

Extended Kalman Filter

Lab 6 – INS as model (4)

General non-linear perturbation with random noise

$$\Delta \dot{\mathbf{x}} = \underbrace{\left[\frac{\partial f(\cdot)}{\partial \mathbf{x}} \right]_{\mathbf{x}=\mathbf{x}^*}}_{\mathbf{F}} \Delta \mathbf{x} + \mathbf{u}(t)$$

2D IMU perturbation with random noise

- **F + noise** - together per element
- **case** : all errors in 'deltas' are modeled as a white noise e.g. $\delta \dot{\alpha} = \delta \omega_{mb}^b = w_g$

(2) perturbation of 2D INS $\rightarrow \mathbf{F}$:

$$\begin{aligned} \delta \dot{\alpha} &= \delta \omega_{mb}^b \\ \delta \dot{\mathbf{v}}^m &= \mathbf{R}_b^m \boldsymbol{\Omega}_{mb}^b \mathbf{f}^b + \mathbf{R}_b^m \delta \mathbf{f}^b \\ \delta \dot{\mathbf{p}}^m &= \delta \mathbf{v}^m \end{aligned}$$

$$\begin{bmatrix} \delta \dot{\alpha} \\ \delta \dot{v}_n \\ \delta \dot{v}_e \\ \delta \dot{p}_n \\ \delta \dot{p}_e \end{bmatrix} = \underbrace{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ -f_2^m & \cdot & \cdot & \cdot & \cdot \\ f_1^m & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot \end{bmatrix}}_{\mathbf{F}_{11}} \begin{bmatrix} \delta \alpha \\ \delta v_n \\ \delta v_e \\ \delta p_n \\ \delta p_e \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & \cos \alpha & -\sin \alpha \\ \cdot & \sin \alpha & \cos \alpha \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}_{\mathbf{G}_{11}} \begin{bmatrix} w_g \\ w_{a_1} \\ w_{a_2} \end{bmatrix}$$

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Lab 6 – INS as model (7) details

Refer to Lab 6 help and/or black-board $c\alpha \rightarrow \cos \alpha$
 $s\alpha \rightarrow \sin \alpha$

$$\underbrace{\begin{bmatrix} \delta \dot{\alpha} \\ \delta \dot{v}_n \\ \delta \dot{v}_e \\ \delta \dot{p}_n \\ \delta \dot{p}_e \\ \delta \dot{b}_c \\ \delta \dot{b}_g \\ \delta \dot{b}_{a_1} \\ \delta \dot{b}_{a_2} \end{bmatrix}}_{\delta \dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix}}_{\mathbf{F}} + \underbrace{\begin{bmatrix} & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{bmatrix} w_g \\ w_{a_1} \\ w_{a_2} \\ q_b \\ q_{a_1} \\ q_{a_2} \end{bmatrix}}_{\mathbf{w}}$$

$\mathbf{F}_{11}^{5 \times 5}$ (top-left block of $\mathbf{F}$)
 $\mathbf{0}^{4 \times 5}$ (bottom-left block of $\mathbf{F}$)
 $\mathbf{G}_{11}^{5 \times 3}$ (top-left block of $\mathbf{G}$)
 $\mathbf{0}^{5 \times 3}$ (top-right block of $\mathbf{G}$)
 $\mathbf{0}^{4 \times 3}$ (bottom-left block of $\mathbf{G}$)

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Lab 6 – INS as model (9) - flowchart

