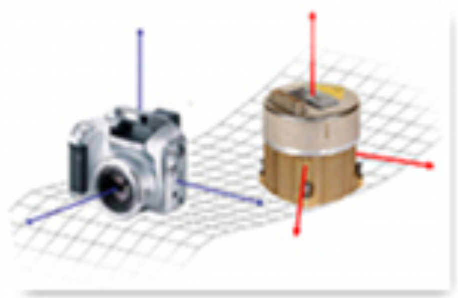
Sensor Orientation Navigation-e-frame



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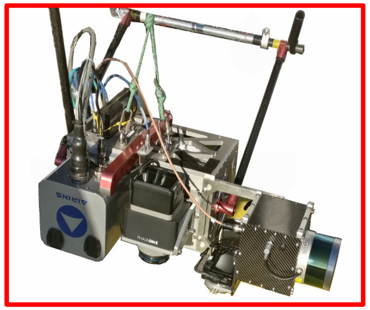
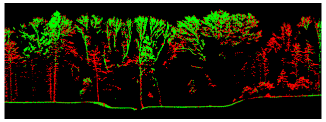
Sensor orientation – main topics

SHOWCASE TODAY !





This translates into three rough big areas

1. Fundamentals
   - How to characterize sensor noise
   - How to transform from the sensed signals to navigation frame?
2. Position, velocity, attitude (navigation)
   - How to formulate navigation equation in different frames?
   - How to resolve them numerically?
3. Sensor fusion
   - How to formulate models for sensor fusion?
   - How to implement it in optimization and use it for mapping?

You need the frames

You need the navigation quantities and the noise properties

Review of objectives

Navigation equations & their solution

- Would like to achieve - continuous (uninterrupted) determination of :
  - Position
  - Velocity
  - Attitude
- Implementation depends on:
  - The *type* of INS (gimbaled vs strapdown)
  - The *choice* of the navigation *frame*

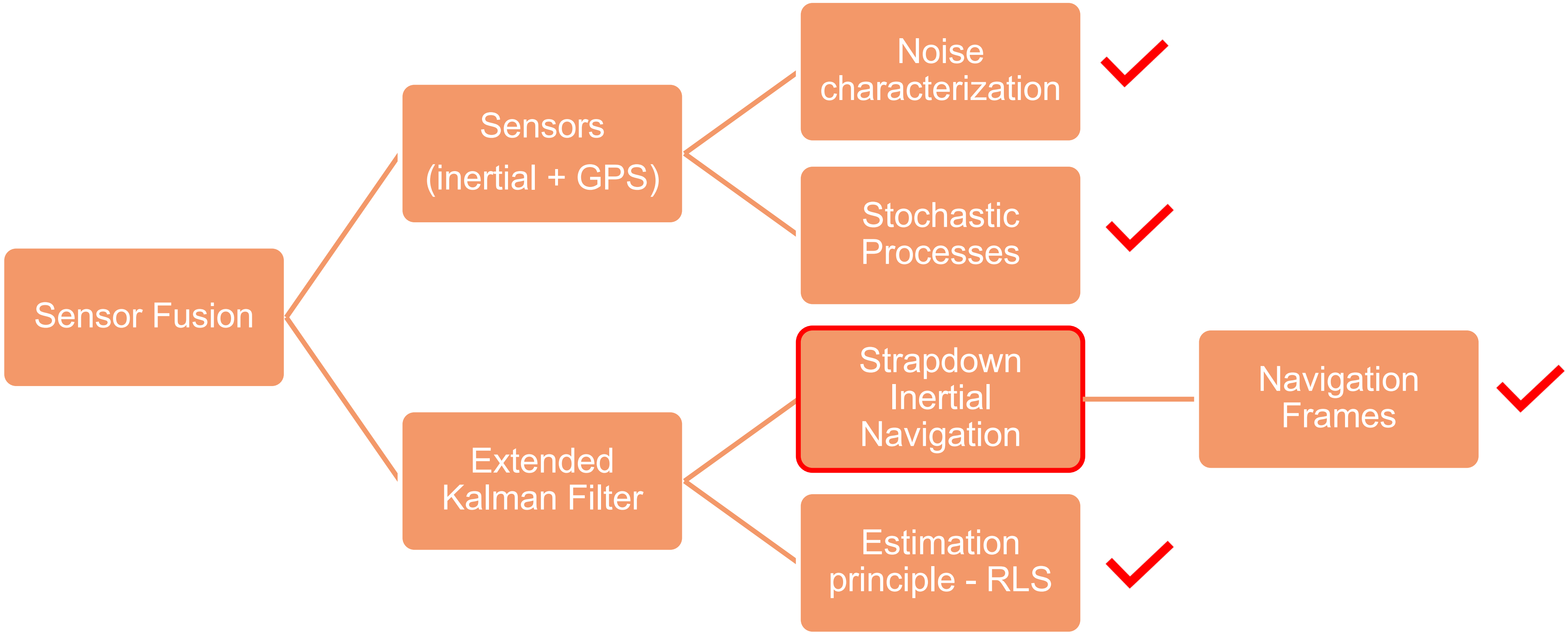
Why (later) to do sensor fusion ? ➔ error compensations of :

- Imperfections in INS initialization
- Imperfections in inertial sensors+ their assembly (1st feelings – Lab 3 today!)

➔ Mathematical derivation …)-:

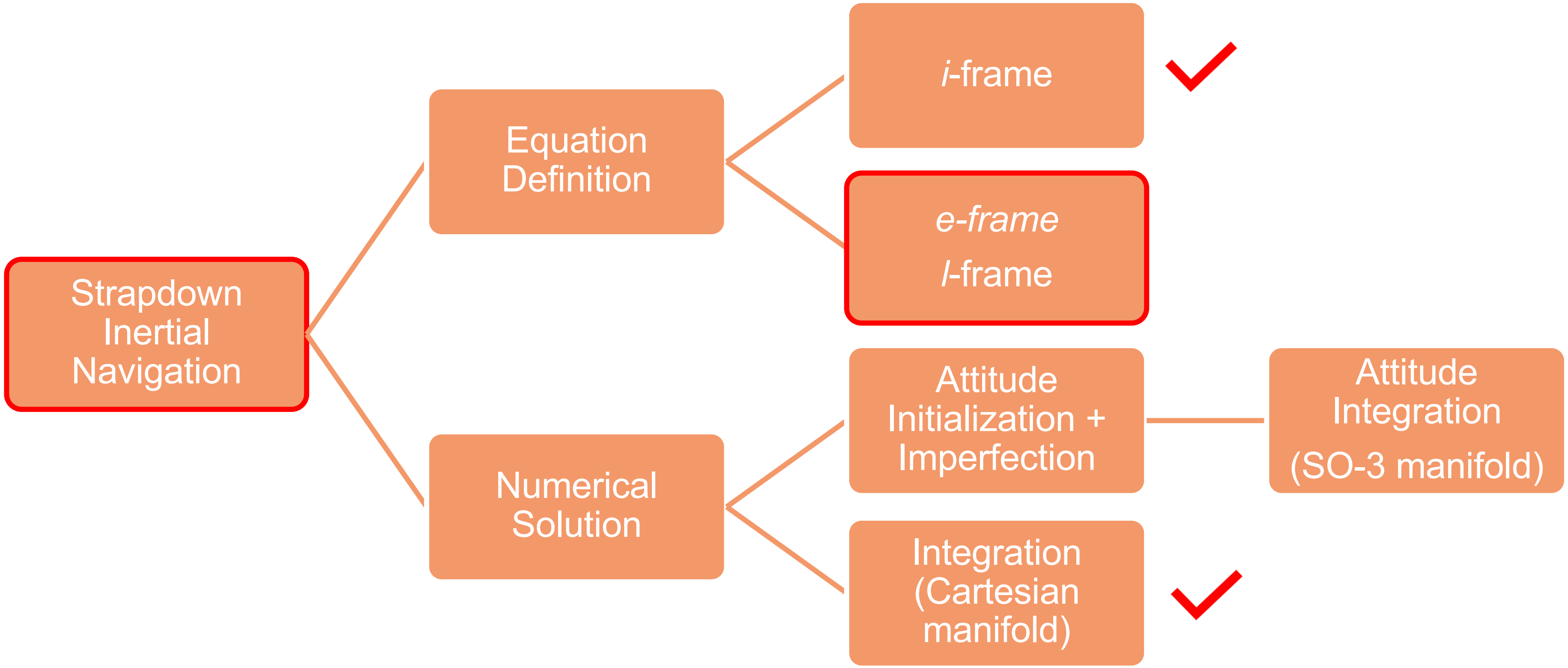
Cockpit view of SO course’s topics

How to reach *integrated* sensor orientation?



Cockpit view of inertial navigation

Prerequisite for reaching *integrated* sensor orientation



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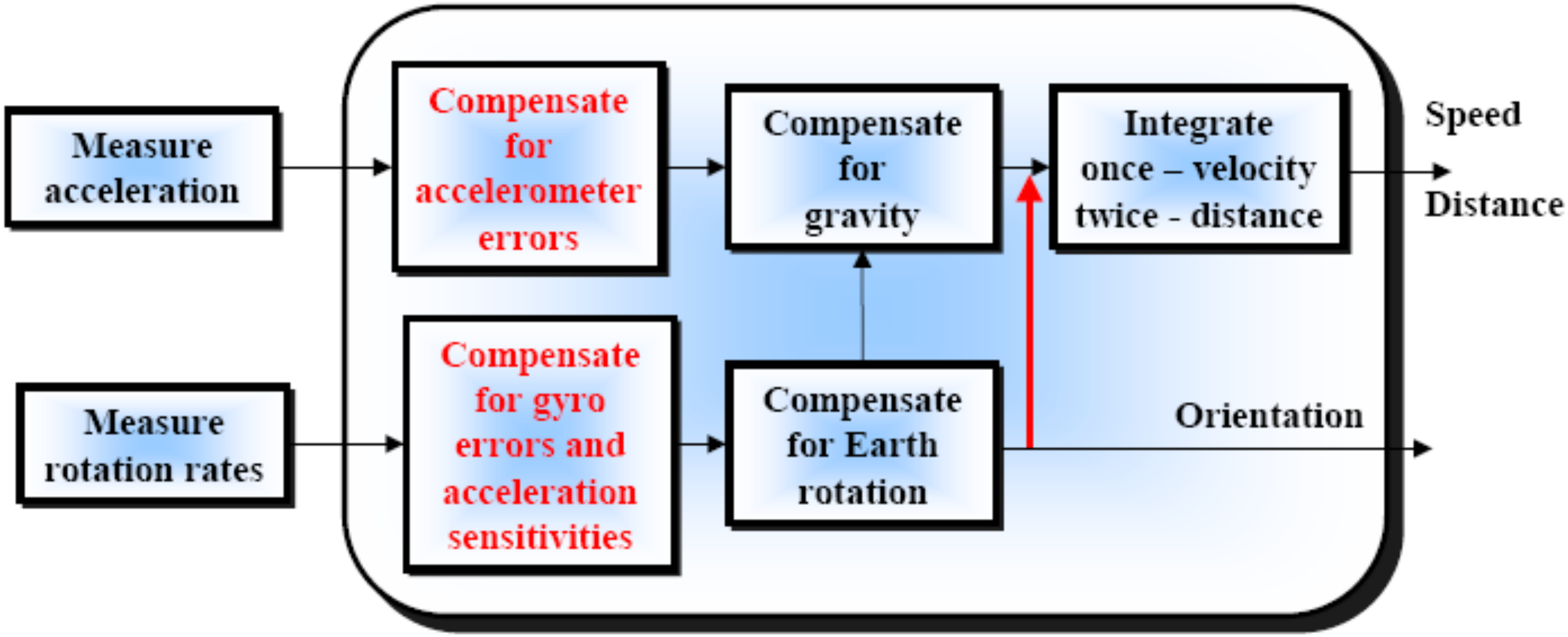
Navigation equations - recall (2/3)

- **Strapdown navigation** – general view



Inertial navigation – agenda

Navigation equations

- *i-frame (Week 5*)
- *e*-frame (Week 6) & SHOW CASE
- *l-frame (local-level)* – polycopié

Attitude (Week 7)

- Initialization – how ?
- Initialization – imperfections & impact

Strapdown inertial navigation (Week 8)

- Attitude solution in 3D
- Review of differences *e-frame, l-frame*
- Impact of error accumulation

Navigation equations in *e*-frame (1/5)

- Relation *e*-frame to *i*-frame

  $$\mathbf{r}^i = \mathbf{R}_e^i \mathbf{r}^e$$ … is a function of time

- 1st Time derivative $\partial \mathbf{x}^i / \partial t$ … $\text{and substitude } \dot{\mathrm{R}} = \mathrm{R}\Omega$

- 2nd Time derivative … (control: results in the general *a*-frame)

J. Skaloud, ESO

Navigation equations in *e*-frame (2/5)

- Control: 2nd time derivative in the general *a*-frame $\ddot{r}^i = R_a^i [\,(\Omega_{ia}^a \Omega_{ia}^a + \dot{\Omega}_{ia}^a)\, r^a + 2\Omega_{ia}^a v^a + \dot{v}^a\,]$
- 2nd Time derivative in e-frame

$$\ddot{\mathbf{r}}^i = \mathbf{R}_e^i \left(\ddot{\mathbf{r}}^e + 2\Omega_{ie}^e \dot{\mathbf{r}}^e + \Omega_{ie}^e \Omega_{ie}^e \mathbf{r}^e + \dot{\Omega}_{ie}^e \mathbf{r}^e \right)$$

3 Apparent forces: Coriolis ($2\Omega_{ie}^e \dot{\mathbf{r}}^e$), Centrifugal ($\Omega_{ie}^e \Omega_{ie}^e \mathbf{r}^e$), Tangential ($\dot{\Omega}_{ie}^e \mathbf{r}^e$)

$\omega_{ie}^e \approx \text{const.} \longrightarrow \dot{\omega}_{ie}^e = 0$

$\bar{\mathbf{g}}$ – gravitation (mass attraction)

$\mathbf{g}$ – gravity definition :

$$\mathbf{g} = \bar{\mathbf{g}} - \Omega_{ie}^e \Omega_{ie}^e \mathbf{r}^e$$

Navigation equations in *e*-frame (3/5)

- 2nd Time derivative in e-frame

$$\ddot{\mathbf{r}}^i = \mathbf{f}^i + \mathbf{g}^i = \mathbf{R}_e^i\left(\ddot{\mathbf{r}}^e + 2\Omega_{ie}^e\dot{\mathbf{r}}^e + \Omega_{ie}^e\Omega_{ie}^e\mathbf{r}^e + \dot{\Omega}_{ie}^e\mathbf{r}^e\right)$$

included in gravity negligible

- Re-express for acceleration *e*-frame

$$\ddot{\mathbf{r}}^e = \mathbf{R}_i^e\mathbf{f}^i + \mathbf{R}_i^e\mathbf{g}^i - 2\Omega_{ie}^e\dot{\mathbf{r}}^e$$

- Specific force **f** is observed in *b*-frame

$$\ddot{\mathbf{r}}^e = \mathbf{R}_b^e\mathbf{f}^b - 2\Omega_{ie}^e\mathbf{v}^e + \mathbf{g}^e$$

- Rate of changes in position and velocity in the **e**-frame

$$\dot{\mathbf{r}}^e = \mathbf{v}^e$$

$$\dot{\mathbf{v}}^e = \ddot{\mathbf{r}}^e$$

Navigation equations in *e*-frame (4/5)

- Rate of change of a rotation matrix in **e**-frame

$$\dot{\mathbf{R}}_b^e = \mathbf{R}_b^e \Omega_{eb}^b$$

- Angular rates observed (gyro) and know (Earth rotation)

$$\Omega_{ib}^b = \left[\omega_{ib}^b \times\right]$$ gyroscope signal in skew-symetric matrix

$$\Omega_{ie}^b = \left[\left(\mathbf{R}_e^b \omega_{ie}^e\right) \times\right]$$ skew-symetric matrix of earth rotation expressed in body-frame

$$\omega_{ie}^e = [0 \;, 0 \;, \omega_e]^T$$ mean Earth rotation rate in e-frame

- Re-expressing the desired angular speed as a difference between 3 know angular speeds

$$\dot{\mathbf{R}}_b^e = \mathbf{R}_b^e \underbrace{\left(\Omega_{ib}^b - \Omega_{ie}^b\right)}_{\Omega_{eb}^b}$$

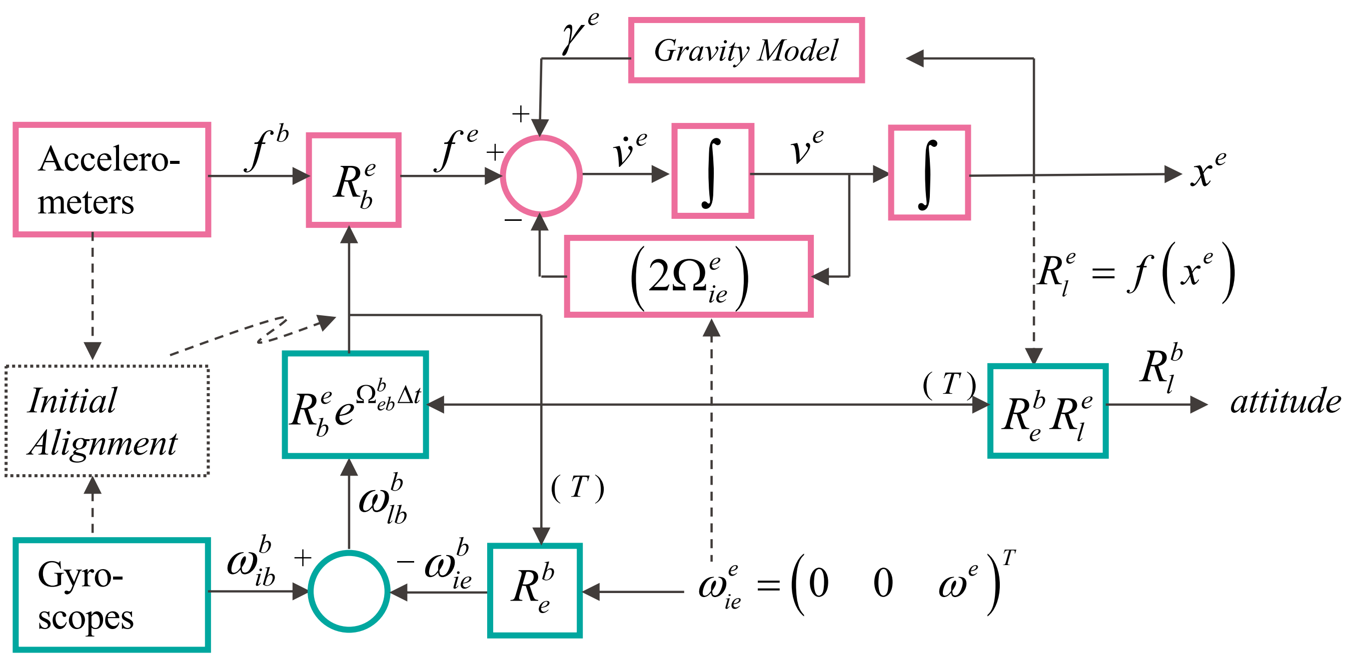
Navigation equations in *e*-frame (5/5)

- Regroup all differential relations for position, velocity and attitude

$$
\begin{aligned}
\dot{\mathbf{r}}^e &= \mathbf{v}^e \\
\dot{\mathbf{v}}^e &= \mathbf{R}_b^e \mathbf{f}^b - 2\Omega_{ie}^e \mathbf{v}^e + \mathbf{g}^e \\
\dot{\mathbf{R}}_b^e &= \mathbf{R}_b^e \left(\Omega_{ib}^b - \Omega_{ie}^b\right)
\end{aligned}
$$

- Where is the forcing input?
- What else is needed to solve them?

Earth-Fixed (e)-frame strapdown INS



Normal gravity model
for information

Somigliana (1929)

- Normal gravity at the sea level (h = 0) on an ellipsoid of revolution

$$\gamma_0(\varphi) = \frac{a \cdot \gamma_a \cdot \cos^2 \varphi + b \cdot \gamma_b \cdot \sin^2 \varphi}{\sqrt{a^2 \cdot \cos^2 \varphi + b^2 \cdot \sin^2 \varphi}}$$

with

- γ_a = Normal gravity at Equator
- γ_b = Normal gravity at Poles
- a = semi-major axis (Equator radius)
- b = semi-minor axis (Pole radius)
- φ = latitude

Normal gravity – for information

Somigliana formula approximated by series of expansion

- At the sea level

$$\gamma_0(\varphi) = \gamma_a \cdot (1 + \beta \cdot \sin^2\varphi + \beta_1 \cdot \sin^2 2\varphi + \ldots)$$

with the following coefficients for (GRS80*)

$$\gamma_a = 9.780327 \frac{\mathrm{m}}{\mathrm{s}^2} \quad \beta = 5.3024 \cdot 10^{-3} \quad \beta_1 = -5.8 \cdot 10^{-6}$$

- At height *h>0* above ellipsoid (GRS80*)

$$g(\varphi, h) = g_0(\varphi) \cdot (1 - (k_1 - k_2 \cdot \sin^2\varphi) \cdot h + k_3 \cdot h^2)$$

with the parameters derived from GSR80:

- $k_1 = 2 \cdot (1 + f + m)/a = 3.15704 \cdot 10^{-7} \mathrm{m}^{-1}$
- $k_2 = 4 \cdot f/a = 2.10269 \cdot 10^{-9} \mathrm{m}^{-1}$
- $k_3 = 3/(a^2) = 7.37452 \cdot 10^{-14} \mathrm{m}^{-2}$

*Geodetic Reference System 1980

Anomalous gravity field
for information

Better approximation of a the Earth geopotential field

- In navigation practically needed only for the most precise IMUs (>100kCHF) operating over larger areas (without satellite navigation)

Global Earth Gravity Models: EGM96, EGM2008, EGM2020, …

National models – e.g swisstopo (0.03 m – 1-3 mGal)