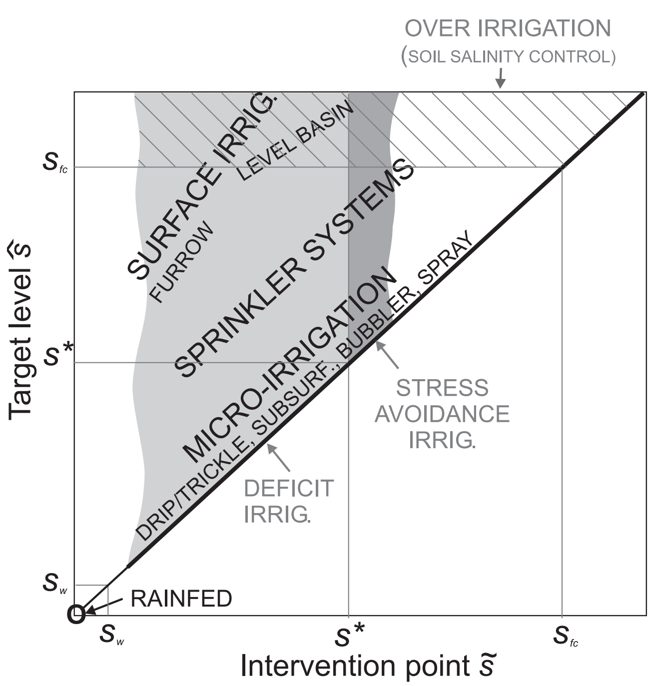


**Fig. 1.** Diagram summarizing irrigation schemes and irrigation methods as a function of the irrigation parameters 'intervention point' $\tilde{s}$ ($x$-axis) and 'target level' $\hat{s}$ ($y$-axis). The irrigation parameter space is limited by the condition $\hat{s} \geqslant \tilde{s}$. The parameter space can be further subdivided into deficit-irrigation (light shaded area) and stress-avoidance irrigation (dark shaded area); white areas in the $\hat{s} \geqslant \tilde{s}$ parameter space identify unrealistic combinations of parameters (see text for details). Target levels above soil field capacity (hatched area) correspond to over-irrigation for soil salinity control. Regarding the irrigation methods, the whole shaded area corresponds to traditional irrigation schemes, with the difference $\hat{s} - \tilde{s}$ being proportional to the amount of water supplied at each application. The thick solid line ($\tilde{s} = \hat{s}$) represents micro-irrigation. The corresponding irrigation methods are also qualitatively indicated.

during the growth stages in which crops are the least sensitive to water stress (sometimes also called 'regulated deficit irrigation'; see, e.g., [45,46]). On the other hand, a slight over-irrigation, corresponding to setting $\tilde{s} > s^*$ (Fig. 1, dark shaded area), may be sometimes implemented to avoid possible water stress originating from uncertainties and/or variability in the stress level $s^*$ due to heterogeneities in soil and crop characteristics. Regarding the target level, $\hat{s}$, traditional irrigation often entails water application bringing soil moisture back to field capacity, i.e. $\hat{s} = s_{fc}$. However, a generic target level may be considered to include other application depths, bringing soil moisture either above or below soil field capacity. Lower target levels ($\hat{s} < s_{fc}$) may be beneficial to limit losses by runoff and deep percolation, should a rainfall event immediately follow the irrigation application. Conversely, water applications that bring soil moisture to levels above soil field capacity, where soil deep percolation becomes non negligible, may be useful for salinity control, as they may contribute to soil solute flushing from the rooting zone (Fig. 1, hatched area).

Clearly, all of these demand-based irrigation strategies can be framed within a 'generalized irrigation scheme' where the key parameters are the intervention point $\tilde{s}$ and the target level $\hat{s}$. Accordingly, the continuum between the traditional irrigation and micro-irrigation can be explored by progressively reducing the amount of applied water per treatment (and thus increasing treatment frequency), as apparent from Fig. 1. For example, micro-irrigation corresponds to the case $\tilde{s} = \hat{s}$ (thick line), while the case of rainfed (or dryland) agriculture is obtained for $\tilde{s} \to 0$ and $\hat{s} \to 0$ (i.e., no irrigation is ever applied). The irrigation parameter space is limited to the combinations of parameters such that $\hat{s} \geqslant \tilde{s}$ (i.e., the target level needs to be larger than intervention point), and may be further subdivided into deficit-irrigation and stress-avoidance irrigation on the basis of the intervention point as detailed above (Fig. 1). Obviously, very low intervention points (left blank area in Fig. 1) are not realistically employed in practice, unless the timing of water availability is significantly constrained, because of the high risk of relevant water stress damages to crops, even with massive but very infrequent irrigation applications. Similarly, extremely high intervention points $\tilde{s}$ (i.e., well above $s^*$; right blank area in Fig. 1) are associated with high irrigation requirements, without significant benefits in terms of yields, and hence are generally avoided.

From the practical point of view, these irrigation schemes are attained in the field by employing various irrigation methods (e.g., [47–49]), as qualitatively indicated in Fig. 1. Traditional irrigation is often carried out by means of surface or sprinkler irrigation. Surface irrigation, where water is redistributed by gravity in basins or furrows, is particularly suited for larger applications (exceeding 50 mm per treatment), even though it generally has relatively low application efficiencies and distribution uniformity. Sprinkler or spray systems are able to mimic rainfall events as small as 5 mm in the case of center pivot systems [48], and hence are effective for light and frequent applications. Nevertheless, sprinkler systems tend to have higher energy demand for operation and higher investment costs when compared to surface irrigation systems [49]. Conversely, the micro-irrigation scheme, requiring almost continuous water applications at low rates, needs more sophisticated systems such as drip or trickle irrigation (also termed localized irrigation). These localized irrigation systems are often automated and are capable of high-frequency irrigation, with high application efficiencies, but require high initial investments, frequent maintenance, and proper operation.

Currently, surface irrigation is the predominant irrigation system worldwide, while sprinkler irrigation is the most common system in the USA, followed by gravity systems (Fig. 2). Despite its costs, localized irrigation is becoming more and more common. Such systems are being used predominantly for high-value

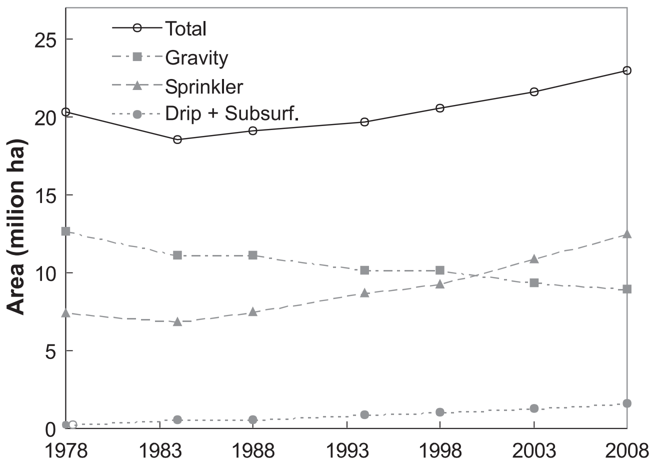


**Fig. 2.** Changes in irrigated areas by irrigation methods in the USA over the period 1978–2008. Source of data: Table 4 of the US Census of Agriculture, Farm and Ranch Irrigation Survey for the corresponding years. Over the period 1978–1994, reported areas refer to the USA conterminous states only.

horticultural crops, because of their high efficiency and water control, while they are still at the research stage for row crops [50,51]. The use of localized irrigation and its extension to other crops is expected to be reinforced by water scarcity and increase in crop and water prices, as well as search for higher water productivity ('more food per drop').

## 3. Soil water balance

The temporal dynamics of soil moisture in an irrigated parcel of land can be effectively described by the following equation [37]

$$nZ_r \frac{ds(t)}{dt} = R(t) + I(s(t)) - ET(s(t)) - LQ(s(t)), \quad (1)$$

where the state variable $s(t)$ is the relative soil moisture, ranging from 0 for perfectly dry soils to 1 at saturation. The soil water balance is averaged over the active soil depth (i.e., where most of the roots are located), $Z_r$; $n$ is soil porosity. Input to the soil water balance are rainfall, $R(t)$, and irrigation, $I(s(t))$, while the main soil water losses are evapotranspiration $ET(s(t))$ and the combination of deep percolation and runoff $LQ(s(t))$. Physical processes are interpreted at a daily time scale. The main simplifying assumptions implicit in Eq. (1), which allow a parsimonious description of soil water balance, are no interactions between soil moisture in the rooting area and the underlying water table, negligible lateral subsurface water redistribution and uniform soil features and rooting depth [52]. As a first approximation, most of these simplifications are appropriate for a variety of agricultural settings, where tillage practices and use of monoculture homogenize rooting depth and soil properties, and flat or gently sloping fields, which present limited later water redistribution, are preferred.

For the purpose of a parsimonious statistical description of hydrologic variability, rainfall $R(t)$ is modeled as instantaneous events occurring according to a marked Poisson process of rate (mean frequency of rainfall events) $\lambda$, and with exponentially distributed depths with mean $\alpha$ [52,53]. This rainfall description can account for intra-seasonal and, at least in part, for inter-annual variability in rainfall regimes (in case of strong interannual variability the analysis can be extended as in Porporato et al. [54], but this will be the subject of a subsequent contribution). The rainfall input $R(t)$ may be purged of canopy interception, which may reduce both the frequency of effective rainfall and its depth [53,55], but for the sake of simplicity, we neglect it here. Mean depth of rainfall events will be normalized as $1/\gamma = \alpha/(nZ_r)$.

Irrigation is assumed to occur according to the demand-based generalized traditional irrigation scheme (Section 2), consisting of concentrated applications of water when soil moisture reaches a given intervention level $\tilde{s}$. At each treatment, a fixed amount of water is applied bringing soil moisture back to a target level $\hat{s}$. Such a scheme is more general than the irrigation schemes discussed in Vico and Porporato [37], which were limited to stress-avoidance micro- or traditional irrigation, with deep irrigation applications in case of traditional irrigation. Conversely, here, through a proper definition of $\tilde{s}$ and $\hat{s}$ with respect to vegetation and soil parameters such as incipient stomatal closure $s^*$ and soil field capacity (above which soil deep percolation becomes relevant), this generalized scheme allows us to assess the water requirements for both deficit and stress-avoidance irrigation, as well as over-irrigation for soil salinity control.

Daily soil water evaporation and plant transpiration per unit surface area, combined here in the term $ET(s(t))$, are assumed to vary in time only through soil moisture temporal variability, thus accounting for plant response to variable water availability. This simplified approach neglects the effect of day-to-day variability in soil evaporation and transpiration rates, caused by either plant development during the growing season or variable environmental conditions (e.g., air temperature and humidity, solar irradiance). While the latter cause has a secondary effect when compared to the impact of rainfall-induced soil moisture variability [18], the former one may be relevant, in particular when annual crops are considered. Nevertheless, as a first approximation, the dependence of transpiration rate on soil moisture, $ET(s(t))$, is assumed constant in time, and interpreted as an average over the growing season. This simplified approach can be at least partly justified by considering that, while transpiration increases during the first part of the growing season after plant emergence, soil evaporation simultaneously declines due to increasing canopy cover (a quantitative assessment of these approximations will be included in a separate contribution). In the following, we will often refer to the evapotranspiration rate normalized by the active soil depth, i.e., $\rho(s(t)) = ET(s(t))/(nZ_r)$. Finally, possible changes in rooting depth with plant development are not included, hence $Z_r$ is to be interpreted as an average value over the growing season.

While for the general results reported below the specific form of $ET(s(t))$ is not relevant, in the following quantitative applications a piecewise linear function with constant losses under well-watered conditions will be assumed [53]

$$\rho(s(t)) = \begin{cases} \eta \frac{s(t)}{s^*} & 0 \leqslant s < s^*, \\ \eta & s^* \leqslant s \leqslant s_1. \end{cases} \quad (2)$$

In Eq. (2), $\eta = ET_{\max}/(nZ_r)$ represents the normalized loss rate under well-watered conditions (averaged over the growing season), and $s^*$ is the point of incipient stomatal closure, below which experimental evidence (e.g., [56–59]) suggests a linear decrease in plant transpiration to basically zero at plant wilting point (here set to zero for simplicity). Both $ET_{\max}$ and $s^*$ depend on crop and soil features.

Finally, losses by deep percolation and runoff, $LQ(s(t))$, are treated here in a simplified manner, by assuming that they take place instantaneously (at the daily time scale) whenever soil moisture reaches a threshold $s_1$, typically around or slightly above soil field capacity [60,61]. This simplified description is particularly suitable for soils with medium to high hydraulic conductivities. Because of this simplified treatment of $LQ(s(t))$, the soil moisture process is bound at $s_1$ [37].

Numerically generated soil moisture time series for different choices of the irrigation parameters $\tilde{s}$ and $\hat{s}$ are presented for illustration in Fig. 3. The choice of $\hat{s}$ strongly impacts the most probable

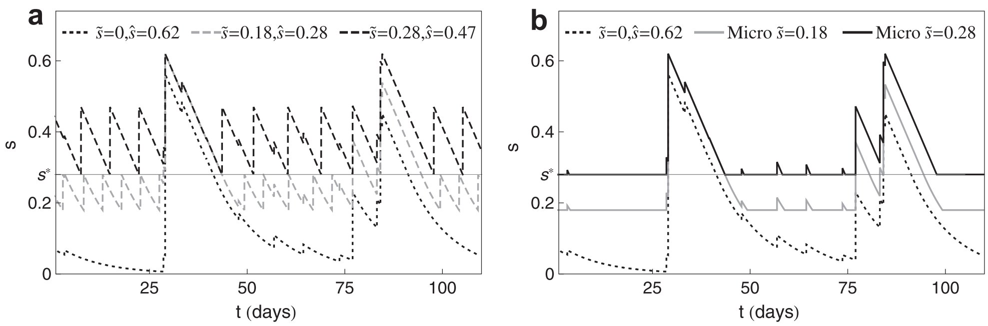


**Fig. 3.** Examples of soil moisture time series for different choices of the parameters $\tilde{s}$ and $\hat{s}$; the case of rainfed agriculture is reported for comparison (dotted line). The thin horizontal line corresponds to $s = s^*$. Soil is sandy loam with $n = 0.43$ [52] and $s_1 = 0.62$. Plant features refer to *Zea mays*: evapotranspiration rate is $ET_{max} = 0.55$ cm day$^{-1}$; $s^* = 0.28$ was estimated from data in [59], and $Z_r = 50$ cm after [64]. Average precipitation depth is $\alpha = 15$ mm and rainfall frequency $\lambda = 0.15$ day$^{-1}$.

values assumed by soil moisture over the course of time: the closer the target level and intervention point are, the smaller the soil moisture range in which the process fluctuates, with infrequent excursions above $\hat{s}$ driven by rainfall occurrence (e.g., black dashed lines in Fig. 3a). Higher $\hat{s}$ values result in wider oscillations of soil moisture, driven by both irrigation application and rainfall occurrence (gray dashed line in Fig. 3a). Micro-irrigation is represented by the extreme case of $\hat{s} \to \tilde{s}$, in which the system spends finite amounts of time at $\tilde{s}$ (solid lines in Fig. 3b). The case of rainfed agriculture is reported for reference (dotted lines in Fig. 3).

## 4. Analytical solutions for the generalized irrigation scheme

The soil water balance described by the nonlinear stochastic differential Eq. (1) has been extensively analyzed elsewhere, both in absence of irrigation ([52] and references therein) and with stress-avoidance micro-irrigation and soil-saturating traditional irrigation [37]. We now extend these previously obtained results to the case of the generalized traditional irrigation described above, and offer exact solutions of the stochastic steady state soil moisture probability density function (pdf), as well as expressions for average irrigation frequencies and required water volumes.

### 4.1. Soil moisture probability density functions

The temporal evolution of soil moisture pdf, $p(s,t)$, for $\tilde{s} \leqslant s \leqslant s_1$, with the generalized irrigation schemes described above, can be represented by the master or differential Chapman–Kolmogorov equation, similar to those reported in [52,53], but with two additional terms to account for irrigation applications. Because a general solution of such equation is not possible, we limit our analyses to stochastic steady-state conditions, i.e., we assume that $\partial p(s,t)/\partial t = 0$. To obtain the desired pdf of soil moisture under stochastic steady state, $p(s)$, we consider the crossing properties of the soil moisture process, which have been obtained before both for rainfed agriculture [40,52] and for stress-avoidance irrigation [37]. We focus here on the frequency of excursions of the soil moisture process below and above a generic threshold $\xi$, i.e., on the frequencies of downcrossing, $\nu_\xi^\downarrow$, and upcrossing, $\nu_\xi^\uparrow$. Because irrigation does not alter the crossing during a soil moisture dry down, the frequency of downcrossings of a generic threshold $\xi \geqslant \tilde{s}$ is as in [40,62]

$$\nu_\xi^\downarrow = \rho(\xi)p(\xi). \tag{3}$$

Conversely, the frequency of upcrossings needs to account for jumps in soil moisture caused by either rainfall events or irrigation applications (see Fig. 3). Following [40], the frequency of soil moisture jumps caused by the occurrence of rainfall is $\lambda\int_{\tilde{s}}^{s} e^{-\gamma(\xi-u)}p(u)du$, while the frequency of soil moisture jumps caused by irrigation is the same as the frequency of downcrossing of the threshold $\xi = \tilde{s}$, i.e., $\rho(\tilde{s})p(\tilde{s})$ [37]. Because of the assumed irrigation scheme, the upcrossings caused by irrigation applications can occur only when the threshold $\xi$ is such that $\tilde{s} \leqslant \xi \leqslant \hat{s}$. Hence, by combining the two cases, the following frequency of upcrossing for a generic soil moisture level $\tilde{s} \leqslant \xi \leqslant s_1$ can be obtained

$$\nu_\xi^\uparrow(\xi) = \theta(\hat{s}-\xi)\rho(\tilde{s})p(\tilde{s}) + \lambda\int_{\tilde{s}}^{\xi} e^{-\gamma(\xi-u)}p(u)du, \tag{4}$$

where $\theta(\cdot)$ is the Heaviside function. Because under stochastic steady-state the frequency of upcrossing of a generic soil moisture threshold $\xi = s$ equals the frequency of downcrossing of the same threshold [63], using Eqs. (3) and (4), the following equation can be written

$$\rho(s)p(s) = \theta(\hat{s}-s)\rho(\tilde{s})p(\tilde{s}) + \lambda\int_{\tilde{s}}^{s} e^{-\gamma(s-u)}p(u)du. \tag{5}$$

Multiplying Eq. (5) by $e^{\gamma s}$ and differentiating with respect to $s$, a first-order ordinary linear differential equation is obtained. Its solution is the desired pdf of soil moisture $s$

$$p(s) = \frac{e^{-\int_{s_L}^{s}\left(\gamma-\frac{\lambda}{\rho(u)}\right)du}}{\rho(s)} \times\left\{C+\rho(\tilde{s})p(\tilde{s})\int_{s_L}^{s}[\gamma\theta(\hat{s}-u)-\delta(\hat{s}-u)]e^{\int_{s_L}^{u}(\gamma-\frac{\lambda}{\rho(y)})dy}du\right\}, \tag{6}$$

where $C$ is a normalizing constant, $\rho(s)$ is a generic normalized evapotranspiration loss function, and $\delta(\cdot)$ is the Dirac delta function. The choice of the lower limit of integration, $s_L$, is arbitrary and only influences the value of the normalizing constant. However, it is convenient to set $s_L = \tilde{s}$ because in this case $C = \rho(\tilde{s})p(\tilde{s})$, so that the pdf of soil moisture $s$ can be written in closed form as

$$p(s) = C\frac{e^{-\int_{\tilde{s}}^{s}\left(\gamma-\frac{\lambda}{\rho(u)}\right)du}}{\rho(s)}\left\{1+\int_{\tilde{s}}^{s}[\gamma\theta(\hat{s}-u)-\delta(\hat{s}-u)]e^{\int_{\tilde{s}}^{u}\left(\gamma-\frac{\lambda}{\rho(y)}\right)dy}du\right\}. \tag{7}$$

Upon integration, the Heaviside and Dirac delta functions allow the description of the piecewise pdf over the different ranges of soil moisture, when the normalizing constant $C$ is obtained by imposing

$$\int_{\tilde{s}}^{s_1} p(u)du = 1. \tag{8}$$

Finally, the cumulative probability function, representing the average fraction of time spent by the soil moisture process below a generic $s$, is given by

$$P(s) = \int_{\tilde{s}}^{s} p(u)du. \tag{9}$$

For $\tilde{s} \to 0$ and $\hat{s} \to 0$, Eq. (7) simplifies to the case of rainfed agriculture (e.g., [52]). For $\hat{s} \to \tilde{s}$, the case of micro-irrigation is retrieved, in which the finite duration of irrigation applications produces an atom of probability in $\tilde{s}$. A more straightforward derivation which naturally includes the atom of probability in $\tilde{s}$ can be obtained by extending the method presented in Vico and Porporato [37] to a generic $\tilde{s}$ (see details in the Appendix).

We now particularize the previous general solution for the loss function in Eq. (2). With appropriate substitutions and integrations, and for the case of deficit irrigation ($\tilde{s} < s^*$), Eq. (7) becomes

$$p(s) = \begin{cases} \frac{C}{\eta}\frac{s^*}{s} e^{-\gamma(s-\tilde{s})} \left(\frac{s}{\tilde{s}}\right)^{\frac{\lambda}{\eta}s^*} \left\{1 + e^{-\gamma\tilde{s}}\tilde{s}^{\frac{\lambda}{\eta}s^*}[h_1(s) - h_1(\tilde{s})]\right\} & \tilde{s} \leqslant s < s^*, \\ \frac{C}{\eta} e^{-\gamma(s-\tilde{s})+\frac{\lambda}{\eta}(s-s^*)} \left(\frac{s^*}{\tilde{s}}\right)^{\frac{\lambda}{\eta}s^*} \left\{1 + e^{-\gamma\tilde{s}}\tilde{s}^{\frac{\lambda}{\eta}s^*}[h_1(s^*) - h_1(\tilde{s})] \right. & \\ \quad \left. + e^{\gamma(s^*-\tilde{s})} e^{-\left(\gamma-\frac{\lambda}{\eta}\right)s^*} \left(\frac{\tilde{s}}{s^*}\right)^{\frac{\lambda}{\eta}s^*} [h_2(s) - h_2(s^*)]\right\} & s^* \leqslant s \leqslant s_1, \end{cases} \tag{10}$$

where

$$\begin{aligned} h_1(s) &= \int^{s} [\gamma\theta(\hat{s} - u) - \delta(\hat{s} - u)] e^{\gamma u} u^{-\frac{\lambda}{\eta}s^*} du, \\ h_2(s) &= \int^{s} [\gamma\theta(\hat{s} - u) - \delta(\hat{s} - u)] e^{(\gamma-\frac{\lambda}{\eta})u} du. \end{aligned} \tag{11}$$

In case of stress-avoidance or over-irrigation (i.e., $\tilde{s} \geqslant s^*$), Eq. (7) simplifies to

$$p(s) = \frac{C}{\eta} e^{-\left(\gamma-\frac{\lambda}{\eta}\right)(s-\tilde{s})} \left\{1 + e^{-\left(\gamma-\frac{\lambda}{\eta}\right)\tilde{s}} [h_2(s) - h_2(\tilde{s})]\right\}. \tag{12}$$

The integrals listed in Eq. (11) are analytically solvable, but for generic parameters they result in rather cumbersome expressions describing the pdf over the three ranges of soil moisture limited by $\tilde{s}$, $s^*$ and $\hat{s}$, which will not be reported here (a Wolfram Mathematica code for computing and plotting the pdf is available from the authors). Conversely, whenever $\tilde{s} < s^*$ (traditional deficit irrigation), obtaining the normalization constant $C$ requires the numerical integration of Eq. (8), which represents the only non-analytical step in our approach.

Examples of steady state soil moisture pdfs are reported in Fig. 4 for different choices of the parameters $\tilde{s}$ and $\hat{s}$. In the most general case, the pdf of $s$ exhibits different behaviors over the three ranges of soil moisture limited by $\tilde{s}$, $s^*$ and $\hat{s}$ (clearly, some of these ranges may not exist with particular choices of irrigation parameters). Above $\hat{s}$ the pdf has the same (though rescaled) shape as the one pertaining to rainfed agriculture, because in that range of soil moisture the distribution of probability is not directly impacted by irrigation applications.

### 4.2. Irrigation frequency and volumes

The crossing properties of the soil moisture process are instrumental for determining the average frequency of irrigation applications and the total water volume required over the growing season. An irrigation application is triggered whenever the soil moisture process reaches the intervention point $\tilde{s}$; hence, as mentioned above, the frequency of irrigation treatment is the same as the frequency of downcrossing of the threshold $\xi = \tilde{s}$, i.e. $\nu_{\tilde{s}} = \rho(\tilde{s})p(\tilde{s})$. Furthermore, because the cumulative probability function $P(\xi)$ (Eq. (9)) represents the fraction of time spent by the process below the threshold $\xi \geqslant \tilde{s}$, the mean time between a downcrossing and the subsequent upcrossing (i.e., the mean time spent below the same threshold) can be obtained as [40]

$$\overline{T}_{\xi}^{\downarrow} = \frac{P(\xi)}{\nu_{\xi}^{\downarrow}} = \frac{P(\xi)}{\rho(\xi)p(\xi)}. \tag{13}$$

Similarly, the mean time spent by the process above the threshold is

$$\overline{T}_{\xi}^{\uparrow} = \frac{1 - P(\xi)}{\nu_{\xi}^{\uparrow}} = \frac{1 - P(\xi)}{\rho(\xi)p(\xi)}. \tag{14}$$

In our conceptual scheme, each irrigation treatment is instantaneous. However, in the limiting case of micro-irrigation ($\hat{s} \to \tilde{s}$) the treatments tend to become infinitely frequent, thus resulting in periods of irrigation of non-zero duration [37]. Each irrigation treatment needs to supply the volume (per unit area) $nZ_r(\hat{s} - \tilde{s})$ to bring soil moisture back to the target level $\hat{s}$. Hence, the average volume per unit cultivated area required for irrigation purposes over a growing season of duration $T_{seas}$ is

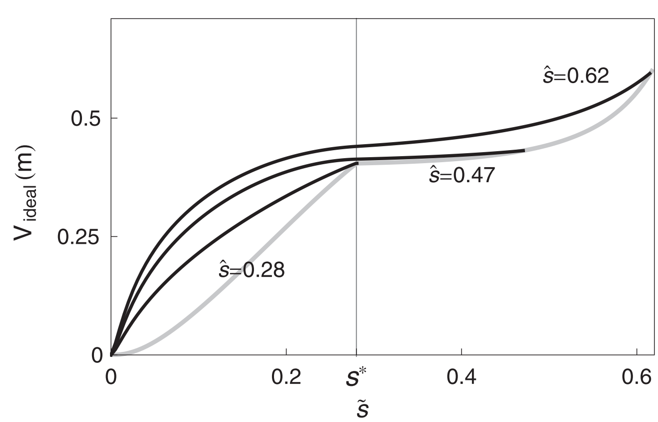


**Fig. 5.** Average required irrigation volumes (Eq. (15)), as a function of intervention point $\tilde{s}$, for different choices of the parameter $\hat{s}$, over a 110-day growing season. Gray line refers to the extreme case of $\hat{s} \to \tilde{s}$, i.e., micro-irrigation. The thin vertical line corresponds to $\tilde{s} = s^*$. All the other parameters are as in Fig. 3.

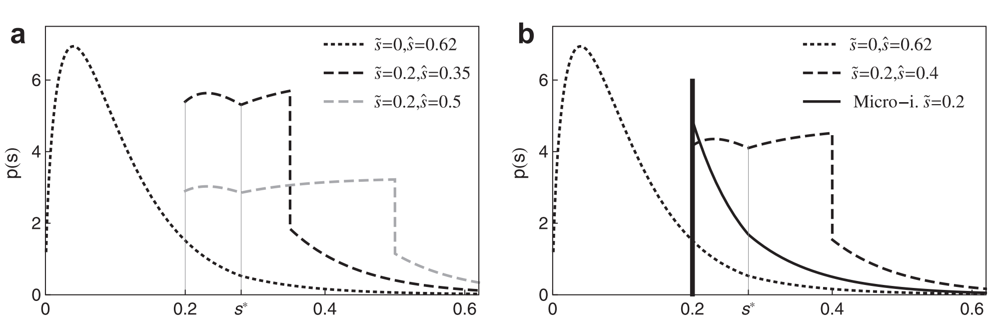


**Fig. 4.** Example of steady state probability density functions of plant available soil moisture relative to different choices of the parameters $\tilde{s}$ and $\hat{s}$. The soil moisture pdf for rainfed agriculture is presented for comparison (dotted lines), along with the case of micro-irrigation (solid line in (b)). In (b) the atom of probability in $s$ = 0.2 (thick vertical line) is not to scale. All the other parameters are as in Fig. 3.

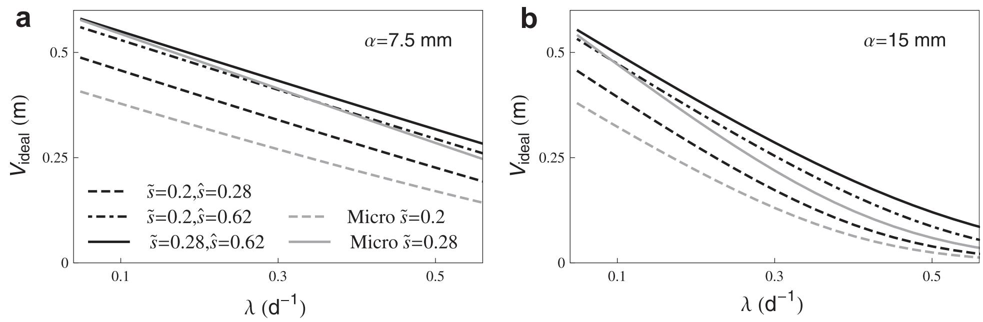


**Fig. 6.** Average required irrigation volumes (Eq. (15)) for a 110-day growing season, as a function of rainfall average frequency for two average event depths ((a) $\alpha$ = 7.5 mm; (b) $\alpha$ = 15 mm), for a variety of irrigation strategies (black lines refer to traditional irrigation, gray lines to micro-irrigation; solid lines to stress-avoidance schemes). All other parameters are as in Fig. 3.

$$V_{ideal}(\tilde{s},\hat{s}) = nZ_r(\hat{s}-\tilde{s})\nu_{\tilde{s}}T_{seas} = nZ_r(\hat{s}-\tilde{s})\rho(\tilde{s})p(\tilde{s})T_{seas}. \tag{15}$$

This required irrigation volume already accounts for losses to runoff and deep percolation caused by any input of water exceeding $s_1$, through its dependence on soil moisture water balance. Nonetheless, as it will be discussed in the companion paper [38], the water requirement in Eq. (15) represents an ideal situation, with irrigation application efficiency equal to 1. Accounting for typical irrigation application efficiency will allow obtaining a more realistic estimate of water requirements.

Fig. 5 represents the average required irrigation volumes, for different choices of irrigation parameters, when rainfall statistics are kept constant. The analysis is limited to combination of parameters for which $\tilde{s} \leqslant \hat{s}$, where the equality corresponds to the case of micro-irrigation (gray line in Fig. 5). For completeness, we consider also extremely low and high intervention points, even though, as discussed in Section 2, they are not commonly employed. In general, when all the other parameters are fixed, the higher $\hat{s}$, the higher the irrigation volumes will also be, due to larger losses at the higher soil moisture levels caused by irrigation. Furthermore, the required volume always increases with the intervention level $\tilde{s}$, as higher $\tilde{s}$ cause soil moisture to be kept at higher values throughout the growing season. For any given target level $\hat{s}$, the maximum required volume is reached for $\tilde{s} = \hat{s}$ (i.e., micro-irrigation), because this scheme forces soil moisture to be always at or above $\hat{s}$ (with potential beneficial effects on yield). Similar patterns are observed for different rainfall amounts and frequency, even though, as expected, a decrease in seasonal rainfall causes an increase in required irrigation volume to compensate for the lower natural inputs.

The role of rainfall timing and amount on irrigation water requirements is explored in Fig. 6, for a range of irrigation strategies. Climate change projections suggest decreases in rainfall frequencies will be the most commonly impacted. Along these lines, Fig. 6 shows the average irrigation volumes over a growing season as a function of rainfall frequency $\lambda$, while keeping average event depth $\alpha$ constant (i.e., in this way, the total rainfall over the growing season depends linearly on $\lambda$). As expected, for less frequent precipitation events, the system relies more and more on irrigation, with rather infrequent alterations caused by precipitation, and the smaller inputs through rainfall need to be compensated through irrigation, regardless of the selected irrigation strategy. Regarding the irrigation strategy, in agreement with Fig. 5, micro-irrigation requires less water than traditional irrigation for the same intervention point, regardless of precipitation frequency. However, stress avoidance micro-irrigation appears to be more sensitive to changes in rainfall frequency than other irrigation schemes. Interestingly, for extremely low rainfall frequencies, water requirements for stress-avoidance irrigation become less dependent on the employed scheme (compare solid lines in Fig. 6). In fact, being ET losses independent of soil moisture above $s^*$ (Eq. (2)), deeper irrigation applications (characterized by higher $\hat{s}$) result in longer periods between subsequent irrigation applications, without requiring significantly higher volumes over the entire growing season. When considering also the role of average precipitation depth, more pronounced increases in required irrigation volumes for similar changes in $\lambda$ are to be expected in locations with higher mean event depths (and hence higher total precipitations; Fig. 6(a) vs. (b)), thus potentially requiring larger adjustments in irrigation strategies in face of climate change. The quantitative characterization of irrigation volumes as a function of climate, crop, and irrigation strategy is relevant for long-term water infrastructure planning. In particular, depending on the expected water availability per unit cultivated area, Fig. 6 suggests which irrigation strategy may prove to be the most suitable to the projected changes in rainfall amount and timing.

### 4.3. Long-term soil moisture balance

The components of the soil water balance are the long-term averages of the soil moisture dynamics (Eq. (1) [see [37]]). The inputs to the system are average daily rainfall, $\langle R\rangle = \alpha\lambda$, and average daily irrigation, $\langle I\rangle = V_{ideal}T_{seas}^{-1}$. Average daily losses through evapotranspiration, $\langle ET\rangle$ , are obtained as $\langle ET\rangle = nZ_r\int_{\tilde{s}}^{s_1}\rho(u)p(u)du$, while runoff and deep percolation losses can be obtained by difference, as $\langle LQ\rangle = \langle R\rangle + \langle I\rangle - \langle ET\rangle$. Under the assumption of stochastic steady-state conditions, a similar balance holds at the growing season time scale of length $T_{seas}$, with total average input of $\alpha\lambda T_{seas} + V_{ideal}$, and total average evapotranspiration $ET_{seas} = \langle ET\rangle T_{seas}$. The latter will be linked to crop yield in the companion paper [38].

## 5. Concluding remarks

The proposed approach allows us to obtain a probabilistic characterization of soil moisture under random rainfall variability for the entire range of irrigation schemes indicated in Fig. 1. The knowledge of the whole statistical ensemble of soil moisture and irrigation values, rather than single soil moisture trajectories driven by a particular realization of the rainfall stochastic forcing, enables a rigorous analysis of hydrologic risk in agriculture, which will be pursued in a future contribution. The obtained mostly analytical formulas are widely applicable across different crops, soil types, and climatic conditions, thus allowing exploration of water requirements under a variety of crop and climatic scenarios, without requiring time-consuming direct simulations. As such, they may be employed to investigate the impacts of the projected climate changes on irrigation requirements for water-management purposes. Moreover, the analytical results for different irrigation strategies embedded in a common framework facilitate comparison among the most commonly employed demand-based irrigation schemes.

For a more complete assessment of demand-based irrigation strategies, the proposed model can be easily combined with a crop productivity function, e.g., describing dependence of crop yield on plant transpiration, as well as economic balance, including crop price and fixed and variable costs. A first step along these lines is presented in a companion paper [38]. In this manner, the model becomes useful for fully assessing irrigation choices, including their productivity, in terms of crop yield, sustainability, and economic feasibility, both in the short and long term.

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## Appendix. Soil moisture probability density function for micro-irrigation

The rate of micro-irrigation application exactly compensates losses through evapotranspiration at a given $\tilde{s}$, thus maintaining soil moisture at $\tilde{s}$ until the next effective rainfall event occurs. Mathematically, this is equivalent to setting the loss function $\rho(s)$ to zero for any $s \leqslant \tilde{s}$. Because soil moisture stays for a finite time at $\tilde{s}$, to obtain the soil moisture probability density function it is convenient to proceed as in [37], writing the master equation and then splitting it into a part for the continuous distribution and a part for the atom of probability in $\tilde{s}$. Since the time spent by the process between any two soil moisture levels is not changed by the presence of the atom of probability in $\tilde{s}$, the master equation admits the same solution as the one relative to the case of rainfed agriculture (as obtained, e.g., in [52]) and only the normalization constant is changed. Hence the continuous part of the pdf reads

$$p_m(s) = \frac{C_m}{\rho(s)} \exp\left(-\gamma s + \lambda \int_{\tilde{s}}^{s} \frac{du}{\rho(u)}\right), \tag{A.1}$$

and the atom of probability in $\tilde{s}$ is

$$p_0 = \frac{\rho(\tilde{s}) p_m(\tilde{s})}{\lambda} = \frac{C_m}{\lambda} e^{-\gamma \tilde{s}}. \tag{A.2}$$

The normalization constant, $C_m$, can be obtained analytically for any $\tilde{s}$ by imposing

$$1 - p_0 = \int_{\tilde{s}}^{s_1} p_m(u) du. \tag{A.3}$$

Finally, because in micro-irrigation water applications perfectly balance ET losses, the mean water volume per unit area required over a growing season of duration $T_{seas}$ is $V_m = nZ_r \rho(\tilde{s}) p_0 T_{seas}$, with mean frequency of application $v_m = \lambda p_0$ (see [37] for details).

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