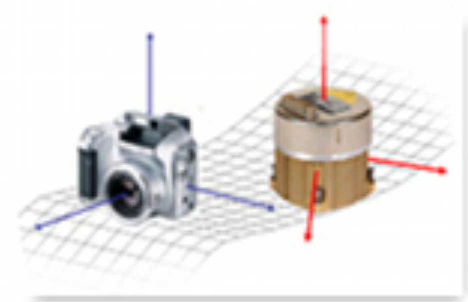
Sensor Orientation Initial Attitude



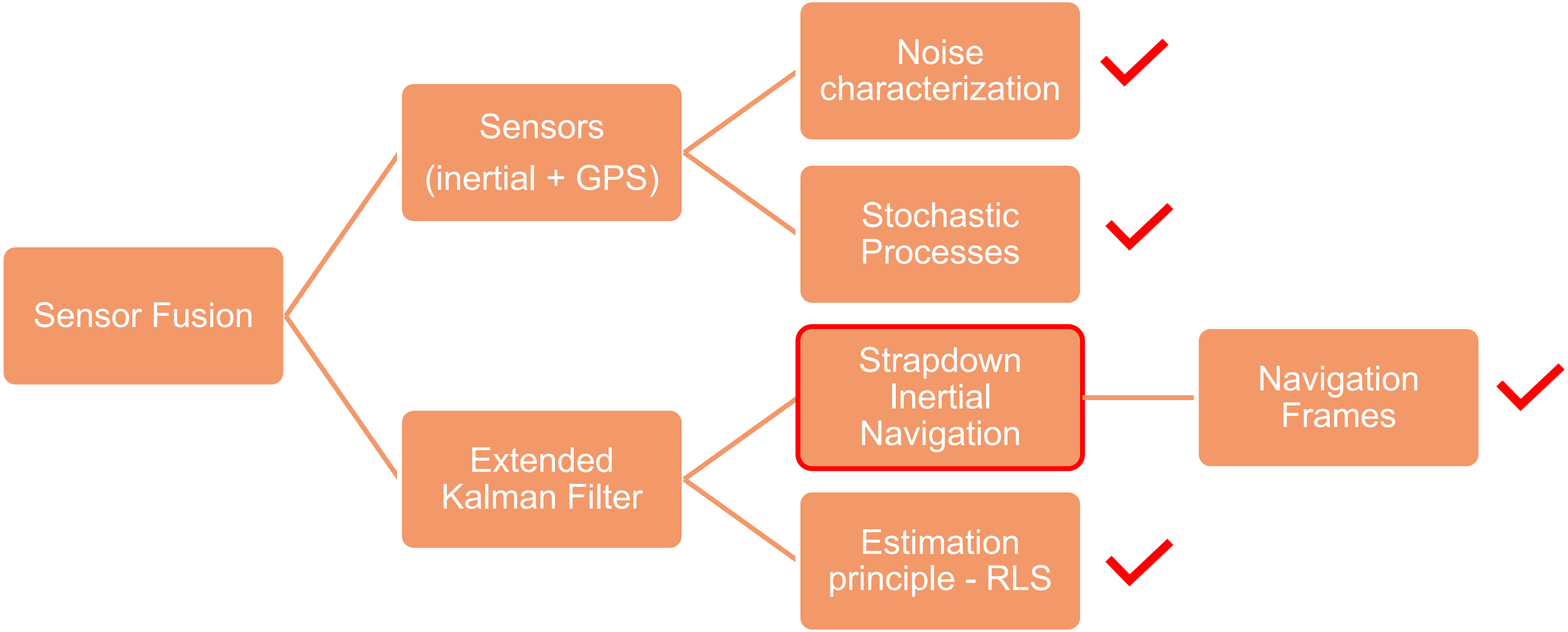
Jan SKALOUD

EPFL, spring semester

École polytechnique fédérale de Lausanne

Cockpit view of SO course’s topics

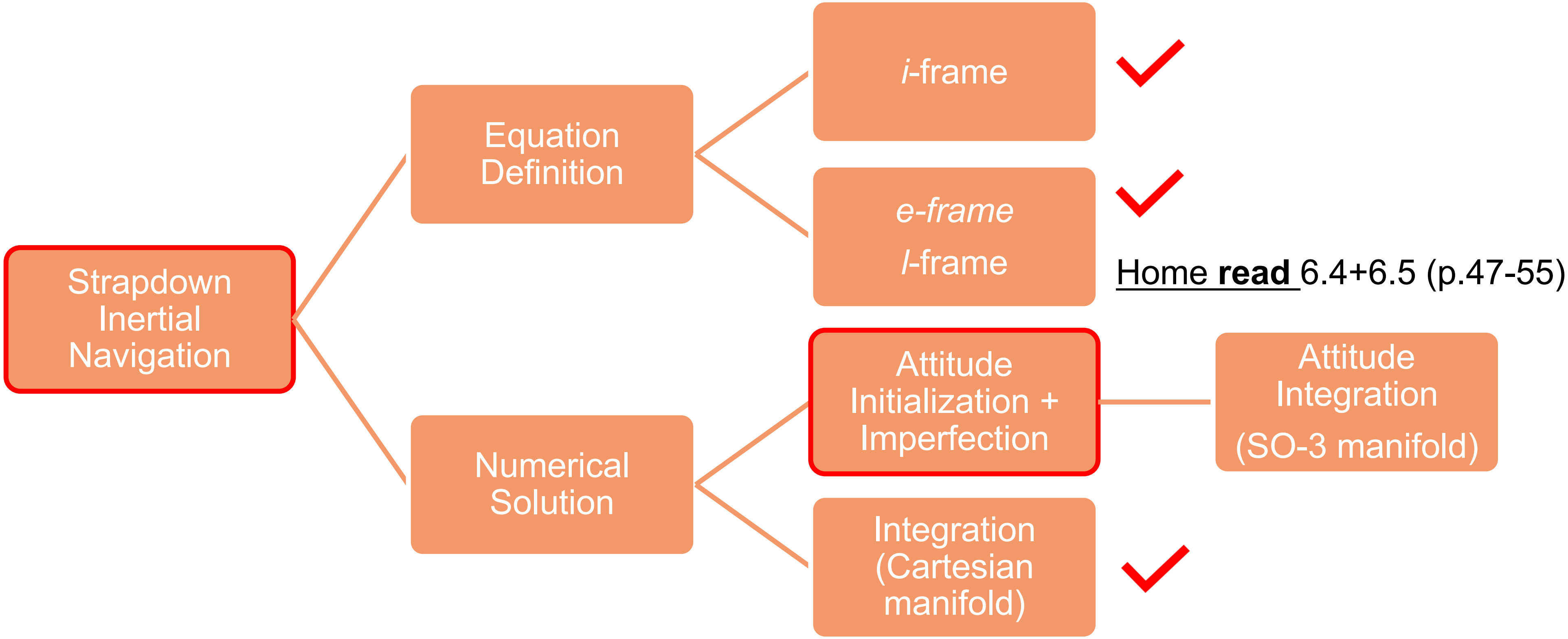
How to reach *integrated* sensor orientation?



J. Skaloud, ESO

Cockpit view of inertial navigation

Prerequisite for reaching *integrated* sensor orientation



Inertial navigation – agenda

Navigation equations

- *i-frame (Week 5*)
- *e*-frame (Week 6) & SHOW CASE
- *l-frame (local-level)* – polycopié (6.4) to read for Week 8 (8 pages)!

Attitude (Week 7)

- Initialization – how ?
- Initialization – imperfections & impact

Strapdown inertial navigation (Week 8)

- Attitude solution in 3D
- Review of differences *e-frame, l-frame*
- Strapdown inertial navigation – polycopié (6.5) to read before !
- Impact of error accumulation

Attitude initialization (in a navigation jargon = Initial Alignment Ch.7)

- Goal: to determine

$$\boxed{\mathbf{R}_b^e \text{ or } \mathbf{R}_b^\ell}$$

since $\mathbf{R}_b^e = \mathbf{R}_\ell^e(\varphi, \lambda) \cdot \mathbf{R}_b^\ell$

recall $\mathbf{R}_\ell^e(\varphi, \lambda) = [\mathbf{n}^e, \mathbf{e}^e, \mathbf{d}^e] = \ldots$ is known!

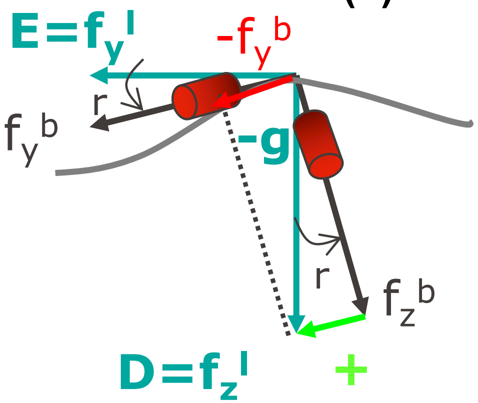
- Methods
  - Kinematic ... external aid! — e.g. GPS velocity, several GPS antennas, certain conditions, …
  - Static
    - Coarse (approx.)
      - A. Roll & Pitch Leveling — Input: Accels
      - B. Azimuth/yaw Gyro-compass. — Input: Gyros
    - Fine (precise) e.g. Kalman Filter

J. Skaloud, ESO

Attitude initialization – A. Leveling (1) approximation(2) for small angles

- Intuitive determination of roll (r) and pitch (p) angles from **static triad** of accelerometers, trough the **projection of local gravity** (**g**) to the sensed specific force (**f**)

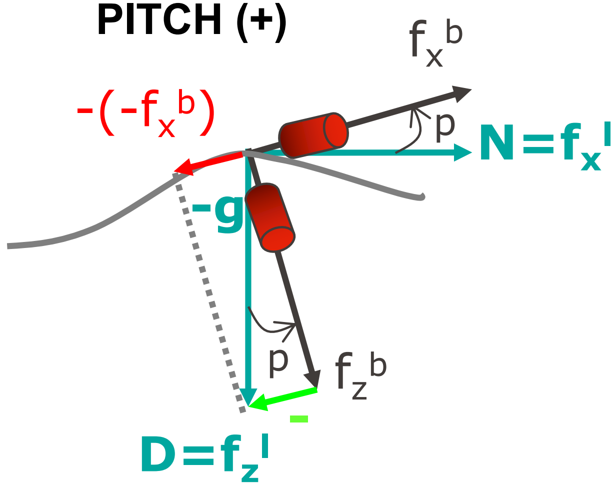
ROLL (+)



$$\tan(r) = \frac{-f_y}{-f_z}$$

$$\sin(r) = \frac{-f_y}{||f||}$$

PITCH (+)



$$\sin(p) = \frac{f_x}{||f||}$$

Notes: 1. for obtaining a correct sign it is better/safer to use function *atan2(nominator, denominator)*
2. approximative because the drawing does not consider a rotational sequence!

Attitude initialization – A. Leveling (2) starting relation (accelerometers)

- Recall from the previous lecture

$$\cancel{\dot{\mathbf{v}}^e} = \mathbf{R}_b^e \mathbf{f}^b - 2\cancel{\Omega_{ie}^e} \mathbf{v}^e + \mathbf{g}^e$$ … is a function of time

- Under static (non-moving) conditions $\mathbf{v}^e = 0\,;\ \dot{\mathbf{v}}^e = 0$, the above eq. reduces:

$$0 = \mathbf{R}_b^e \mathbf{f}^b + \mathbf{g}^e$$

- By multiplying the above eq. from left by $\mathbf{R}_e^\ell = (\mathbf{R}_\ell^e)^T$ (see slide 7)

$$-\mathbf{R}_e^\ell \mathbf{R}_b^e \mathbf{f}^b = \mathbf{R}_e^\ell \mathbf{g}^e$$

- The above term is equivalent to

$$-\mathbf{R}_b^\ell \mathbf{f}^b = \mathbf{g}^\ell$$

$$\boxed{-\mathbf{f}^b = \mathbf{R}_\ell^b \mathbf{g}^\ell}$$

readings from 3 accelerometers

?

known from a model, e.g. Eq. 6.45 $\mathbf{g}^\ell = [0,\ 0,\ \gamma(\varphi,\ h)]^T$

Attitude initialization – A. Leveling (3) final relation (accelerometers)

- Expressing $-\mathbf{f}^b = \mathbf{R}_\ell^b \mathbf{g}^\ell$ in the individual components, while considering yaw=0:

$$\bar{\ell} \quad - \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}^b = \mathbf{R}_1(r)\mathbf{R}_2(p)\,\mathbf{I}_3(yaw=0) \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}^\ell$$

- After substituting $\mathbf{R}_1(r)$, $\mathbf{R}_2(p)$ matrices and performing a multiplication of the right side:

$$- \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}^b = \begin{bmatrix} -g\sin(p) \\ g\sin(r)\cos(p) \\ g\cos(r)\cos(p) \end{bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

- We obtain 3 equations for two unknowns, several possibilities:

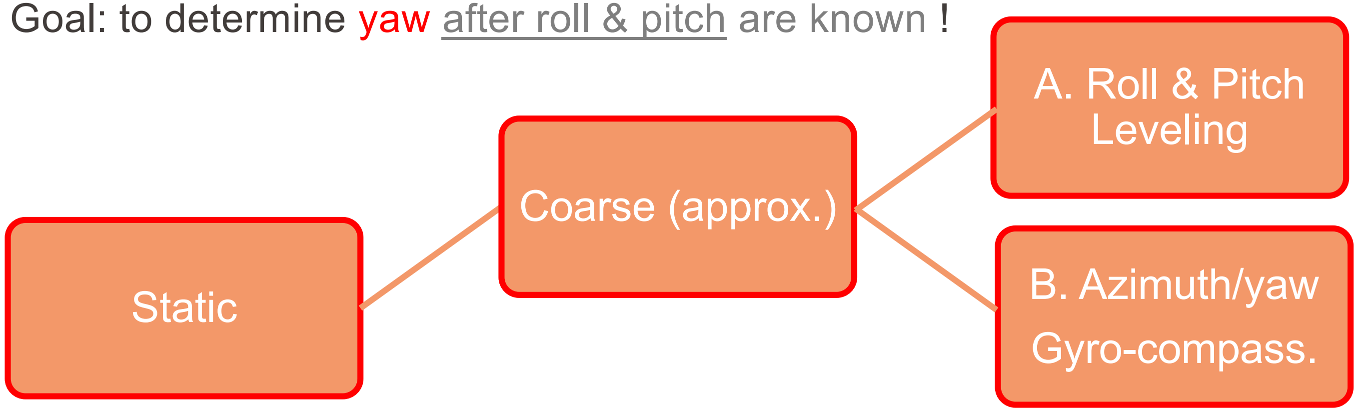
(1)/[(2)^2 +(3)^2]

$$\boxed{(2)/(3):\ \tan(r) = \frac{-f_y}{-f_z} \qquad (1):\ \sin(p) = \frac{f_x}{||f||} \qquad \tan(p) = \frac{f_x}{\sqrt{f_y^2+f_z^2}}}$$

Note: absolute knowledge of gravity is not needed – all is known from the accelerometers !

Attitude initialization – B. gyro compassing (1)

- Goal: to determine yaw after roll & pitch are known !



- How?
  1. *Project gyro observations* to “leveled” plane (e.g. plane where $r = p = 0$)
  2. Determine yaw *from the projected gyro data* (formulas via I. drawing, vs. II. equations)

- Transformation to “leveled” (but not yet oriented to North), symbol ($\bar{\ell}$) :

$$\mathbf{R}_b^{\bar{\ell}} = [\mathbf{R}_1(r)\mathbf{R}_2(p)]^T$$

Note: in such a $\bar{\ell}$ -plane z axis – points down, but x, y axes do not necessary align with North, East

Attitude initialization – B. gyro compassing (2)

1. Projection of gyro readings to a “leveled” $\bar{\ell}$ -plane (using previously determined roll & pitch):

$$\omega_{ib}^{\bar{\ell}} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}^{\bar{\ell}} = \mathbf{R}_b^{\bar{\ell}} \, \omega_{ib}^b \qquad \mathbf{R}_b^{\bar{\ell}} = [\mathbf{R}_1(r)\mathbf{R}_2(p)]^T$$

Note: in a static (non-rotating with respect to Earth) case

$$\omega_{ib}^b = \omega_{ie}^b + \underbrace{\omega_{eb}^b}_{=0}$$

Therefore, the norm

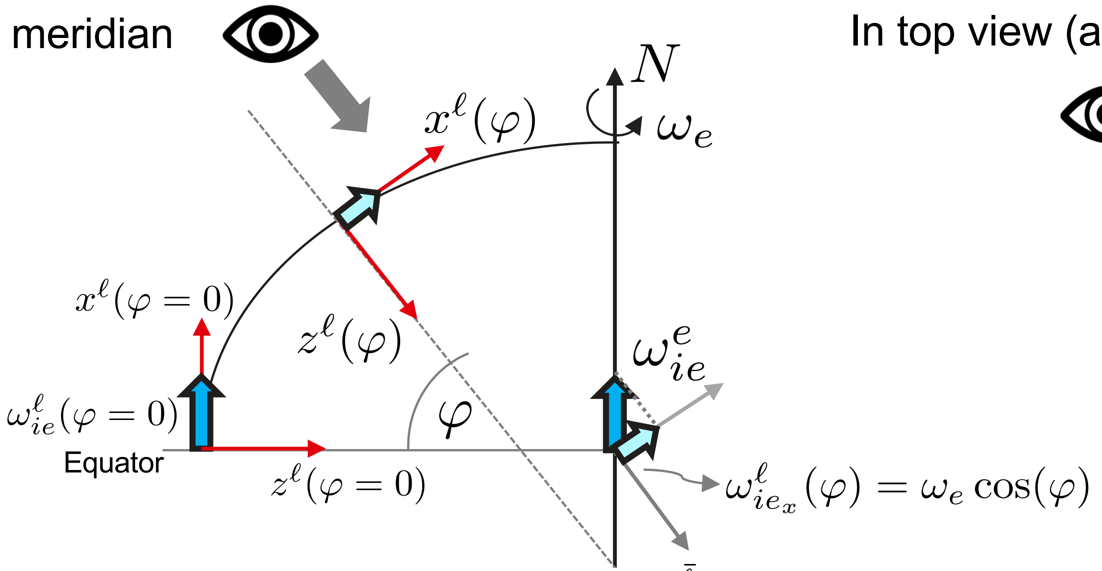
$$||\omega_{ib}^{\bar{\ell}}|| = ||\omega_{ib}^b|| = \omega_e$$

J. Skaloud, ESO

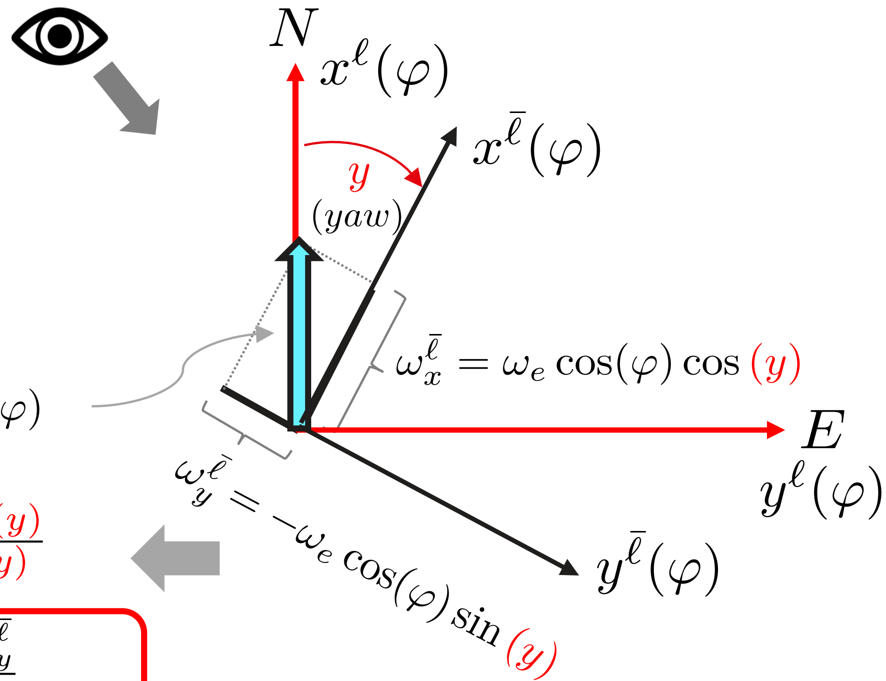
Attitude initialization – B. gyro compassing (3)

- 2. Determine yaw from the projected gyro data via I. deduction from drawing

Local meridian



In top view (above local-level)



$$\frac{\omega_y^{\bar\ell}}{\omega_x^{\bar\ell}} = \frac{-\omega_e\cos(\varphi)\sin(y)}{\omega_e\cos(\varphi)\cos(y)}$$

Note: knowledge about Earth rotation rate value is not needed!

$$\Longrightarrow \boxed{\tan(y) = \frac{-\omega_y^{\bar\ell}}{\omega_x^{\bar\ell}}}$$

Attitude initialization – B. gyro compassing (4)

- 2. Determine yaw from the projected gyro data via II. <u>equations</u>

Here: main idea – details in polycopié (Ch. 7)

$$\omega_{ib}^{b} = \mathbf{R}_{\ell}^{b}\,\omega_{ie}^{\ell} = \mathbf{R}_{\ell}^{b}\,\mathbf{R}_{e}^{\ell}\,\omega_{ie}^{e}$$

Multiplying the above equation from left by $\mathbf{R}_{b}^{\ell}$

$$\mathbf{R}_{b}^{\ell}\,\omega_{ib}^{b} = \mathbf{R}_{e}^{\ell}\omega_{ie}^{e}$$

Eq. 3.19

Decomposition: $$\mathbf{R}_{3}^{T}\underbrace{\mathbf{R}_{2}^{T}\,\mathbf{R}_{1}^{T}\,\omega_{ib}^{b}}_{\omega_{ib}^{\bar{\ell}}} = \begin{bmatrix} \cdots & \cdots & \cos\varphi \\ \cdots & \cdots & 0 \\ \cdots & \cdots & -\sin\varphi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega_e \end{bmatrix}$$

$$\begin{bmatrix} \cos(y) & -\sin(y) & 0 \\ \sin(y) & \cos(y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_x^{\bar{\ell}} \\ \omega_y^{\bar{\ell}} \\ \omega_z^{\bar{\ell}} \end{bmatrix} = \begin{bmatrix} \omega_e\cos\varphi \\ 0 \\ -\omega_e\sin\varphi \end{bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

from 2nd Eq.:

$$\boxed{\tan(y) = \frac{-\omega_y^{\bar{\ell}}}{\omega_x^{\bar{\ell}}}}$$

Accel. leveling – limiting factors

Q1 – What is the expected accuracy derived from leveling process?

- in NED:

$$\sin(r) = \frac{-f_y}{||f||} \longrightarrow \text{roll}$$

$$\sin(p) = \frac{f_x}{||f||} \longrightarrow \text{pitch}$$

- The roll, pitch accuracy is governed by accel's. accuracy (mainly bias *b* = Δf). For small angles:

$$\Delta r = -\Delta f_y / g$$

$$\Delta p = -\Delta f_x / g$$

- Example: 10 mg bias → leveling error of …?

Gyro compassing – limiting factors

- Considering NED and small error in yaw (δ_y):
  - $\sin(\delta_y) \approx \delta_y$ and $\cos(\delta_y) \approx 1$ in

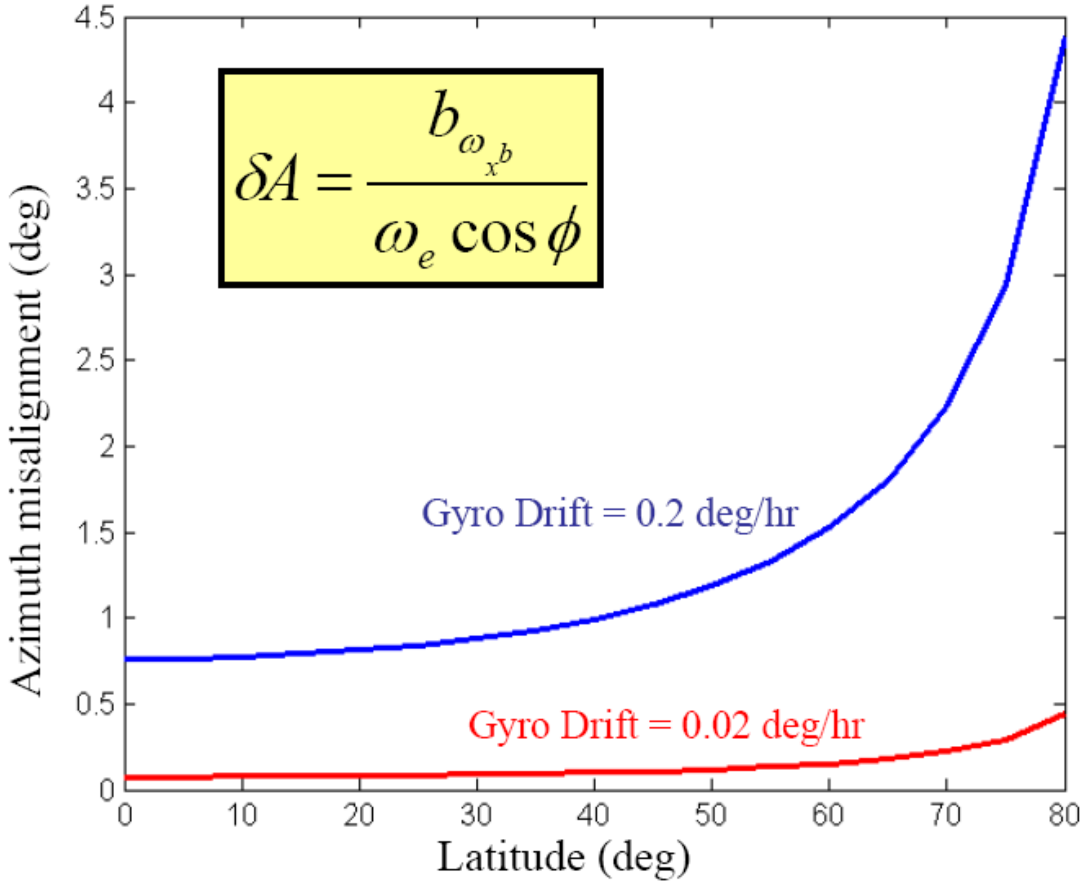
$$\omega_x^{\bar{\ell}} = \omega_e \cos(\varphi)\cos(y) \sim \omega_e \cos\varphi$$
$$\omega_y^{\bar{\ell}} = -\omega_e \cos(\varphi)\sin(y) \sim \omega_e \cos\varphi \cdot (-\delta y)$$

  - $\rightarrow$ Yaw accuracy will mainly depend on the gyro accuracy – bias (b), e.g.:

$$\delta y = \frac{b_{\omega_y^{\bar{\ell}}}}{\omega_e \cos\varphi}$$

  - Example: 45 deg latitude, gyro bias 0.2 deg/h
    - $\rightarrow$ What is the error in yaw ?

Azimuth (=yaw) accuracy vs. gyro bias (drift)



Azimuth (=yaw) accuracy vs. gyro noise

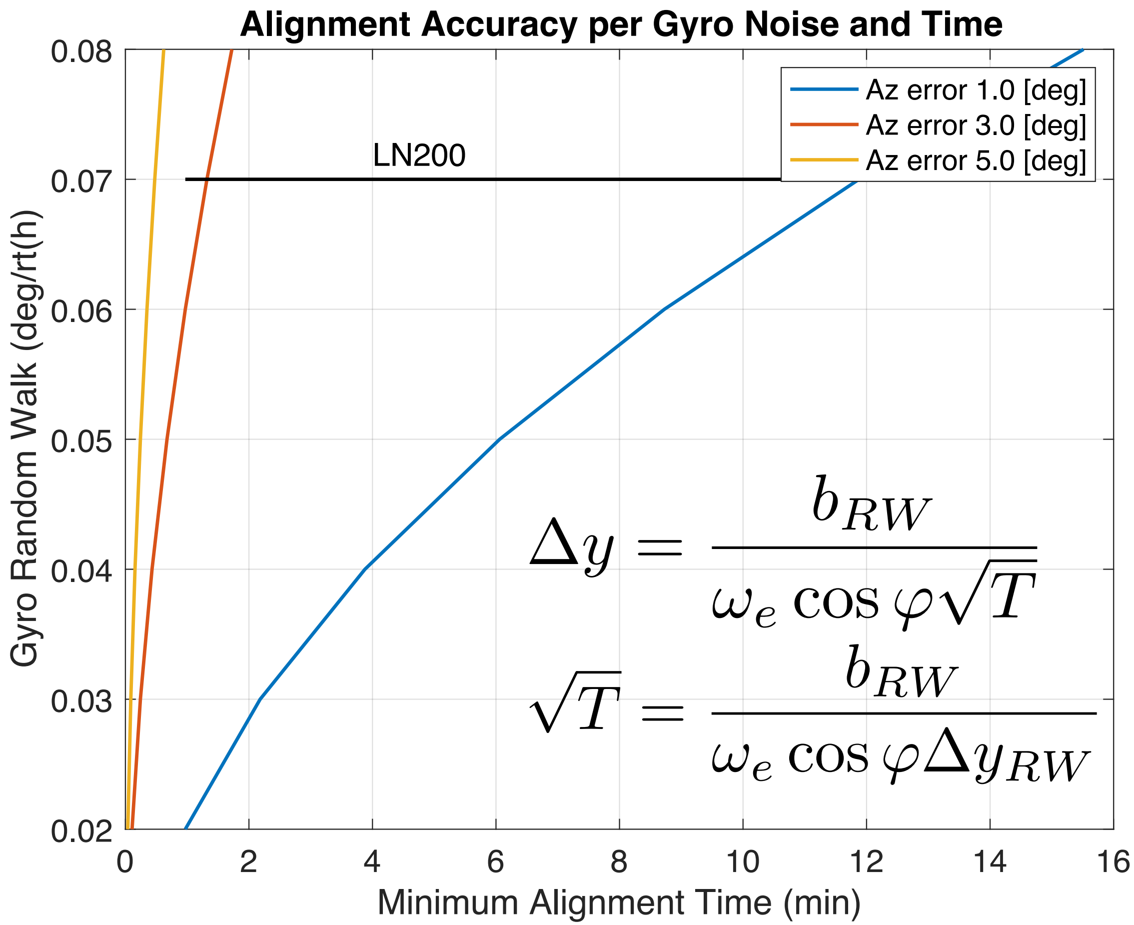
- Integrated gyro noise produces Angular Random Walk (RW) bias (a_{RW})

- a_{RW} depends on noise level and the integration time!

$$\Delta y_{RW} = \frac{b_{RW}}{\omega_e \cos\varphi \sqrt{T}}$$

- -> For a given a_{RW} we can achieve different azimuth alignment accuracy as a function of averaging time T!

Effect of gyro angular RW on alignment time



$$\Delta y = \frac{b_{RW}}{\omega_e \cos\varphi \sqrt{T}}$$

$$\sqrt{T} = \frac{b_{RW}}{\omega_e \cos\varphi \Delta y_{RW}}$$