

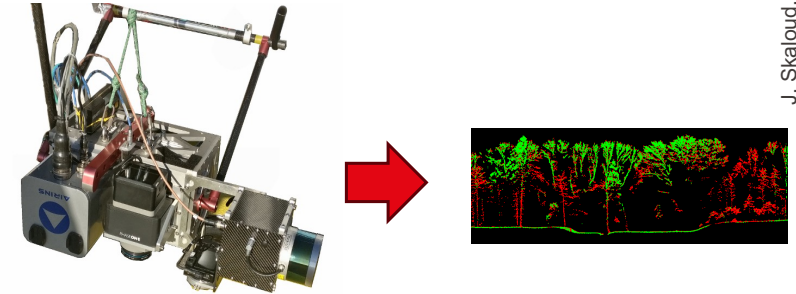


Sensor Orientation – Reference Frames

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Sensor orientation – main topics

This translates into three rough big areas



1. Fundamentals

- How to characterize sensor noise
- How to transform from the sensed signals to navigation frame?

2. Position, velocity, attitude (navigation)

- How to formulate navigation equation in different frames?
- How to resolve them numerically?

3. Sensor fusion

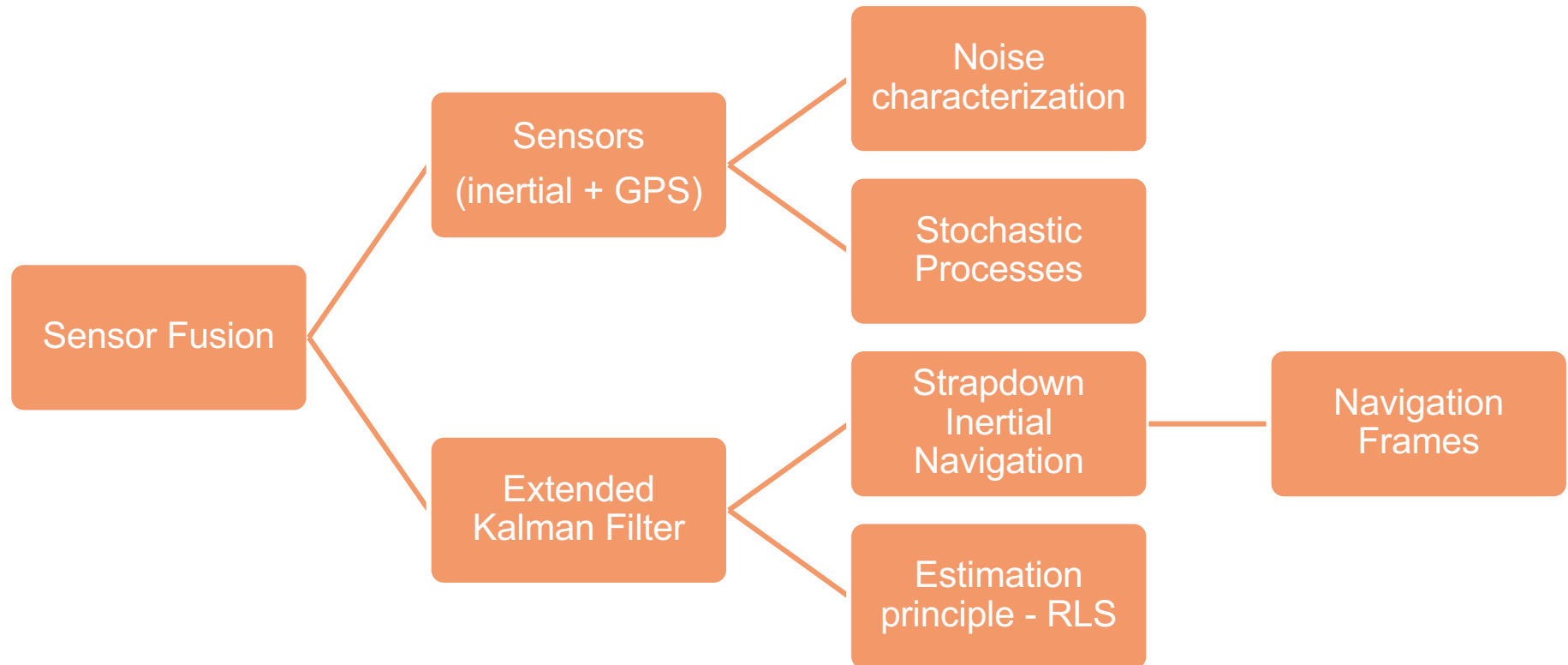
- How to formulate models for sensor fusion?
- How to implement it in optimization and use it for mapping?

You need the frames

You need the navigation quantities and the noise properties

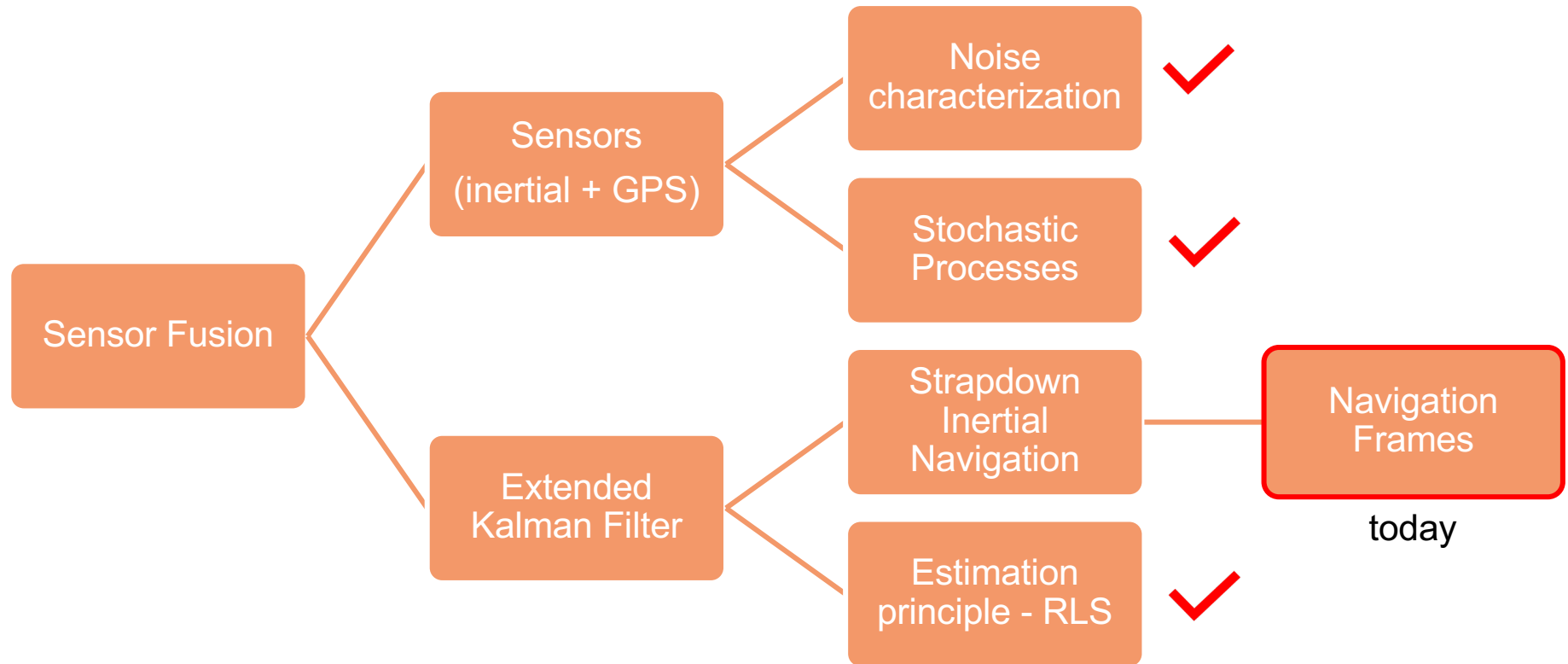
Cockpit view of SO course's topics

How to reach *integrated* sensor orientation?



Cockpit view of SO course's topics

How to reach *integrated* sensor orientation?



Reference Frames – agenda (Ch 3)

- Definition
 - Coordinate / reference systems & frames
 - Right-handed Cartesian system

 - Frames in navigation
 - Inertial
 - ECEF
 - Local-level
 - Body

 - Frames in mapping
 - Sensor frame
 - Mapping frame
 - Mapping projection
 - Mathematical vs. physical height
 - Transformations
- } Later (Chapter 10)

Definitions – theory

Coordinate system

- Parameterization of space
- Example: geocentric (X,Y,Z), geodetic (ϕ,λ,h), map projection (E,N,H)

Reference system

- A set of conventions with models required to define at any time a triad of axes.
- Example: **ITRS** (Inertial Terrestrial Reference Systems)

Reference frame

- Realization of a reference system, i.e. list of “points” with their position and velocity coordinates
- Example: **ITRF96** (Inertial Terrestrial Reference Frame established in 1996)

Reference datum

- Reference frame with defined body (e.g. ellipsoid) of mass and rotation
- Example: **WGS84** (World Geodetic System established in 1984)

Definition - practice

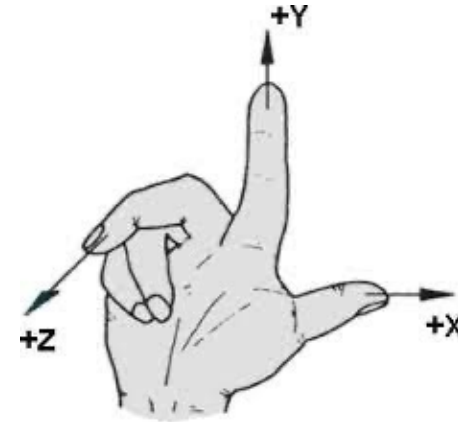
Notion of reference *frame* and *systems* is very often exchanged

Further distinction of coordinate systems/frames (**more in Chap. 10**)

- Global
 - Geocentric $(X,Y,Z)^e$
 - Geodetic (for a given ellipsoid) $(\varphi,\lambda,h)^e$
 - Map-projection $(\varphi,\lambda)^e \rightarrow (E,N)^e = f(\varphi,\lambda)^e$
- Local (with respect to a fixed-point in a reference frame)
 - Local-level geodetic (local-geodetic vertical)
 - Local-level astronomic (local-astronomic vertical)

Cartesian frame

The right-handed system – axes 1-2-3



The right-handed system – rotation (+)



Cartesian frame transformation

Two frames, p and q with identical origins: $\mathbf{x}^q = \mathbf{R}_p^q \mathbf{x}^p$

- $\mathbf{R}_p^q$ is a rotation matrix that rotates p -frame into q -frame
- Properties: inverse = transposition

$$\mathbf{R}_p^{qT} \mathbf{R}_p^q = \mathbf{R}_p^q \mathbf{R}_p^{qT} = \mathbf{I} \quad \text{and} \quad \det(\mathbf{R}_p^q) = 1$$

- Notation - rotation from p -frame into q -frame via intermediate s -frame

$$\mathbf{R}_p^q = \mathbf{R}_s^q \mathbf{R}_p^s$$

Rotation matrix – 3 angles

Composed of 3 successive rotations (3.15)

$$\mathbf{R}_p^q = \mathbf{R}_3(\alpha_3) \mathbf{R}_2(\alpha_2) \mathbf{R}_1(\alpha_1)$$

$$\mathbf{R}_1(\alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_1 & \sin \alpha_1 \\ 0 & -\sin \alpha_1 & \cos \alpha_1 \end{bmatrix}$$

$$\mathbf{R}_2(\alpha_2) = \begin{bmatrix} \cos \alpha_2 & 0 & -\sin \alpha_2 \\ 0 & 1 & 0 \\ \sin \alpha_2 & 0 & \cos \alpha_2 \end{bmatrix}$$

$$\mathbf{R}_3(\alpha_3) = \begin{bmatrix} \cos \alpha_3 & \sin \alpha_3 & 0 \\ -\sin \alpha_3 & \cos \alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Derivation of $\mathbf{R}_1 \mathbf{R}_2 \mathbf{R}_3$ later

Rotation matrix – rules application

Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation

- Inertial (i) 
- Earth-fixed (e)
- Local-level (l)
- Body (b) or platform
- ...
- Wind (w)
- Magnetic

Frames in navigation

□ Inertial frame (*i*)

- Newton's mechanics "holds"
 - Non-accelerating
 - Non-rotating

$$\mathbf{F}^i = m_i \ddot{\mathbf{x}}^i$$

- In a gravitation field

$$m_i \ddot{\mathbf{x}}^i = \mathbf{F}^i + m_g \mathbf{g}^i$$

- Under Einstein principle of equivalence
 - Dividing both sides by $m_i = m_g$

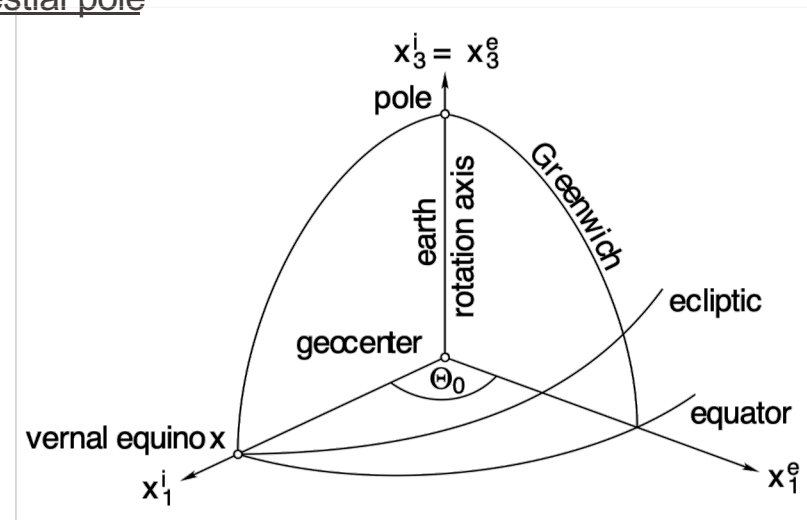
$$\ddot{\mathbf{x}}^i = \mathbf{f}^i + \mathbf{g}^i$$

└─ Specific force

Where to find an inertial frame?

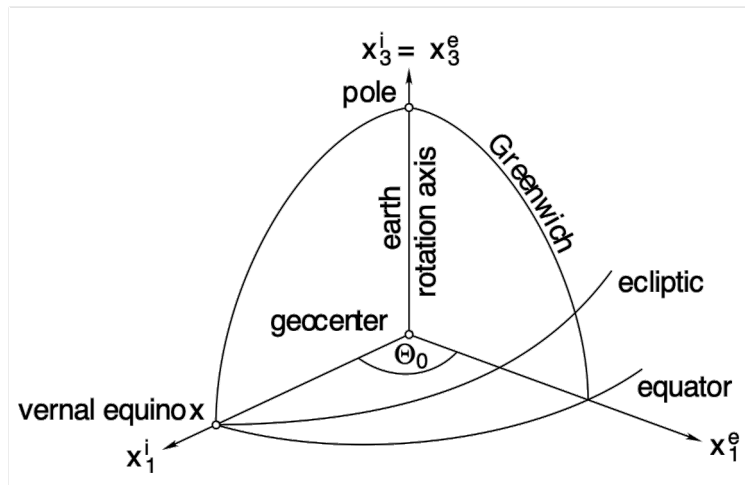
Frames in navigation

- **Inertial frame (i)**
 - Non-rotating
 - Non-accelerating
 - (Literally non-existing!, only approximation)
 - Origin at the center of mass of the Earth
 - x_1^i points towards the Vernal equinox
 - x_3^i points towards the mean celestial pole



Frames in navigation

- **Earth frame (ECEF) (e)**
 - Origin at the center of mass of the Earth
 - x_1 points towards the Greenwich meridian
 - x_3 points towards the mean celestial pole
 - Cartesian coordinates (x_1^e, x_2^e, x_3^e)



■ $(e) \rightarrow (i)$

$$R_e^i = R_3(-\Theta) = R_3(-\omega_{ie}\Delta t)$$

$$\omega_{ie} = 7.292115 \times 10^{-5} \text{ rad} / s$$

Transformation inertial to ECEF

Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
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Frames in navigation

□ Ellipsoidal coordinates (φ, λ, h)

■ Conversion: $(\varphi, \lambda, h) \rightarrow (x_1^e, x_2^e, x_3^e)$

$$\mathbf{x}^e = \begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \end{bmatrix} = \begin{bmatrix} (N + h) \cos \varphi \cos \lambda \\ (N + h) \cos \varphi \sin \lambda \\ \left(\frac{b^2}{a^2} N + h \right) \sin \varphi \end{bmatrix}$$

■ Conversion: $(x_1^e, x_2^e, x_3^e) \rightarrow (\varphi, \lambda, h)$

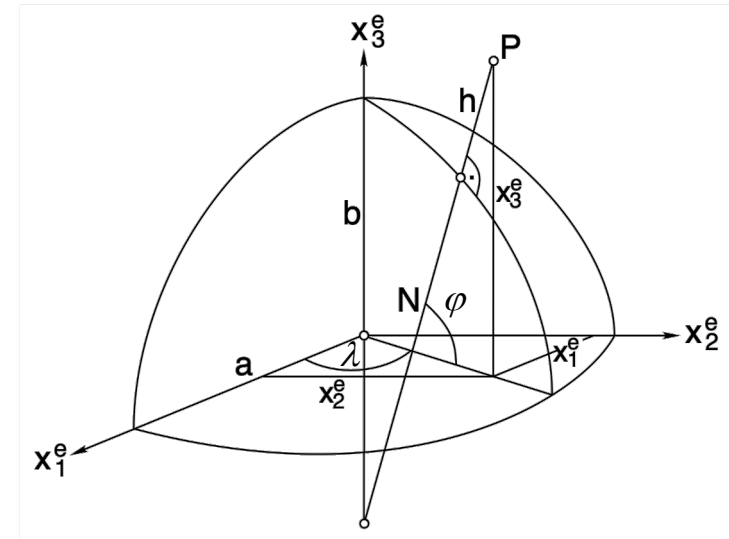
$$\lambda = \arctan(x_2^e / x_1^e)$$

$$p = \sqrt{x_1^{e^2} + x_2^{e^2}} = (N + h) \cos \varphi$$

$$\psi = \arctan(x_3^e a / p b)$$

$$\varphi = \arctan \frac{x_3^e + e'^2 b \sin^3 \psi}{p - e^2 a \cos^3 \psi}$$

$$h = p / \cos \varphi - N$$



$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$$

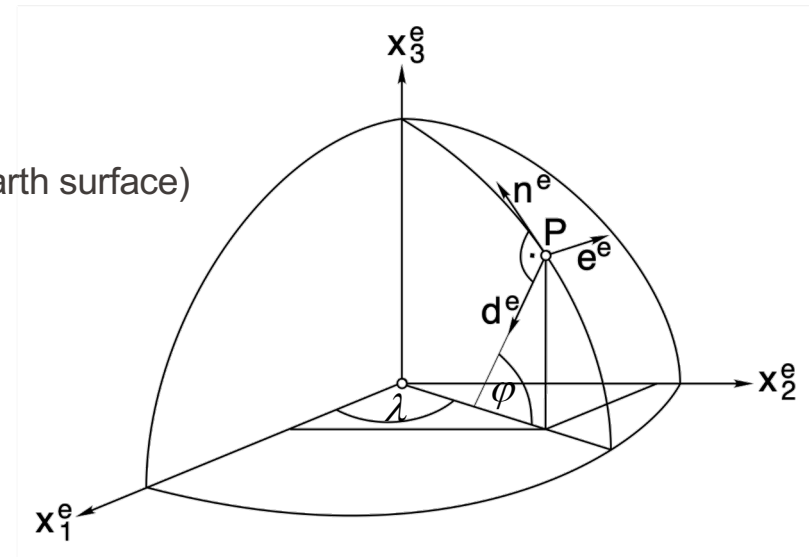
$$e^2 = (a^2 - b^2) / a^2$$

$$e'^2 = (a^2 - b^2) / b^2$$

Frames in navigation

Local-level frame (*l*)

- Local geodetic frame
- Arbitrary origin (e.g., a point on the Earth surface)
- x_1 points to North
- x_2 points to East
- x_3 points to the local nadir (down)
→ NED (North, East, Down)



$$\mathbf{d}^e = \begin{bmatrix} -\cos \varphi \cos \lambda \\ -\cos \varphi \sin \lambda \\ -\sin \varphi \end{bmatrix}, \quad \mathbf{n}^e = -\frac{\partial \mathbf{d}^e}{\partial \varphi}, \quad \mathbf{e}^e = -\frac{1}{\cos \varphi} \frac{\partial \mathbf{d}^e}{\partial \lambda}$$

Frames in navigation

Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation

- Inertial (i)
- Earth-fixed (e)
- Local-level (l) 
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Frames in navigation

Local-level frame (*l*)

- Local geodetic frame (North – East – Down)

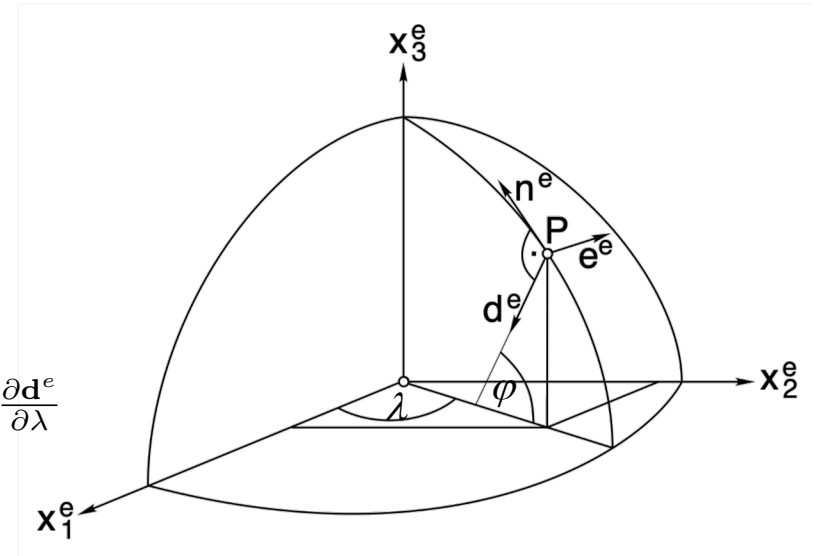
$$\mathbf{d}^e = \begin{bmatrix} -\cos \varphi \cos \lambda \\ -\cos \varphi \sin \lambda \\ -\sin \varphi \end{bmatrix}, \quad \mathbf{n}^e = -\frac{\partial \mathbf{d}^e}{\partial \varphi}, \quad \mathbf{e}^e = -\frac{1}{\cos \varphi} \frac{\partial \mathbf{d}^e}{\partial \lambda}$$

$$\mathbf{R}_{l-NED}^e = \begin{bmatrix} \mathbf{n}^e & \mathbf{e}^e & \mathbf{d}^e \end{bmatrix}$$

$$\mathbf{R}_{l-NED}^e = \begin{bmatrix} -\sin \varphi \cos \lambda & -\sin \lambda & -\cos \varphi \cos \lambda \\ -\sin \varphi \sin \lambda & \cos \lambda & -\cos \varphi \sin \lambda \\ \cos \varphi & 0 & -\sin \varphi \end{bmatrix} \quad (3.19)$$

- Attention: some software (systems) uses East – North – Up convention!

$$\mathbf{R}_{l-ENU}^e = \begin{bmatrix} \mathbf{e}^e & \mathbf{n}^e & -\mathbf{d}^e \end{bmatrix}$$



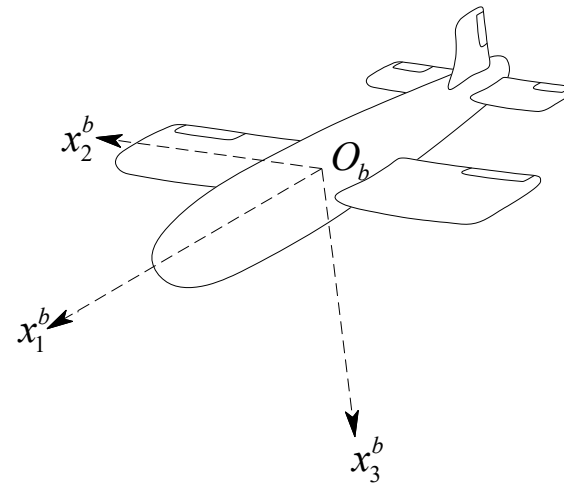
Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m)
- Mapping projection (p)

Navigation

- Inertial (i)
- Earth-fixed (e)
- Local-level (l)
- Body (b) or platform ←
- ...
- Wind (w)
- Magnetic



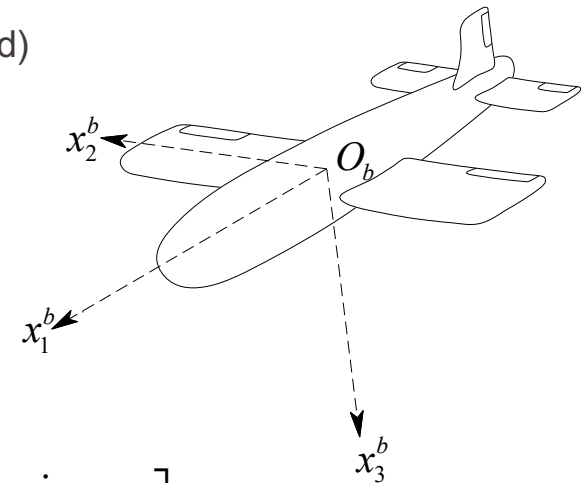
Frames in navigation

□ Body frame

- Attached to the body
- Origin at an arbitrary point (center of mass is preferred)
- x_1 points forward
- x_2 points to right
- x_3 points down
- Three angles: roll (r), pitch (p), yaw (y)
- (l) $\rightarrow$ (b)

$$\mathbf{R}_l^b = \mathbf{R}_1(r) \mathbf{R}_2(p) \mathbf{R}_3(y)$$

$$\mathbf{R}_l^b = \begin{bmatrix} \cos p \cos y & \cos p \sin y & -\sin p \\ \sin r \sin p \cos y & \sin r \sin p \sin y & \sin r \cos p \\ -\cos r \sin y & +\cos r \cos y & \\ \cos r \sin p \cos y & \cos r \sin p \sin y & \cos r \cos p \\ +\sin r \sin y & -\sin r \cos y & \end{bmatrix} \quad (3.21)$$



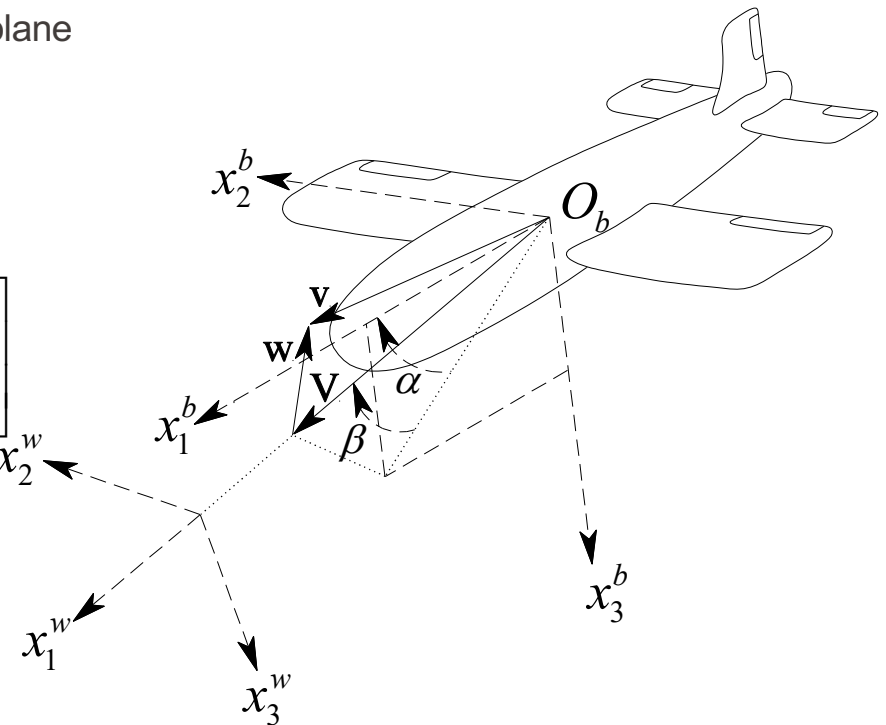
Frames in navigation

Frames in navigation

□ Wind frame (only for curiosity!)

- Origin coincides with body frame origin
- x_1 points to the direction of air speed
- x_3 points to normal vector of $x_2^b - x_1^w$ plane
- x_2 completes the right-handed frame
- Transformation $(b) \rightarrow (w)$

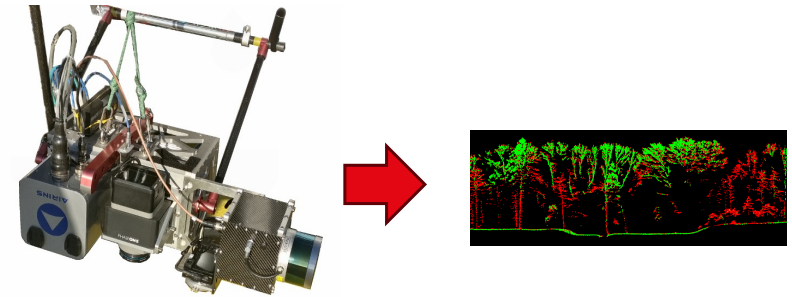
$$\begin{aligned}
 R_b^w &= R_3(\beta) R_2^T(\alpha) \\
 &= \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}
 \end{aligned}$$



Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor ←
- Mapping local (m)
- Mapping projection (p)



Navigation

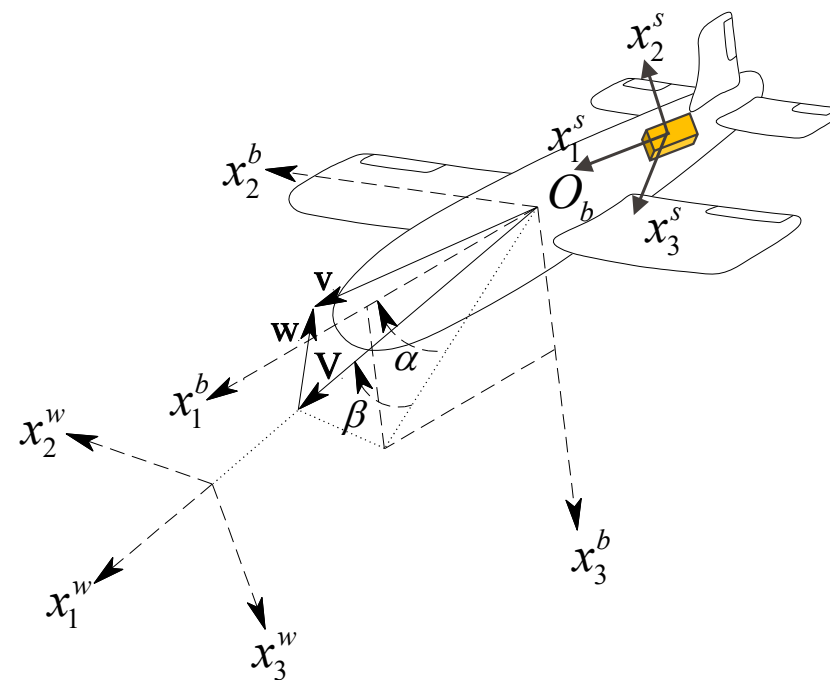
- Inertial (i)
- Earth-fixed (e)
- Local-level (l)
- Body (b) or platform
- ...
- Wind (w)
- Magnetic

Frames in mapping

□ Sensor frame

- Attached to a sensor (camera, laser scanner, IMU, etc.)
- Arbitrary origin and axes orientation (preferably in the principal directions of the sensor)
- $(s) \rightarrow (b)$

$$x^b = R_s^b(x^s + o_b^s)$$



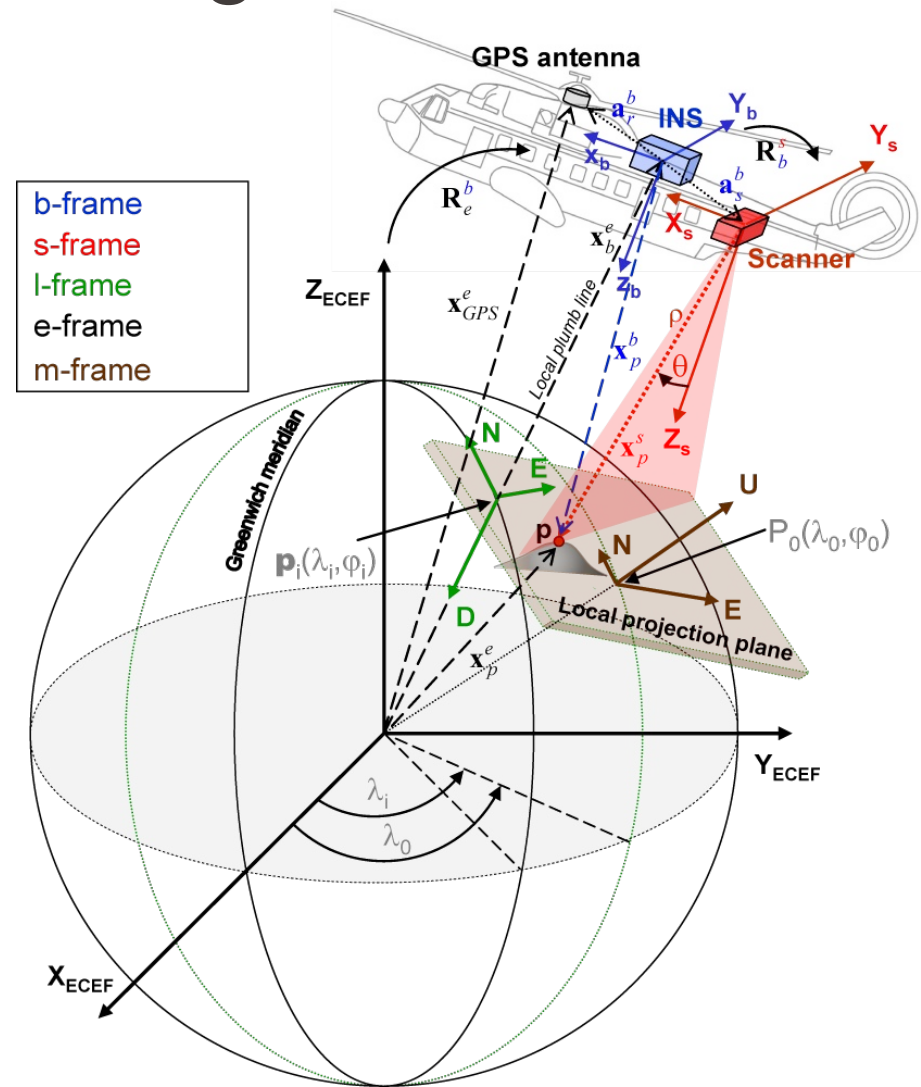
Frames in mapping & navigation

Mapping

- Sensors (s) – e.g. optical sensor
- Mapping local (m) ←
- Mapping projection (p)

Navigation

- Inertial (i)
- Earth-fixed (e)
- Local-level (l)
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- ...
- Wind (w)
- Magnetic



Adding national frames & projection

Id	Description
s	Sensor frame, defined by the principal axes of an image sensor (either camera or a laser-scanner)
b	Body frame, realized by the triad of accelerometers within an INS
l	Tangent frame of the global earth ellipsoid (“local-level frame”), spanned by the ellipsoidal north-, east-, and down-axes (NED)
e	Earth-centered earth-fixed (ECEF) frame, realized by a variant of the International Terrestrial Reference Frame (ITRF)
$\bar{l}$	Tangent frame of the national ellipsoid (NED)
n	Eccentric earth-fixed frame, defined for the national datum
p	Projection frame established by the national map projection, spanned by the grid east-, north-, and up-axes (ENU)

Later
Chap. 10