LAB 1 – Stochastic Processes

1. Generate 3 random sequences (i.e., '**white noise**' WN) each comprising 200'000 samples. Set the standard deviation of these sequences to 2 [units]. For the sake of simplicity, assume that the generated sequences are sampled at 1 Hz.
   Hint: in Matlab, the function "randn" generates white noise with variance equaling unity; but you may use also Python ("numpy.random.randn") for your assignment.

2. Use these sequences to generate three realizations of **'random walk' (**RW).

3. Use the previously generated random sequences to generate three realizations each of 1st order **Gauss-Markov** (GM) process for two correlation times (e.g. τ_1=2000 and τ_2=500 samples).
   Hint: In MATLAB use *'exp()'* and for python use *'numpy.exp()'*.

By now, you should have 12 different sequences. Save all the realizations in a text file (1 file per stochastic sequence: 1 for WN, 1 for RW, 1 for each GM). The data should be stored as 3 columns with 8 decimals.

4. Compute the noise characteristics for each sequence by a: **autocorrelation** (AC) function; b: **power-spectral-density** (PSD); c: **Allan Variance**

5. Compute the parameters of stochastic processes and compare your findings with the values determined by the online tool for noise characterization via GMWM[1]. It is sufficient to upload one sequence for each noise.

[1] https://smac-group.github.io/gui4gmwm/

Report content:

Please keep the report brief and provide only what is asked.

1. Figures to be provided :
   - **a.** Figure 1
     - i. Subplot 1: 3 WN realizations with different color and legends
     - ii. Subplot 2: 3 RW realizations with different color and legends
     - iii. Subplot 3: 3 GM (τ=2000) realizations with different color and legends
     - iv. Subplot 4: 3 GM (τ=500) realizations with different color and legends
   - **b.** Figure 2 - Repeat the same figure structure as Fig.1 for the Auto Correlation
   - **c.** Figure 3 - Repeat the same figure structure as Fig.1 for the PSD
   - **d.** Figure 4 - Plot Allan Variance for WN, RW and two GMs in separate subplots. For each stochastic processes, only consider one realization
2. Answers to the following questions[2]:
   - **a.** How does the shape of the empirically determined autocorrelation function **correspond** to the theoretical ones in all cases? Explain the differences, if any.
   - **b.** How do the *empirically* determined values of **standard deviation** (i.e. calculated from all realizations) and **correlation length** (derived from the plot) **deviate** from the *true* values (i.e. those used in simulation)? **Make a quantitative comparison using a table.**
   - **c.** Which noise characteristic best identified the underlying stochastic process?
3. Submit your report in addition to the MATLAB/Python code via Moodle in a single zip file. Please make sure that your code is commented and runs!

Lab weight: 5%
Deadline without penalty: 10/03/2024

Autocorrelation:
xcorr.m (MATLAB)
numpy.correlate (Python)

Power Spectral Density (PSD):
pwelch.m (MATLAB)
scipy.signal.welch (Python)

Allan Variance:
allanvar (MATLAB)
https://pypi.org/project/AllanTools/ (Python)

[2]The answers and the comments should be relevant, short and consistent as would be expected during an oral exam. In other words, large number of pages does not prove that you well understood the subject.