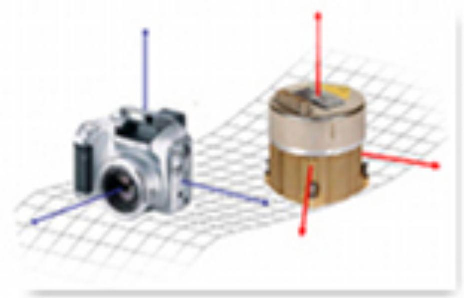

Sensor Orientation – Reference Frames

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EPFL, spring semester 2024

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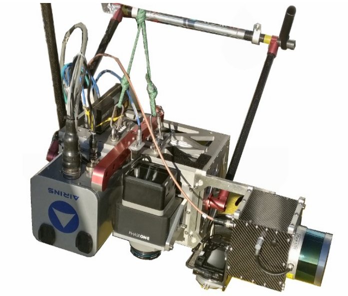
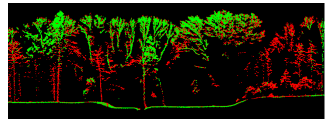
Sensor orientation – main topics

This translates into three rough big areas





1. Fundamentals
   - How to characterize sensor noise
   - How to transform from the sensed signals to navigation frame?

You need the frames

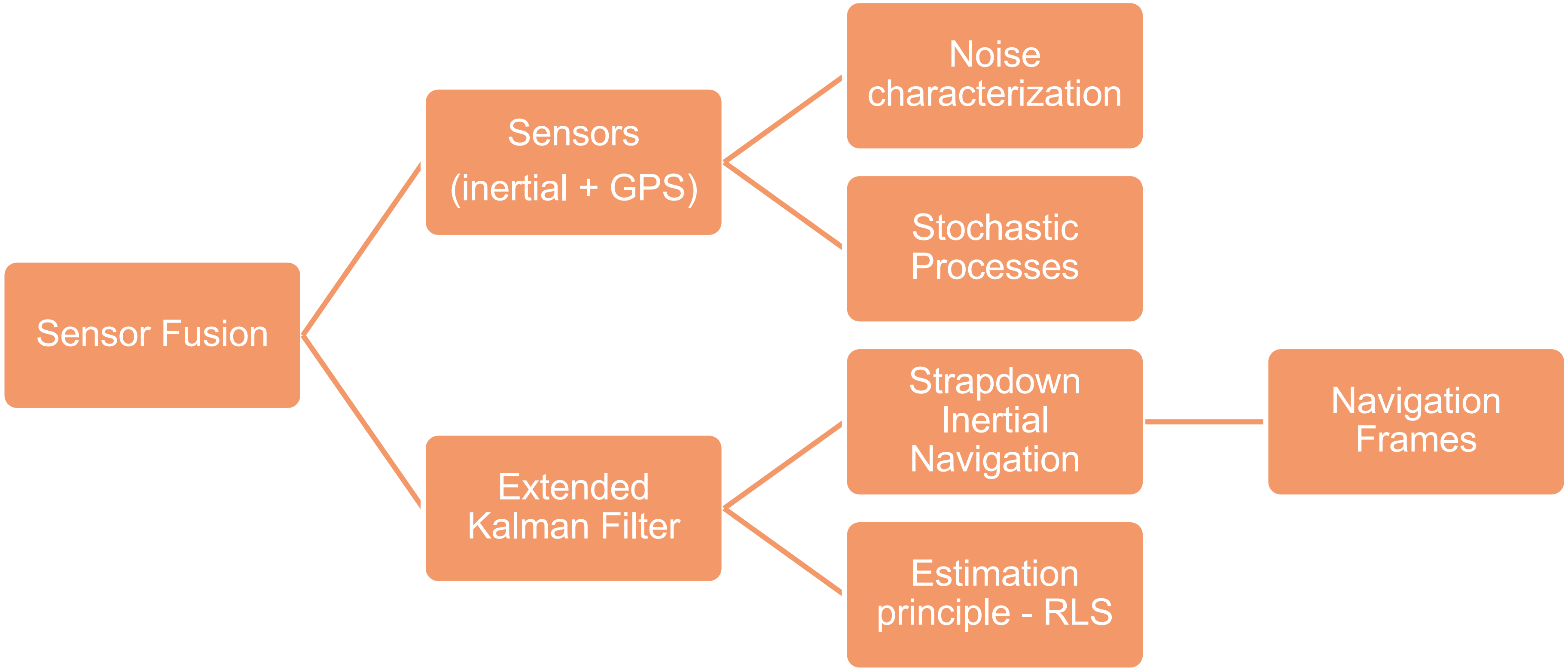
2. Position, velocity, attitude (navigation)
   - How to formulate navigation equation in different frames?
   - How to resolve them numerically?

You need the navigation quantities and the noise properties

3. Sensor fusion
   - How to formulate models for sensor fusion?
   - How to implement it in optimization and use it for mapping?

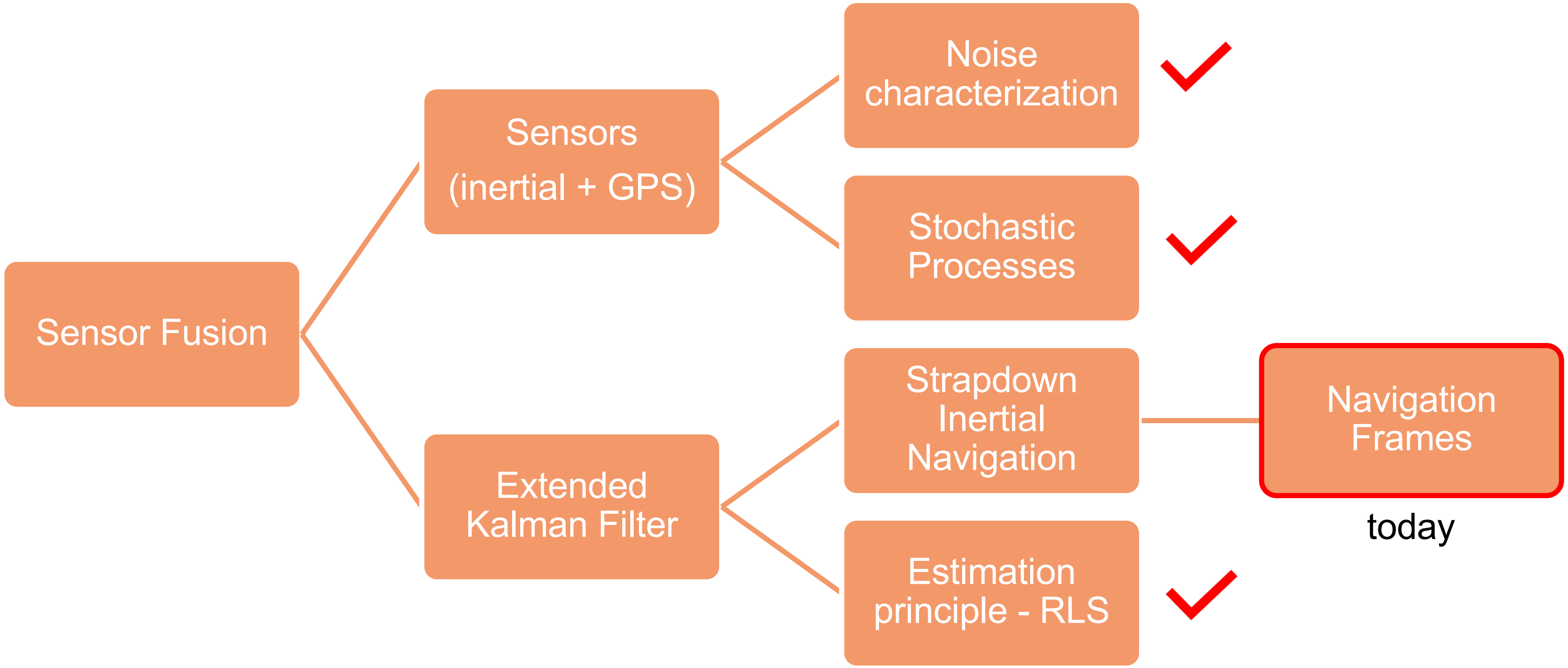
Cockpit view of SO course’s topics

How to reach *integrated* sensor orientation?



Cockpit view of SO course's topics

How to reach *integrated* sensor orientation?



Reference Frames – agenda (Ch 3)

- Definition
  - Coordinate / reference systems & frames
  - Right-handed Cartesian system
- Frames in navigation
  - Inertial
  - ECEF
  - Local-level
  - Body
- Frames in mapping
  - Sensor frame
  - Mapping frame
  - Mapping projection
  - Mathematical vs. physical height
  - Transformations

Later (Chapter 10)

Definitions – theory

Coordinate system

- Parameterization of space
- Example: geocentric (X,Y,Z), geodetic (φ,λ,h), map projection (E,N,H)

Reference **system**

- A set of conventions with models required to define at any time a triad of axes.
- Example: ITRS (Inertial Terrestrial Reference Systems)

Reference **frame**

- Realization of a reference system, i.e. list of “points” with their position and velocity coordinates
- Example: ITRF96 (Inertial Terrestrial Reference Frame established in 1996)

Reference **datum**

- Reference frame with defined body (e.g. ellipsoid) of mass and rotation
- Example: WGS84 (World Geodetic System established in 1984)

Definition - practice

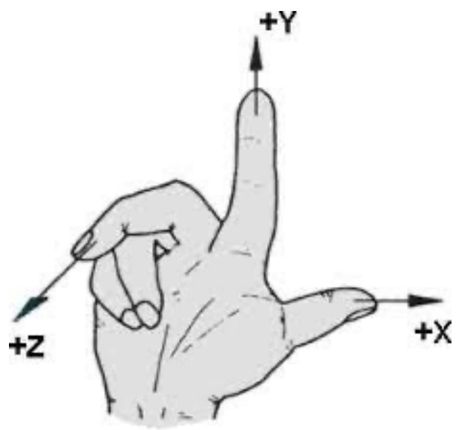
Notion of reference *frame* and *systems* is very often exchanged

Further distinction of coordinate systems/frames (more in Chap. 10)

- Global
  - Geocentric $(X,Y,Z)^e$
  - Geodetic (for a given ellipsoid) $(\varphi,\lambda,h)^e$
  - Map-projection $(\varphi,\lambda)^e \rightarrow (E,N)^e = f(\varphi,\lambda)^e$
- Local (with respect to a fixed-point in a reference frame)
  - Local-level geodetic (local-geodetic vertical)
  - Local-level astronomic (local-astronomic vertical)

Cartesian frame

The right-handed system – axes 1-2-3



The right-handed system – rotation (+)

Cartesian frame transformation

Two frames, p and q with identical originis: $\mathbf{x}^q = \mathbf{R}_p^q \mathbf{x}^p$

- $\mathbf{R}_p^q$ is a rotation matrix that rotates p-frame into q-frame
- Properties: inverse = transposition

$$\mathbf{R}_p^{q\,\mathrm{T}} \mathbf{R}_p^q = \mathbf{R}_p^q \mathbf{R}_p^{q\,\mathrm{T}} = \mathbf{I} \quad \text{and} \quad \det\left(\mathbf{R}_p^q\right) = 1$$

- Notation - rotation from p-frame into q-frame via intermediate s-frame

$$\mathbf{R}_p^q = \mathbf{R}_s^q \, \mathbf{R}_p^s$$

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Rotation matrix – 3 angles

Composed of 3 successive rotations

$$\mathbf{R}_p^q = \mathbf{R}_3(\alpha_3)\,\mathbf{R}_2(\alpha_2)\,\mathbf{R}_1(\alpha_1)$$

$$\mathbf{R}_1(\alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$\mathbf{R}_2(\alpha_2) = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix}$$

$$\mathbf{R}_3(\alpha_3) = \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Derivation of $R_1\, R_2\, R_3$ later

Rotation matrix – rules application

Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation

- Inertial (i)
- Earth-fixed (e)
- Local-level (l)
- Body (b) or platform
- …
- Wind (w)
- Magnetic

Frames in navigation

- **Inertial** frame (*i*)
  - Newton's mechanics "holds"
    - Non-accelerating
    - Non-rotating

$$\mathbf{F}^i = m_i \ddot{\mathbf{x}}^i$$

  - In a gravitation field

$$m_i \ddot{\mathbf{x}}^i = \mathbf{F}^i + m_g \mathbf{g}^i$$

  - Under Einstein principle of equivalence
    - Dividing both sides by $m_i = m_g$

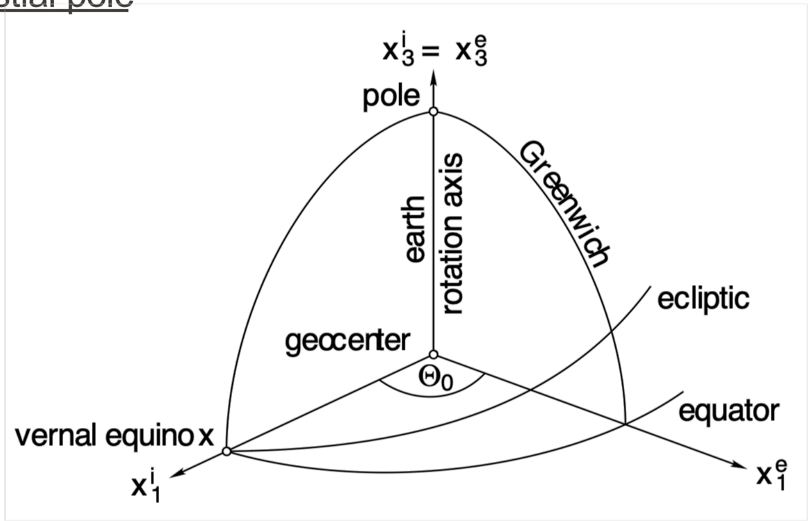
$$\ddot{\mathbf{x}}^i = \mathbf{f}^i + \mathbf{g}^i$$

Specifc force

Where to find an inertial frame?

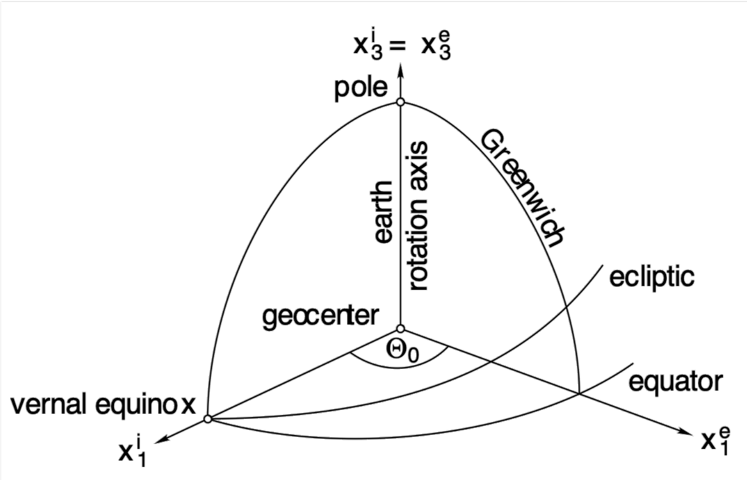
Frames in navigation

- **Inertial** frame (*i*)
  - Non-rotating
  - Non-accelerating
  - (Literally non-existing!, only approximation)
  - Origin at the center of mass of the Earth
  - x_1 points towards the vernal equinox
  - x_3 points towards the mean celestial pole



Frames in navigation

- **Earth** frame (ECEF) (e)
  - Origin at the center of mass of the Earth
  - x_1 points towards the Greenwich meridian
  - x_3 points towards the mean celestial pole
  - Cartesian coordinates (x_1^e, x_2^e, x_3^e)



- (e) → (i)

$$R_e^i = R_3(-\Theta) = R_3(-\omega_{ie}\Delta t)$$

$$\omega_{ie} = 7.292115\times10^{-5}\, rad\,/\,s$$

Transformation inertial to ECEF

Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation

- Inertial (i)
- Earth-fixed (e)
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- Magnetic

Frames in navigation

- Ellipsoidal coordinates (φ, λ, h)
  - Conversion: $(\varphi, \lambda, h) \rightarrow (x_1^e, x_2^e, x_3^e)$

$$\mathrm{x}^e = \begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \end{bmatrix} = \begin{bmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ \left(\frac{b^2}{a^2}N+h\right)\sin\varphi \end{bmatrix}$$

  - Conversion: $(x_1^e, x_2^e, x_3^e) \rightarrow (\varphi, \lambda, h)$

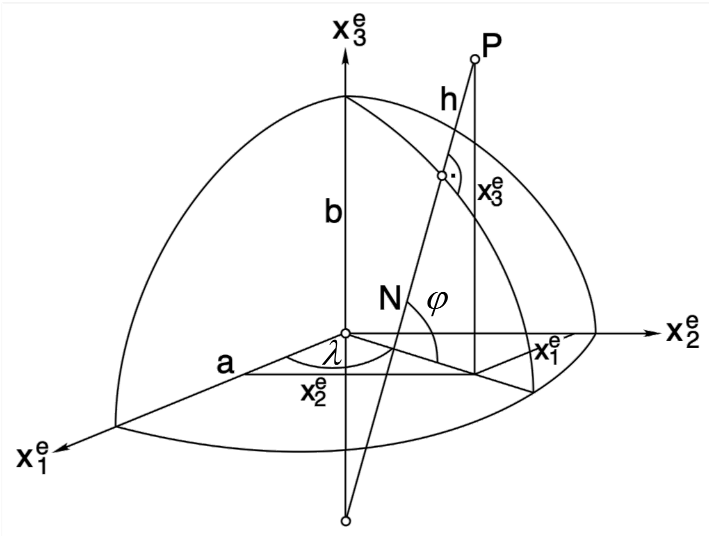
$$\lambda = \arctan\left(x_2^e / x_1^e\right)$$

$$p = \sqrt{x_1^{e^2} + x_2^{e^2}} = (N+h)\cos\varphi$$

$$\psi = \arctan\left(x_3^e a / pb\right)$$

$$\varphi = \arctan\frac{x_3^e + e'^2 b\sin^3\psi}{p - e^2 a\cos^3\psi}$$

$$h = p / \cos\varphi - N$$



$$N = \frac{a}{\sqrt{1 - e^2\sin^2\varphi}}$$

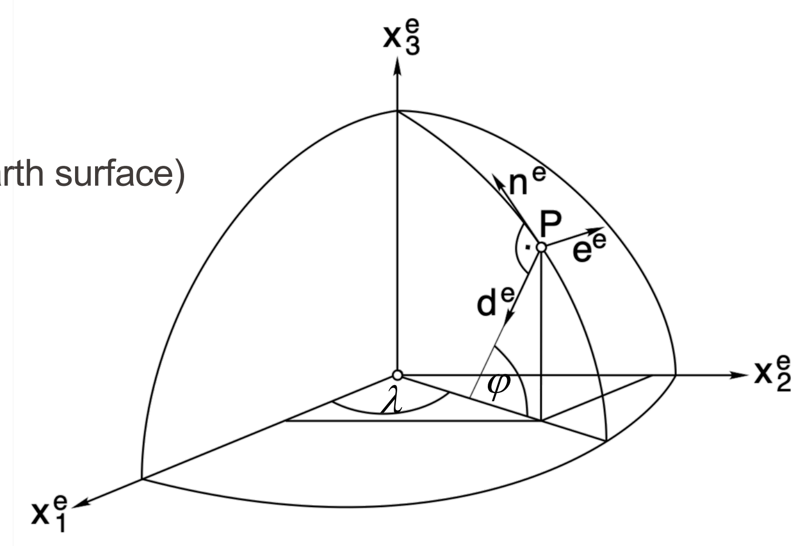
$$e^2 = \left(a^2 + b^2\right) / a^2$$

$$e'^2 = \left(a^2 - b^2\right) / b^2$$

Frames in navigation

Local-level frame (l)

- Local geodetic frame
- Arbitrary origin (e.g., a point on the Earth surface)
- x_1 points to North
- x_2 points to East
- x_3 points to the local nadir (down)
  →NED (North, East, Down)



$$\mathbf{d}^e = \begin{bmatrix} -\cos\varphi\ \cos\lambda \\ -\cos\varphi\ \sin\lambda \\ -\sin\varphi \end{bmatrix},\ \mathbf{n}^e = -\frac{\partial \mathbf{d}^e}{\partial \varphi},\ \mathbf{e}^e = -\frac{1}{\cos\varphi}\frac{\partial \mathbf{d}^e}{\partial \lambda}$$

Frames in navigation

Frames in mapping & navigation

Mapping
- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation
- Inertial (i)
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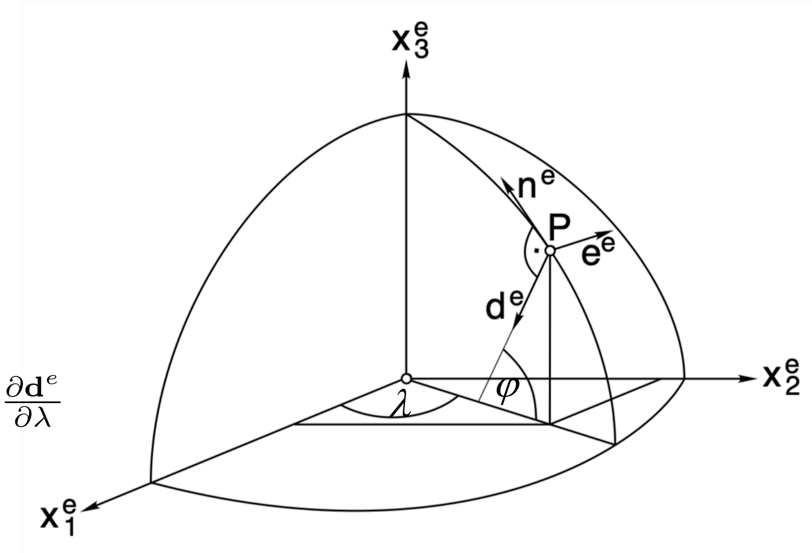
Frames in navigation



Local-level frame (*l*)

- Local geodetic frame (North – East - Down)

$$\mathbf{d}^e = \begin{bmatrix} -\cos\varphi\,\cos\lambda \\ -\cos\varphi\,\sin\lambda \\ -\sin\varphi \end{bmatrix},\ \mathbf{n}^e = -\frac{\partial \mathbf{d}^e}{\partial\varphi},\ \mathbf{e}^e = -\frac{1}{\cos\varphi}\frac{\partial \mathbf{d}^e}{\partial\lambda}$$

$$\mathbf{R}^e_{l-NED} = \begin{bmatrix} \mathbf{n}^e & \mathbf{e}^e & \mathbf{d}^e \end{bmatrix}$$

$$\mathbf{R}^e_{l-NED} = \begin{bmatrix} -\sin\varphi\,\cos\lambda & -\sin\lambda & -\cos\varphi\,\cos\lambda \\ -\sin\varphi\,\sin\lambda & \cos\lambda & -\cos\varphi\,\sin\lambda \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix}$$

- Attention: some software (systems) uses East – North – Up convention!

$$\mathbf{R}^e_{l-ENU} = \begin{bmatrix} \mathbf{e}^e & \mathbf{n}^e & -\mathbf{d}^e \end{bmatrix}$$

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Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation

- Inertial (i)
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- Local-level (l)
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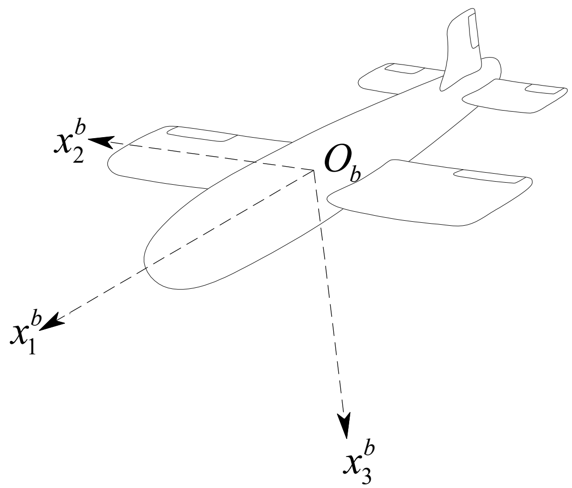
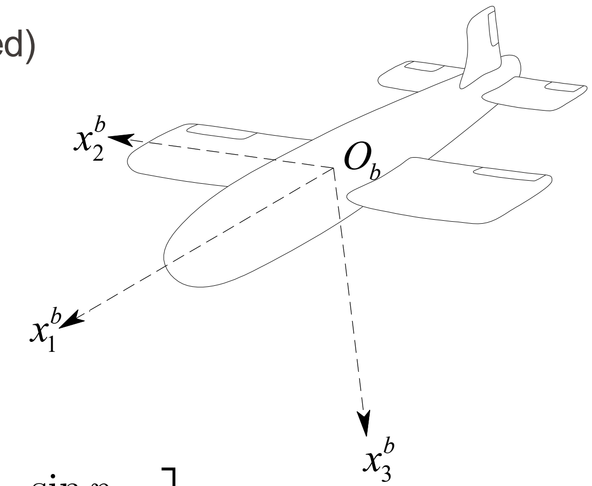


Frames in navigation

- **Body** frame
  - Attached to the body
  - Origin at an arbitrary point (center of mass is preferred)
  - x_1 points forward
  - x_2 points to right
  - x_3 points down
  - Three angles: roll (r), pitch (p,) yaw (y)
  - $(l) \rightarrow (b)$



$$\mathbf{R}_l^b = \mathbf{R}_1(r)\,\mathbf{R}_2(p)\,\mathbf{R}_3(y)$$

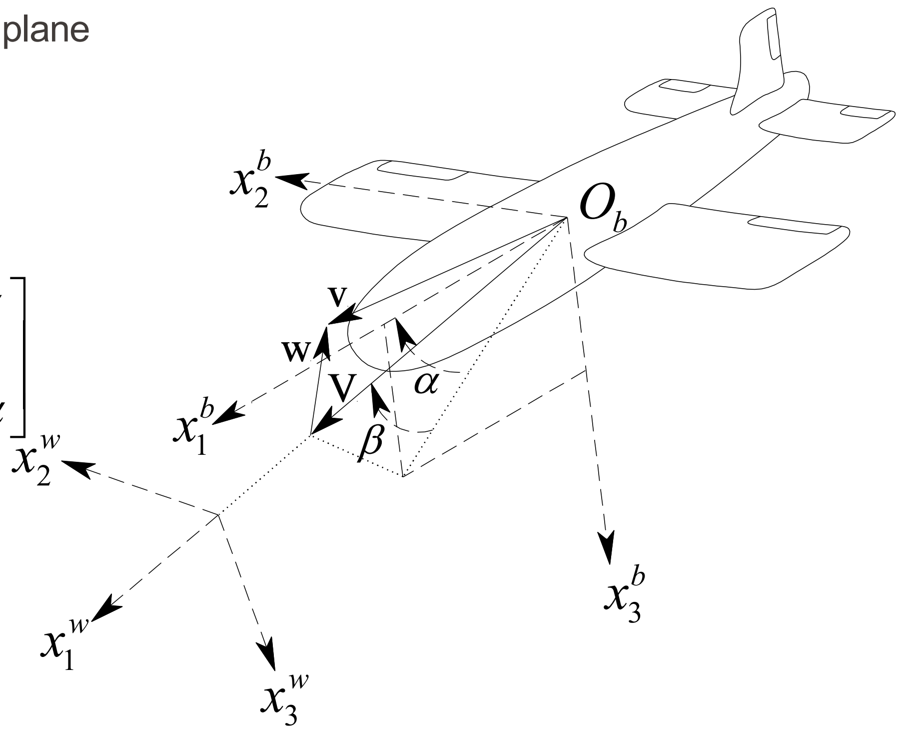
$$\mathbf{R}_l^b = \begin{bmatrix} \cos p\ \cos y & \cos p\ \sin y & -\sin p \\ \sin r\ \sin p\ \cos y - \cos r\ \sin y & \sin r\ \sin p\ \sin y + \cos r\ \cos y & \sin r\ \cos p \\ \cos r\ \sin p\ \cos y + \sin r\ \sin y & \cos r\ \sin p\ \sin y - \sin r\ \cos y & \cos r\ \cos p \end{bmatrix}$$

Frames in navigation

Frames in navigation

- **Wind** frame (only for curiosity!)
  - Origin coincides with body frame origin
  - x_1 points to the direction of air speed
  - x_3 points to normal vector of $x_2^b - x_1^w$ plane
  - x_2 completes the right-handed frame
  - Transformation (b) -> (w)

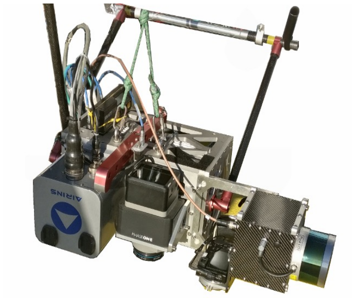
$$
\begin{aligned}
R_b^w &= R_3(\beta) R_2^T(\alpha) \\
&= \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}
\end{aligned}
$$

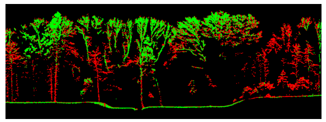


Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)





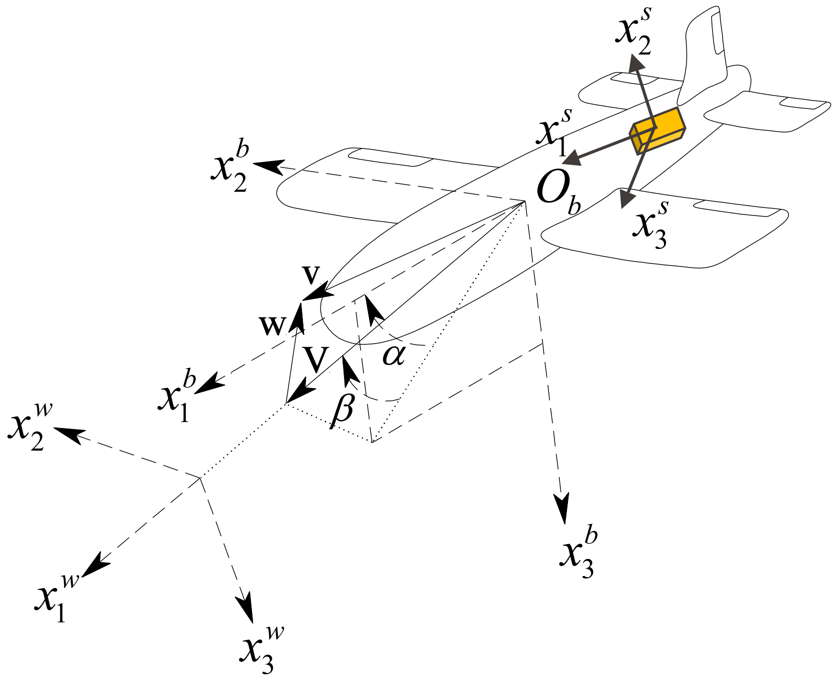
Navigation

- Inertial (i)
- Earth-fixed (e)
- Local-level (l)
- Body (b) or platform
- …
- Wind (w)
- Magnetic

Frames in mapping

- **Sensor** frame
  - Attached to a sensor (camera, laser scanner, IMU, etc.)
  - Arbitrary origin and axes orientation (preferably in the principal directions of the sensor)
  - $(s) \rightarrow (b)$

$$x^b = R_s^b(x^s + o_b^s)$$



Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
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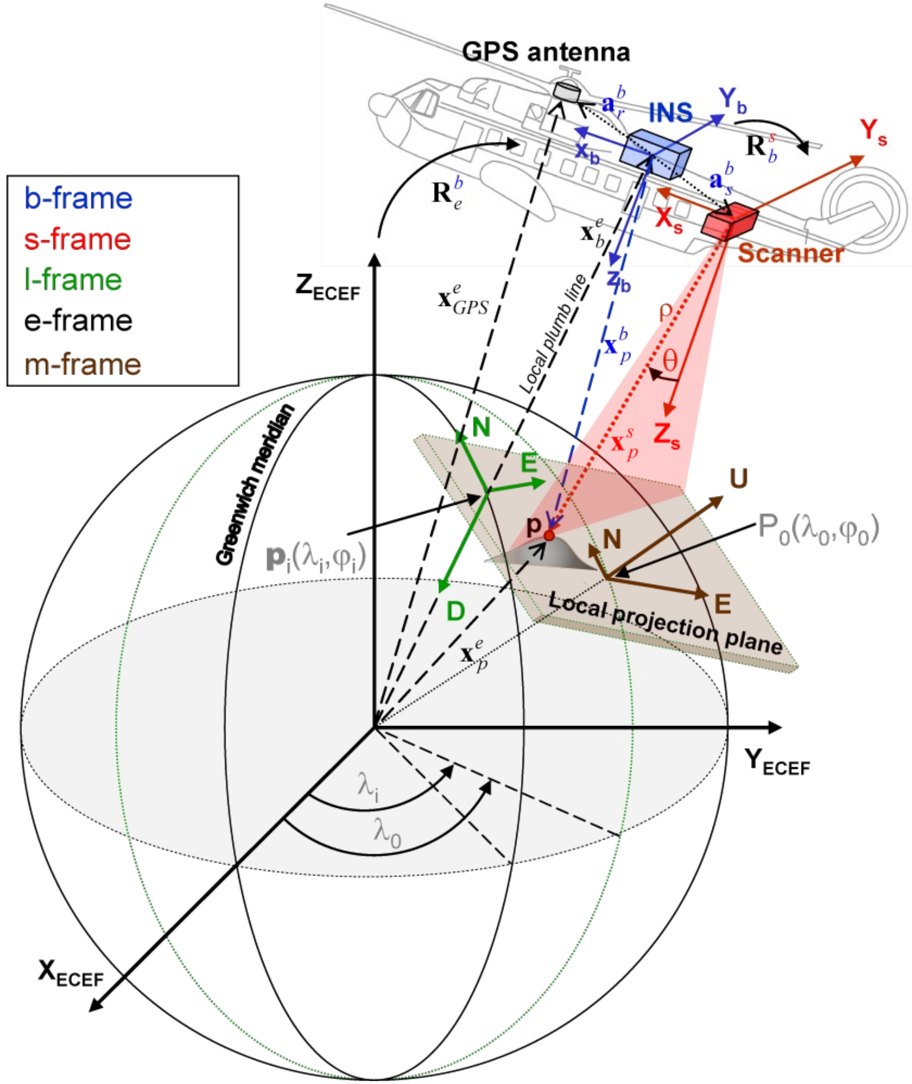

Adding national frames & projection

Id	Description
s	Sensor frame, defined by the principal exes of an image sensor (either camera or a laser-scanner)
b	Body frame, realized by the triad of accelerometers within an INS
l	Tangent frame of the global earth ellipsoid (“local-level frame”), spanned by the ellipsoidal north-,east-, and down-axes (NED)
e	Earth-centered earth-fixed (ECEF) frame, realized by a variant of the International Terrestrial Reference Frame (ITRF)
$\bar{l}$	Tangent frame of the national ellipsoid (NED)
n	Eccentric earth-fixed frame, defined for the national datum
p	Projection frame established by the national map projection, spanned by the grid east-, north-, and up-axes (ENU)

Later Chap. 10