Lab 5 - Kalman Filtering with simulated GPS data: simple (a=0) model (2 weeks)

Objective

Filter "2D GPS-like" positions on a circle by a simple (and suboptimal) Kalman Filter.

Tasks

1. Use your Matlab code from Lab 2 and re-generate the reference trajectory of the virtual vehicle along the circular path. Note: use radius $r = 25$ m, angular rate $\omega = \frac{\pi}{100}$ rad/s and time interval $\Delta t = 1$ s.

2. Simulate 'GPS-position' measurements along this path by adding random white noise: $\sigma_{gps,x} = 0.5$ m, $\sigma_{gps,y} = 0.5$ m, to the reference trajectory (separately for each coordinate).

3. Calculate the Kalman-filtered trajectory based on GPS-position observations, assuming the following initial conditions and motion model:

   (a) Assume an initial uncertainty in the vehicle's initial position $\sigma_{x_0} = 10$ m, and in initial velocity $\sigma_{v_0} = 0.1$ m/s.

   (b) Uniform linear motion of constant velocity (i.e. acceleration a $= 0$ m/s^2).

   (c) Consider the uncertainty of the motion model (noise covariance matrix based on $\sigma_{\dot{v}} = 0.05$ m/s^2/Hz).

4. Repeat Tasks 2 and 3 five times. Assuming the error in the estimate to be a white noise with a mean of 0. For each realization, calculate the standard deviation of the error:

   (a) Empirical standard deviations characterizing **real** GPS positioning quality $\sigma_{xy}^{GPS_{emp}} = \sqrt{\sigma^2_{x,mean} + \sigma^2_{y,mean}}$ using the difference: $E(p_{ref} - p_{gps})^2$

   (b) Empirical standard deviations characterizing **filtered** positioning quality $\sigma_{xy}^{KF_{emp}} = \sqrt{\sigma^2_{x,mean} + \sigma^2_{y,mean}}$, using the differences: $E(p_{ref} - p_{kf})^2$

   (c) Plot the evolution of the **KF-predicted** positioning quality $\sigma_{xy}^{KF_p} = \sqrt{\sigma_x^2 + \sigma_y^2}$, derived from the diagonal elements of the $\hat{\mathbf{P}}$ post-update σ_x and σ_y). Report the stabilized value in the table. Submission of this plot is not required for the report.
   Process noise =xx ; Frequency =yy

	1	2	3	4	5
$\sigma_{xy}^{GPS_{emp}}$					
$\sigma_{xy}^{KF_{emp}}$					
$\sigma_{xy}^{KF_P}$					

Questions

(i) What is the true overall improvement of the positioning accuracy by the filtering (i.e. comparing $\sigma_{xy}^{KF_{emp}}$ versus $\sigma_{xy}^{GPS_{emp}}$).

(ii) How many measurements does it take to stabilize the predicted accuracy in position?

(iii) Does the evolution of the predicted positioning accuracy depend on the actual measurements? Why?

(iv) How well does the empirically estimated position accuracy $\sigma_{xy}^{KF_{emp}}$ correspond to the predicted accuracy $\sigma_{xy}^{KF_p}$? Which parameters of the filter would you suggest modifying to improve the agreement?

(v) What do you observe when you increase/decrease the process noise 10 times ? What happens to the innovation sequences ?

(vi) What happens to the standard deviation of the estimated position and velocity while filtering at 100 Hz instead of 1Hz?

Deliverables

1. Plot the position errors (N and E separately) and velocity errors (N and E separately) alongside their 3-sigma bounds (obtained from $\hat{\mathbf{P}}$) for **1 realization** each for 1 Hz and 100 Hz at the three *different* process noises.

2. The **innovation sequence**, i.e. the differences between the predicted and the real observation ($z_t^{GPS} - \mathbf{H}\, x_t$) at each update of the North end East coordinates. Plot the histogram of innovation sequences for **1 realization** for 1 Hz (KF) at three different process noises.

3. Filled tables.

4. Answer to questions.

5. Your code.

Lab weight: 10%
Deadline: end of Week 12 (i.e. 18/05/2025) without penalty.